NOTES ON ILLIQUIDITY PANICS

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1. Multiple equilibria

• We now consider a simple version of a financial accelerator model in which panics are possible
• The model lasts 2 periods, \( t = 0, 1 \)
• As in the baseline Kiyotaki-Moore model there is a fixed supply of capital

\[ k^h + k = \bar{k} \]

• Initial net worth

\[ n = pk_0 - d_0 \]

• Invest in capital yields

\[ Ak \]

• Inferior technology

\[ y^h = f(k^h) \]

• Assumption: efficient allocation has all capital in banks

\[ f'(0) \leq A = p^* \]

• Equilibrium:

  – optimality for households (always unconstrained)

\[ p = f'(\bar{k} - k) \]

  – optimality for banks: three cases,

* if \( p > A \) then \( k = 0 \),
* if \( p = A \) then any

\[ 0 \leq k \leq \frac{pk_0 - d_0}{p - \theta A} \]

is optimal
* if \( p < A \) then

\[ k = \frac{pk_0 - d_0}{p - \theta A} \]

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So far we have focused on equilibria where banks are not bankrupt, so \( pk_0 > d_0 \), more below on bankrupt banks

Equilibrium, pair \( p, k \) that satisfies conditions above

Multiple equilibria?

Yes, but only possible if
\[
\frac{pk_0 - d_0}{p - \theta A}
\]
is increasing in \( p \) (for \( p < A \))

This requires
\[
k_0 (p - \theta A) > pk_0 - d_0
\]
or, equivalently,
\[
d_0 \frac{k_0}{k_0} > \theta A
\]

high enough initial leverage

We also have converse result: if \( d_0/k_0 > \theta A \) then we can find a function \( f \) such that the model admits multiple equilibria

Proof:
- Choose \( f \) piecewise linear, \( f'(k^h) = \bar{a} \) if \( k^h \leq \hat{k} \) and \( f'(k^h) = a \) if \( k^h > \hat{k} \)
- Choose \( a, \bar{a} \) so that \( \frac{d_0}{k_0} < a < \bar{a} < A \)
- Then choose \( \bar{k} \) and \( \hat{k} \) so that
\[
\frac{ak_0 - d_0}{a - \theta A} < \hat{k} - \bar{k} < \frac{\bar{a}k_0 - d_0}{\bar{a} - \theta A}
\]

(which can be done since \( \frac{pk_0 - d_0}{p - \theta A} \) is increasing)
- Then there are 2 equilibria, one at \( p = a \) and one at \( p = \bar{a} \)

So far we have ignored the question whether \( d_0 \) is sustainable, that is if, in equilibrium
\[
(pk_0 > d_0)
\]

If we allow for bankrupt banks it is even easier to find multiple equilibria

However it is harder to embed model with bankrupt banks in dynamic model as we need to allow for defaultable debt

Lending of last resort

Suppose there is a good equilibrium with \( p = A \)

Tax consumers and buy capital at price \( A - \epsilon \)

Even if the government only can get \( A_g << A \) the announcement of the policy eliminates the bad equilibrium