1. MPC

- Empirical studies on MPC
- Johnson, Parker, Souleles, 2006

Table 2—The Contemporaneous Response of Expenditures to the Tax Rebate

<table>
<thead>
<tr>
<th>Estimation method</th>
<th>Food</th>
<th>Strictly nondurable goods</th>
<th>Nondurable goods</th>
<th>Food</th>
<th>Strictly nondurable goods</th>
<th>Nondurable goods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rebate</td>
<td>0.109</td>
<td>0.239</td>
<td>0.373</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.056)</td>
<td>(0.115)</td>
<td></td>
<td>(0.115)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Rebate &gt; 0)</td>
<td></td>
<td></td>
<td></td>
<td>51.5</td>
<td>96.2</td>
<td>178.8</td>
</tr>
<tr>
<td>(27.6)</td>
<td>(53.6)</td>
<td></td>
<td>(65.0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>0.570</td>
<td>0.449</td>
<td>1.165</td>
<td>0.352</td>
<td>0.391</td>
<td>1.106</td>
</tr>
<tr>
<td>(0.320)</td>
<td>(0.550)</td>
<td></td>
<td>(0.673)</td>
<td>(0.318)</td>
<td>(0.548)</td>
<td>(0.670)</td>
</tr>
<tr>
<td>Change in adults</td>
<td>130.3</td>
<td>281.5</td>
<td>415.8</td>
<td>131.1</td>
<td>287.7</td>
<td>418.6</td>
</tr>
<tr>
<td>(57.8)</td>
<td>(90.0)</td>
<td></td>
<td>(102.8)</td>
<td>(57.8)</td>
<td>(90.2)</td>
<td>(102.9)</td>
</tr>
<tr>
<td>Change in children</td>
<td>73.3</td>
<td>98.3</td>
<td>178.4</td>
<td>74.0</td>
<td>98.7</td>
<td>179.2</td>
</tr>
<tr>
<td>(45.3)</td>
<td>(82.4)</td>
<td></td>
<td>(98.3)</td>
<td>(45.3)</td>
<td>(82.5)</td>
<td>(98.3)</td>
</tr>
<tr>
<td>RMSE</td>
<td>934</td>
<td>1680</td>
<td>2047</td>
<td>934</td>
<td>1680</td>
<td>2047</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
</tbody>
</table>

- The rebate timing is random, so positive coefficient can be interpreted as a rejection of PIH
- The coefficient cannot interpreted as MPC, JPS warn us (because receipt anticipated)
- Still following literature often interprets them as MPC
- Bottom line: they are large
- Can standard Bewley-Aiyagari model produce large MPCs?
- Answer: it all depends on how you calibrate the asset supply and the credit availability
- What matters is what number you choose for

\[ \int (a_i + \phi) \, di \]

- Smaller supply of assets: higher average MPC
- See Matlab simulations
- How can we choose asset supply?
- Traditional approach (Aiyagari): look at total capital stock \( K \) in economy

Date: Spring 2019.
• Asset supply from 
\[ 1 + r = f'(K) + 1 - \delta \]

• General equilibrium 
\[ \int a_idi = K \]

• This yields a calibration with very low MPC
• How can we reconcile with data?
• What is household wealth: liquid assets (bank deposits) + housing wealth
• If we target \( \int a_idi \) to total wealth same issue as above
• But we can think that housing is less liquid: rich hand-to-mouth consumers (Kaplan and Violante 2014)
• It is essentially analogous to setting \( \int a_idi \) = liquid assets (excluding housing)
• All solved?
• Not really, recent evidence that challenges more deeply optimizing model
• Ganong and Noel, 2018
• Point toward behavioral models (Laibson, 1997)

2. Durable goods

• Consumer maximizes 
\[ E \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \]

• Budget constraint
\[ p_t h_t + a_t + c_t = p_t (1 - \delta) h_{t-1} + (1 + r) a_{t-1} + y_t \]

• \( y_t \) still follows a Markov process
• Euler equation 
\[ u_c(c_t, h_t) = \beta (1 + r) E_t u_c(c_{t+1}, h_{t+1}) \]

• Optimality for \( h_t \)
\[ u_h(c_t, h_t) = p_t u_c(c_t, h_t) - \beta E_t p_{t+1} (1 - \delta) u_c(c_{t+1}, h_{t+1}) \]

• Suppose price of durable is non-stochastic
• Then, using Euler equation 
\[ u_h(c_t, h_t) = p_t u_c(c_t, h_t) - p_{t+1} (1 - \delta) \frac{u_c(c_t, h_t)}{1 + r} \]

or 
\[ \frac{u_h(c_t, h_t)}{u_c(c_t, h_t)} = p_t - \frac{p_{t+1} (1 - \delta)}{1 + r} \]
Figure 1. Spending at expiration of unemployment benefits (from Ganong and Noel, 2018)

- Expression on the RHS is “user cost of capital”. If you have

\[ \rho_t = p_t - \frac{p_{t+1} (1 - \delta)}{1 + r} \]

you can borrow \( p_t - \rho_t \), buy the asset today, resell it tomorrow and use the sale receipt to repay your debt as \( p_{t+1} (1 - \delta) = (1 + r) (p_t - u_t) \), so \( u_t \) is the cost of using the
asset for one period. In a frictionless world in which renting and owning provide the same services $\rho_t$ should be equal to the rental rate

- Assume borrowing constraint is just
  \[ a_t \geq 0 \]

- Define total wealth
  \[ w_t = p_t (1 - \delta) h_{t-1} + (1 + r) a_t \]

- Then budget constraint becomes
  \[ \frac{1}{1 + r} w_{t+1} + c_t + \rho_t h_t = w_t + y_t \]

  and we have essentially an income fluctuation problem with 2 goods

- Now, assume $p_t = p$ constant and utility function
  \[ u(c, h) = \frac{1}{1 - \gamma} \left( c^\alpha h^{1-\alpha} \right)^{1-\gamma} \]

- Define total spending
  \[ x_t = c_t + \rho h_t \]

- Solution of static allocation between $c$ and $h$
  \[ c_t = \alpha x_t \]
  \[ h_t = \frac{1 - \alpha}{\rho} x_t \]

  so indirect utility function is equal (modulo a multiplicative constant) to
  \[ \frac{1}{1 - \gamma} x_t^{1-\gamma} \]

- So we can just solve
  \[ E \sum \beta^t \frac{1}{1 - \gamma} x_t^{1-\gamma} \]
  \[ \frac{1}{1 + r} w_{t+1} + x_t = w_t + y_t \]
  \[ w_{t+1} \geq 0 \]

- Effect of income shock on purchases of durables

- Consider i.i.d. shocks
  \[ x_t = X(w_t + y_t) \]

- Non-durable spending is
  \[ c_t = \alpha x_t \]
• Durable purchases are
\[ h_t - (1 - \delta) h_{t-1} = (1 - \alpha) \frac{x_t}{p} - (1 - \delta) h_{t-1} \]

• % response of non-durable purchases to a small temporary income shock \( dy_t \) is
\[ \frac{dc_t}{c_t} = \alpha \frac{X'(w_t + y_t)}{x_t} dy_t = \alpha \frac{X'(w_t + y_t) y_t}{x_t} \frac{dy_t}{y_t} = X'(w_t + y_t) \frac{y_t}{x_t} \frac{dy_t}{y_t} \]

• % response on durable purchases is
\[ \frac{dh_t}{h_t - (1 - \delta) h_{t-1}} = \frac{h_t}{h_t - (1 - \delta) h_{t-1}} X'(w_t + y_t) \frac{y_t}{x_t} \frac{dy_t}{y_t} = \frac{h_t}{h_t - (1 - \delta) h_{t-1}} \frac{dc_t}{c_t} \]
and if \( h_t \approx h_{t-1} \)
\[ \frac{dh_t}{h_t - (1 - \delta) h_{t-1}} \approx \frac{1}{\delta} \frac{dc_t}{c_t} \]

• Main takeaway: durables are more volatile, the more so the more durables they are (lower \( \delta \))

• The intuition is straightforward: consumers want to adjust durable services and non-durable consumption proportionally; durable services are proportional to the stock of durables, so consumers want to adjust the stock of durables proportionally to non-durable consumption; in steady state we only buy \( \delta \) of the stock each period; so if we want to adjust the stock of, say, 1%, and in steady state we are buying \( \delta = 5\% \) of the stock, that’s a 20% increase in durable spending for a 1% increase in non-durable spending

• A second observation: durable spending is more responsive to interest rate changes

• We will talk about how \( x_t \) responds to changes in \( r_t \) in the next class

• For durables however, on top of the change in \( x_t \) we have the change in the user cost \( \rho_t \)

• Since
\[ \rho_t = p \left(1 - \frac{1 - \delta}{1 + r_t}\right) \approx p (r_t + \delta) \]
we have
\[ \frac{d\rho_t}{\rho_t} = \frac{dr_t}{r_t + \delta} \]

• Since
\[ h_t = (1 - \alpha) \frac{x_t}{\rho_t} \]
we have
\[ \frac{dh_t}{h_t - (1 - \delta) h_{t-1}} = \frac{h_t}{h_t - (1 - \delta) h_{t-1}} \frac{dh_t}{h_t} = \frac{h_t}{h_t - (1 - \delta) h_{t-1}} \left( \frac{dx_t}{x_t} - \frac{d\rho_t}{\rho_t} \right) \]
or
\[ \frac{dh_t}{h_t - (1 - \delta) h_{t-1}} = \frac{h_t}{h_t - (1 - \delta) h_{t-1}} \left( \frac{dx_t}{x_t} - \frac{1}{r_t + \delta} \right) dr_t \]
• The second term in brackets amplifies the (negative) effect of $r_t$ on $x_t$
• Second takeaway: durable spending is more sensitive to changes in the interest rate