DO TARIFFS REDUCE THE TRADE DEFICIT?

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ABSTRACT. Two simple models are used to show that the effect of tariffs on the trade deficit depends on the nature of the deficit and on its persistence. In the first model, a country runs a trade deficit because of a traditional intertemporal motive to borrow in the present to repay in the future, so the deficit is transitory. In this model, a permanent increase in tariffs increases the real interest rate for the country in deficit, leading to less borrowing and to a reduction in the trade deficit. In the second model, the deficit arises because the country acts as a world intermediary, taking gross asset and liability positions towards the rest of the world that lead to a net debtor position. In this second model, we can have a permanent trade deficit. In that case, a tariff increase can actually have a zero effect on the trade deficit.

1. INTRODUCTION

The recent policy debate on the effect of tariffs has revived interest on the connection between tariffs and the trade deficit.[1] The arithmetic of the current account implies that a reduction in the trade deficit can only occur if tariffs affects the net savings of the country affected. In this paper, I argue that if and how the incentives to save are changed by the presence of tariffs depends on what is the underlying nature of the current account deficit.

In particular, I distinguish two different reasons why a country may be running a trade deficit. The first is a basic intertemporal reason: a country runs a deficit in periods in which it wants to spend more than its current income; when the country repays in the future, it will run a trade surplus. A second reason why a country may be running a current account deficit is that the country is acting as a world intermediary, holding both gross asset and gross liability positions against the rest of the world, and earning a higher return on the asset positions than on the liability positions. This view has been formulated in a variety of ways in recent work on the US current account deficit, by, among others, Gourinchas and Rey [2007], Caballero et al. [2008], Maggiori [2013].

I present two simple models that capture these two views of the trade deficit. The first model is a completely standard intertemporal model. The second model, is a model of a country that has an advantage in issuing bonds that are used as world liquidity. The two models are identical in terms of their goods trading structure, but differ in their asset trading structure. I then show that the implications of the two models for the effects of tariffs are strikingly different. In particular, in the first model there is a clear intertemporal channel at work: tariffs increase the real interest rate for the borrowing economy and thus induce consumers in that economy to save more, increasing domestic net savings and reducing the

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[1] See, for example, "Tariffs and the Trade Balance," on Paul Krugman's *New York Times* blog, December 27, 2016.

current account deficit. In the second model, instead, the real interest rate effect is absent if the current account deficit is stationary before the policy intervention. Other effects are present, through valuation channels that affect the net value of the countries' initial gross positions. In some simple cases these valuation effects are also zero and tariffs have no effect at all on the trade balance.

Of course, the models presented are stylized and do not capture a number of other important considerations in international asset trading. In particular, they lack an explicit treatment of risk and risk sharing and they are extremely simplified on the currency and monetary dimension. They are however useful to isolate the real interest channel and to show under what conditions it may or may not work to change current account balances.

A number of recent papers have explored the effect of trade costs on the trading of assets across borders. Fitzgerald [2012] Eaton et al. [2016] and Reyes-Heroles [2016].

2. The intertemporal argument

The starting point is a simple two countries/two goods world economy. In this section, I introduce a simple discrete time, two period version of this economy, with $t = 1, 2$. This will give the main insight of the standard intertermporal approach. The ideas in this section are not new and are found, for example, in Obstfeld and Rogoff [2000]. It is useful to present them here to set the stage for the results in the next section.

There are two countries, home and foreign, and two goods. Each country receives a deterministic sequence of endowments: the home country receives $e_{H,t}$ of the first good, the home good; the foreign country receives $e_{F,t}$ of the other good, the foreign good. The preferences of the home consumer are Cobb-Douglas in the two goods, so home consumption is

$$c_t = \left(c_{H,t}\right)^{\alpha} \left(c_{F,t}\right)^{1-\alpha},$$

where $c_{H,t}$ and $c_{F,t}$ are consumption of the two goods in period t. The preferences of the foreign consumer are symmetric, so foreign consumption is

$$c_t^* = \left(c_{F,t}^*\right)^{\alpha} \left(c_{H,t}^*\right)^{1-\alpha},$$

where asterisks denote the foreign country. Each country's consumption basket is biased towards the domestically produced good, that is, $\alpha > 1/2$.

Intertemporal preferences are described by the utility functions $\sum \beta^t u\left(c_t\right)$ and $\sum \beta^t u\left(c_t^*\right)$, where the function u is CRRA with a coefficient of relative risk aversion γ. Agents enter period $t = 1$ with zero initial financial positions. Agents borrow and lend on the world capital market at the interest rate i_1 and, within each period, they trade home and foreign goods at the prices $p_{H,t}$ and $p_{F,t}$. All prices are denominated in a common unit of account, say in dollars.

Let D_t denote the trade deficit of the home country, denominated in home goods. The intertemporal budget constraint for the home country requires

$$p_{H,1} D_1 + \left(1 + i_1\right)^{-1} p_{H,2} D_2 = 0. \tag{2.1}$$

Suppose the economy's parameters are such that the home country is a net borrower in period 1, so it runs a trade deficit $D_1 > 0$. By the intertemporal budget constraint, the country must run a trade surplus $D_2 < 0$ in period 2. We want to see what happens to the trade deficit D_1 if the domestic government unilaterally introduces a tariff on the foreign good. In particular, let us focus on the effects of a permanent proportional tariff τ, because that is the case in which the intertemporal effects are harder to understand. The receipts of the tariff are rebated lump sum to the domestic consumer.

Cobb-Douglas preferences imply that in each period the home consumer spends a fraction

$$\tilde{\alpha} = \frac{\alpha}{\alpha + \frac{1-\alpha}{1+\tau}} \tag{2.2}$$

of total spending on the home good. The tariff τ distorts spending in favor of the domestic good. For the foreign consumer there is no distortion, so the fraction spent on the foreign good is just α. Given these observations, it is easy to solve for equilibrium relative price of foreign goods in each period:

$$\frac{p_{F,t}}{p_{H,t}} = \frac{(1-\tilde{\alpha})\, e_{H,t} - (\tilde{\alpha} + \alpha - 1)\, D_t}{(1-\alpha)\, e_{F,t}}. \tag{2.3}$$

Let's turn now to intertemporal decisions. The Euler equation of the domestic consumer is

$$u'(c_1) = (1+i_1)\frac{p_1}{p_2}\beta u'(c_2),$$

where p_t is the consumer price index defined as

$$p_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\, p_{H,t}^{\alpha}\left((1+\tau)\, p_{F,t}\right)^{1-\alpha}.$$

The Euler equation of the foreign consumer takes a similar form. Combining the two Euler equations, after some manipulation, gives the relation

$$\frac{u'(c_1)}{u'(c_2)} = \left(\frac{\frac{p_{H,1}}{p_{F,1}}}{\frac{p_{H,2}}{p_{F,2}}}\right)^{2\alpha-1}\frac{u'(c_1^*)}{u'(c_2^*)}. \tag{2.4}$$

This condition shows that the different consumption baskets of the two consumers create a gap between the real interest rates faced by the home and by the foreign consumer. The gap is captured by the first term on the right-hand side. In particular, if the home good is relatively more expensive in period 1, this discourages spending by home consumers in period 1 more than it discourages spending of foreign consumers, because the weight on the home good in the home price index is $\alpha > 1 - \alpha$.

Notice that the tariff does not appear directly in equation (2.4), because the tariff is permanent. However, the tariff does affect intertemporal decisions because it affects differentially the relative prices in the two periods. Differentiating (2.3) with respect to $\tilde{\alpha}$ and using $D_1 > 0 > D_2$, shows that the effects of a positive increase in the tariff, which increases $\tilde{\alpha}$, for given levels of D_1 and D_2, are:[2]

$$\frac{d\left(\frac{p_{H,1}}{p_{F,1}}\right)}{\frac{p_{H,1}}{p_{F,1}}} > \frac{d\tilde{\alpha}}{1-\tilde{\alpha}} > \frac{d\left(\frac{p_{H,2}}{p_{F,2}}\right)}{\frac{p_{H,2}}{p_{F,2}}} > 0. \tag{2.5}$$

In period 1 the home consumer is spending more than his/her current income, so introducing a distortion in favor of the home good increases its price relatively more than in period 2. This increases the ratio $\frac{p_{H,1}}{p_{F,1}} / \frac{p_{H,2}}{p_{F,2}}$, which, as argued above, increases the real interest rate for the home consumer more than for the foreign consumer. Through the Euler equations, this tilts the home consumption path towards period 1. To complete the analysis and derive the response of the trade deficit, requires taking into account the intertemporal budget constraint (2.1). However, once we add that step, the algebra is complicated by wealth effects, making the mechanism less transparent, so I turn to a numerical example.

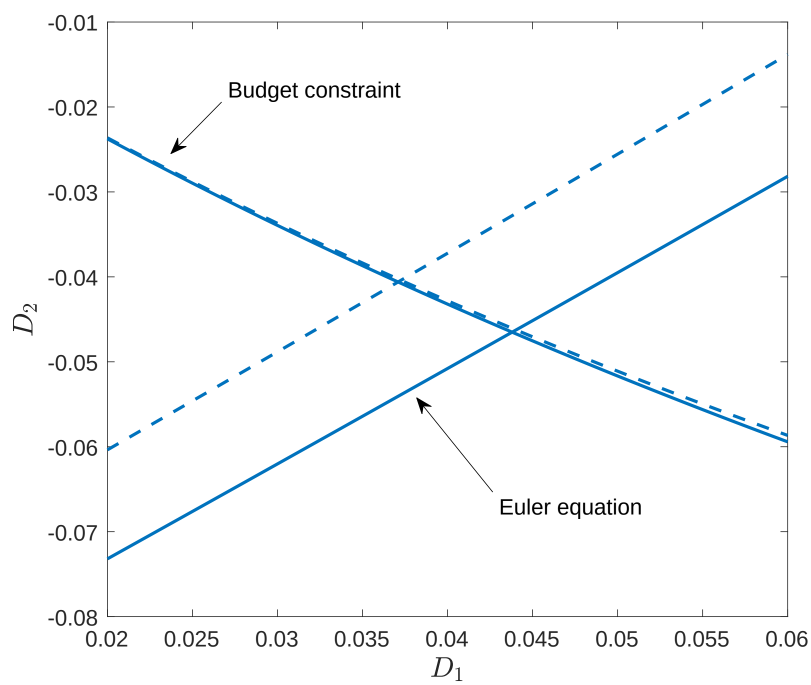

FIGURE 2.1. Intertemporal approach: comparative statics
Note: Solid lines: no tariff; dashed lines: $\tau = 0.25$. See text for model parameters.

Consider a simple numerical example, with the following parameters[3]

$$\beta = 1, \quad \alpha = 0.7, \quad \gamma = 2, \quad e_{H,1} = e_{F,2} = 1, e_{H,2} = e_{F,1} = 2.$$

With this parameters the home country displays an increasing income path, while the foreign country has a decreasing one. This induces the home country to borrow in equilibrium.

[2]The detailed steps are in the appendix.

[3]We cannot assume $\gamma = 1$, because in that case the Cole and Obstfeld result applies and we have $D_1 = D_2 = 0$ for any configuration of the other parameters.

In the appendix, we show that an equilibrium can be found looking at the intersection of two relations between the trade deficits D_1 and D_2. One is an increasing relation obtained from the Euler equation condition (2.4), and the other is a decreasing relation obtained from the intertemporal budget constraint (2.1). Figure 1 shows these relations before and after the introduction of a tariff $\tau = 0.25$. The tariff causes an upward shift of the Euler equation relation. The logic of this shift is in the mechanism described above: the presence of the tariff increases the relative price of the home good today more than tomorrow, discouraging intertemporal trade. At the same time, there is a wealth effect that shifts out the budget constraint curve, as the tariff improves the terms of trade for the home country. This second effect is small in the example. The equilibrium outcome is that the home country borrows less in period 1.

Summing up, we have a simple example in which the tariff does change intertemporal incentives and it does so in such a way as to increase net savings by the country imposing the tariff, thus reducing the current account deficit. The logic of the example is that the tariff affects global spending on different goods with different force depending on the sign of the trade deficit. This explains why the tariff is more effective at increasing the home good price in the first period, in which the country is borrowing, which is the crucial step for the real interest rate channel to work. As we shall see, this mechanism is muted in the liquidity supply model of the next section.

3. A MODEL OF WORLD LIQUIDITY SUPPLY

Let us now modify the model to introduce a motive for trading gross asset positions. In particular, I now add to the model a liquid asset and assume that the home country has a monopoly in the supply of the liquid asset. This allows me to formulate in a simple way the view of the US as a world's financial intermediary laid out in the introduction. It is convenient to formulate this version of the model in an infinite horizon, continuous time setting.

There are two types of bonds, liquid and illiquid bonds. I want to capture the idea that liquid bonds can be used to clear a variety of transactions. I model this transactional benefit by simply assuming that the real value of liquid bonds enters the utility function. In many ways, the model resembles traditional models of the transactional benefits of money balances, e.g. Sidrauski [1967], with two main differences. First, I interpret broadly the liquid assets issued by the home country as including interest-paying treasury instruments, so liquid bonds pay a nominal interest rate i_b that may be larger than zero. Second, I assume that only the domestic government can issue liquid bonds, capturing the special role that dollar assets play in the world economy and the special position of the US government in emitting highly liquid, dollar-denominated assets.

The preferences of the domestic consumer are

$$\int_0^\infty e^{-\rho t} \left(u\left(c\right) + v\left(\frac{b}{p}\right)\right) dt$$

where

$$c = c_H^{\alpha} c_F^{1-\alpha},$$

the utility functions are

$$u(c) = \frac{1}{1-\gamma} c^{1-\gamma}, \quad v\left(\frac{b}{p}\right) = \frac{\psi}{1-\gamma}\left(\frac{b}{p}\right)^{1-\gamma}.$$

The domestic budget constraint, in nominal terms, is

$$p_H c_H + (1+\tau) p_F c_F + \dot{b} + \dot{a} = p_H e_H + ia + i_b b + T,$$

where a and b denote holdings of illiquid and liquid bonds, i and i_b are the interest rates on illiquid and liquid bonds, τ is an ad-valorem tariff on purchases of foreign goods, and T is a lump-sum transfer to domestic consumers. All prices are denominated in a common, world-wide unit of account. The domestic government's budget constraint is

$$\dot{B} + \tau p_F c_F = T + i_b B,$$

where B is the total supply of liquid bonds. The preferences of the foreign consumer are analogous to those of the domestic consumer, with the same discount factor ρ and the same functions u and v. The consumption basket is symmetric $c_t^* = (c_F^*)^{\alpha}(c_H^*)^{1-\alpha}$, as in the model of Section 2 and all foreign variables are denoted with stars.

For now we assume that the rest of the world imposes no tariffs. Moreover, since the foreign government does not issue liquid assets, we can simply ignore the foreign government.

To study the model, we characterize consumer optimality in two steps. First, there is the financial side, that is, there are conditions that determine optimal holdings of liquid and illiquid bonds. These boil down to two equations. The first is the standard Euler equation

$$\gamma \frac{\dot{c}}{c} = i - \pi - \rho$$

where π is the domestic inflation rate, that is, the growth rate of the home price index

$$p = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} p_H^{\alpha} ((1+\tau) p_F)^{1-\alpha}.$$

The second equation is the optimality condition for liquid bonds, which can be written as

$$(i - i_b) u'(c) = v'\left(\frac{b}{p}\right). \tag{3.1}$$

This equation leads to the demand for liquid balances

$$b = \left(\frac{i - i_b}{\psi}\right)^{-\frac{1}{\gamma}} pc, \tag{3.2}$$

which has a standard interpretation: a higher rate of return differential between liquid and illiquid bonds induces agents to economize on liquid bonds. Analgous conditions characterize the foreign demand for liquid bonds. Notice that if $i_b \geq i$ there is an unbounded demand for liquid bonds, so in equilibrium we must have

$$i_b < i.$$

The other side of the model is the goods markets. Given Cobb-Douglas preferences, the demand of domestic consumers for domestic and foreign goods is

$$c_H = \tilde{\alpha} \frac{p_H c_H + p_F c_F}{p_H}, \quad c_F = (1 - \tilde{\alpha}) \frac{p_H c_H + p_F c_F}{p_F}, \tag{3.3}$$

where we use the notation $\tilde{\alpha} = \alpha / \left(\alpha + \frac{1-\alpha}{1+\tau}\right)$ introduced in the previous section.

Analogous conditions apply to the foreign consumer, except for the inverted role of the goods H and F and the absence of the tariff τ.

Market clearing in goods markets requires $c_j + c_j^* = e_j$ for $j = H, F$. Market clearing in asset markets requires $a + a^* = 0$ and $b + b^* = B$.

Having provided a general characterization of an equilibrium in this economy, we now turn to analyze a steady state and, next, to the effect of tariffs.

4. An equilibrium with permanent trade deficits

Suppose now that the endowments e_H and e_F are both growing at the constant rate g. Assume also that the domestic government targets a certain value for the yield on the liquid bond i_b, and adjusts the total supply of bonds B to reach that yield. We want to characterize a stationary equilibrium in which the prices p_H and p_F are constant and all real quantities—consumption levels, liquid and illiquid bond positions—grow at the rate g in both countries. The Euler equations of the home and foreign consumer are both satisfied if the interest rate is

$$i = \rho + \gamma g.$$

Notice that we need to assume that $\rho - (1 - \gamma) g > 0$ to ensure that the utility function is well defined when consumption grows at rate g. This implies that the inequality $i > g$ must hold in a stationary equilibrium.

Adding up the budget constraints of the domestic consumers and of the domestic government and using $b = B - b^*$ (from asset market clearing), after some algebra, gives

$$p_H c_H + p_F c_F = p_H e_H + (i - g) a - (i_b - g) b^*. \tag{4.1}$$

For the foreign consumer, using $a^* = -a$ (from asset market clearing), we get

$$p_H c_H^* + p_F c_F^* = p_F e_F^* - (i - g) a + (i_b - g) b^*. \tag{4.2}$$

In the appendix, we show how to proceed from these two equations to fully characterize a stationary equilibrium. Those derivations lead to the following result.

Proposition 1. *If the vector $(\bar{a}, i, i_b, p_F, p_H)$ satisfies the conditions*

$$i = \rho + \gamma g > i_b,$$

$$\frac{e_F^*}{e_H} > (i - g) \frac{\bar{a}}{p_F} > -\frac{p_H}{p_F}, 1 + (g - i_b) \psi^{\frac{1}{\gamma}} (i - i_b)^{-\frac{1}{\gamma}} > 0, \tag{4.3}$$

$$(4.4)\quad \frac{p_H}{p_F} = \frac{1}{1-\tilde{\alpha}} \left\{ \tilde{\alpha}\,(i-g)\,\frac{\bar{a}}{p_F} + \frac{\tilde{\alpha}\,(g-i_b)\,\psi^{\frac{1}{\gamma}}\,(i-i_b)^{-\frac{1}{\gamma}} + 1 - \alpha}{1+(g-i_b)\,\psi^{\frac{1}{\gamma}}\,(i-i_b)^{-\frac{1}{\gamma}}} \left[\frac{e_F^*}{e_H} - (i-g)\,\frac{\bar{a}}{p_F} \right] \right\},$$

the vector characterizes a stationary equilibrium in which all prices are constant, all quantities grow at rate g, and $a/e_H = \bar{a}$.

The first two conditions in the proposition ensure that, as argued above, the Euler equation is satisfied and the demand for liquid bonds is well defined. The inequalities (4.3) are sufficient conditions that ensure that both domestic and foreign consumers have positive consumption levels. Condition (4.4) gives the relative price that equilibrates the goods market. Notice that absent asset trade—that is, if we set $\bar{a} = 0$ and $\psi = 0$—and absent the tariff—so $\tilde{\alpha} = \alpha$—the last equation would give the standard result $p_H/p_F = e_F^*/e_H$ that arises under Cobb-Douglas preferences. The presence of asset trade introduces a transfer between the two countries, which, due to home bias in consumption tilts the terms of trade in favor of the country receiving the transfer.

Proposition 2. *If the stationary equilibrium of Proposition 1 satisfies $i_b < g$ and*

$$\frac{\psi^{\frac{1}{\gamma}}\,(i-i_b)^{-\frac{1}{\gamma}}}{1+(g-i_b)\,\psi^{\frac{1}{\gamma}}\,(i-i_b)^{-\frac{1}{\gamma}}} \left[\frac{e_F^*}{e_H} - (i-g)\,\frac{\bar{a}}{p_F} \right] > \frac{\bar{a}}{p_F},$$

we have an equilibrium in which the domestic country is a net debtor to the rest of the world, $a - b^* < 0$, runs a permanent trade deficit $p_H e_H - p_H c_H - p_F c_F > 0$, and runs a permanent current account deficit $\dot{b}^* - \dot{a} > 0$.

The current account deficit is the change in the US net foreign asset position and is equal to $\dot{b}^* - \dot{a} = g\,(b^* - a)$ in a stationary equilibrium. The home economy is both issuing debt at a rate g and acquiring foreign assets at rate g and, given that $b^* > a$, this adds up to a net inflow of resources. At the same time, the country has to pay interest on its debt and receives interest on its foreign assets. Absent a liquidity premium, the interest payments would dominate the flow coming from net issuances, so the country would need to run a trade surplus. However, if the liquidity premium is large enough, the net interest payments $i_b b^* - ia$ are smaller than the net asset issuances $g\,(b^* - a)$ and it is possible for a net debtor to run a permanent trade deficit. A sufficient condition for this to be the case is the inequality $i_b < g$ assumed in the proposition.[4]

5. The effects of a tariff increase

Take the stationary economy of the last section and consider what happens if, unexpectedly and permanently, the domestic economy increases the tariff τ. The response to this shock is easy to derive thanks to the fact that the economy immediately jumps to a new stationary equilibrium. The only delicate step is to check what happens to asset positions at the moment of the shock. That adjustment depends on the unit of account in which the

[4]It is easy to write a model in which a net creditor country runs a persistent trade deficit, as that does not require the introduction of a liquid asset. The challenge addressed in this paper is to write a model in which a net debtor country runs a trade deficit.

assets were originally denominated. We begin from the simplest case, which is the case in which both the liquid and illiquid asset are denominated in terms of the foreign good. To formalize this case, we just assume that p_F is 1 in the initial stationary equilibrium and that it remains equal to 1 in the new stationary equilibrium. In this case, we can guess and verify that the asset positions a and b^* remain unchanged at the instant of the shock. The flow value of the foreign country's wealth, excluding the liquid asset, is equal to

$$e_F^* - (i-g)\,a.$$

By our guess the value of a is unchanged. Moreover, the value of i remains unchanged at $\rho + \gamma g$ in a new stationary equilibrium. Since the demand for liquid wealth is proportional to $e_F^* - (i-g)\,a$, we confirm the guess that b^* is unchanged, and we obtain that the value of

$$p^* c^* = e_F^* - (i-g)\,a + (i_b - b)\,b^*$$

is also unchanged. The relative price p_H/p_F in equation (4.4) must clearly be affected by a shock to τ that changes $\tilde{\alpha}$. In particular, an increase in τ increases the demand for the home good and so it increases p_H, since p_F remains equal to 1. This means that the price index of the foreign consumer p^* increses. This, together with the fact that $p^* c^*$ is unaffected, implies that c^* decreases, so the foreign consumer is worse off as its wealth denominated in foreign goods has not changed but its terms of trade have worsened. Turning to the home consumer, setting $p_F = 1$ the budget constraint is

$$p_H c_H + c_F = p_H e_H + (i-g)\,a - (i_b - g)\,b^*.$$

Notice that the terms $(i-g)\,a - (i_b - g)\,b^*$ are not affected by the shock, so $p_H e_H$ and $p_H c_H + c_F$ are increasing exactly by the same amount. We summarize these derivations in the following proposition.

Proposition 3. *If the foreign good is the numeraire, an unexpected, permanent increase in τ leads to:*

- **a:** *Unchanged asset positions a, b, b^*;*
- **b:** *Unchanged paths for the trade deficit and for the current account deficit of the home country;*
- **c:** *An increase in the price of the home good p_H;*
- **d:** *Identical increases in the value of domestic output and domestic spending;*
- **e:** *A reduction in foreign consumption and welfare;*
- **f:** *An increase in domestic consumption and welfare if the tariff is small.*

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6. Appendix

6.1. **Derivations for the two good model of Section 2.** The demand functions for the home good are

$$c_{H,t} = \alpha \frac{p_t c_t}{p_{H,t}},$$
$$c^*_{H,t} = (1-\alpha)\,\frac{p^*_t c^*_t}{p_{H,t}}.$$

The definition of the trade deficit and the countries' budget constraints imply

$$p_{H,t}c_{H,t} + p_{F,t}c_{F,t} = \left(\alpha + \frac{1-\alpha}{1+\tau}\right) p_t c_t = p_{H,t}\left(e_{H,t} + D_t\right), \tag{6.1}$$

and

$$p_{H,t}c^*_{H,t} + p_{F,t}c^*_{F,t} = p^*_t c^*_t = p_{F,t}e_{F,t} - p_{H,t}D_t. \tag{6.2}$$

Combining the conditions above, market clearing in the home good can be written as

$$\tilde{\alpha}\frac{p_{H,t}\left(e_{H,t}+D_t\right)}{p_{H,t}} + (1-\alpha)\,\frac{p_{F,t}e_{F,t} - p_{H,t}D_t}{p_{H,t}} = e_{H,t},$$

which yields the equilibrium relative price (2.3).

The Euler equations of the two consumers are

$$\frac{u'\left(c_1\right)}{\beta u'\left(c_2\right)} = \left(1+i_1\right)\frac{p_1}{p_2} = \left(1+i_1\right)\frac{p^{\alpha}_{H,1}p^{1-\alpha}_{F,1}}{p^{\alpha}_{H,2}p^{1-\alpha}_{F,2}},$$

$$\frac{u'\left(c^*_1\right)}{\beta u'\left(c^*_2\right)} = \left(1+i_1\right)\frac{p^*_1}{p^*_2} = \left(1+i_1\right)\frac{p^{\alpha}_{F,1}p^{1-\alpha}_{H,1}}{p^{\alpha}_{F,2}p^{1-\alpha}_{H,2}}.$$

Combining them yields (2.4). Defining

$$\rho_t = \frac{p_{F,t}}{p_{H,t}}$$

equation (2.4) can be written as follows

$$\left(\frac{c_1}{c_1^*}\right)^{-\gamma}\rho_1^{2\alpha-1}=\left(\frac{c_2}{c_2^*}\right)^{-\gamma}\rho_2^{2\alpha-1}. \tag{6.3}$$

Equations (6.1) and (6.2) allow us to derive c_t and c_t^* as

$$\left(\alpha+\frac{1-\alpha}{1+\tau}\right)c_t=\frac{p_{H,t}}{p_t}\left(e_{H,t}+D_t\right)=\frac{1}{\xi}\rho_t^{\alpha-1}\left(e_{H,t}+D_t\right),$$

$$c_t^*=\frac{1}{p_t^*}\left(p_{F,t}e_{H,t}-p_{H,t}D_t\right)=\frac{1}{\xi}\rho_t^{1-\alpha}\left(e_{F,t}-\frac{1}{\rho_t}D_t\right).$$

Substituting for c_t in the home consumer Euler equation gives the real interest rate in home goods

$$\frac{1}{1+i_1}\frac{p_{H,2}}{p_{H,1}}=\beta\frac{u'\left(c_2\right)}{u'\left(c_1\right)}\frac{p_1}{p_2}\frac{p_{H,2}}{p_{H,1}}=\beta\left(\frac{\rho_2^{\alpha-1}\left(e_{H,2}+D_2\right)}{\rho_1^{\alpha-1}\left(e_{H,1}+D_1\right)}\right)^{-\gamma}\frac{\rho_2^{\alpha-1}}{\rho_1^{\alpha-1}}.$$

The steps above allow us to express the equilibrium conditions compactly in the following three conditions:

$$\left(\frac{\rho_1^{\alpha-1}\left(e_{H,1}+\Delta_1\right)}{\rho_1^{1-\alpha}e_{F,1}-\rho_1^{-\alpha}\Delta_1}\right)^{-\gamma}\rho_1^{2\alpha-1}=\left(\frac{\rho_2^{\alpha-1}\left(e_{H,2}+\Delta_2\right)}{\rho_2^{1-\alpha}e_{F,t}-\rho_2^{-\alpha}\Delta_2}\right)^{-\gamma}\rho_2^{2\alpha-1}, \tag{6.4}$$

$$D_1\left(e_{H,1}+D_1\right)^{-\gamma}\rho_1^{(1-\gamma)(\alpha-1)}+\beta D_2\left(e_{H,2}+D_2\right)^{-\gamma}\rho_2^{(1-\gamma)(\alpha-1)}=0, \tag{6.5}$$

$$\rho_t=\frac{\left(1-\tilde{\alpha}\right)e_{H,t}-\left(\tilde{\alpha}+\alpha-1\right)D_t}{\left(1-\alpha\right)e_{F,t}}. \tag{6.6}$$

Equation (6.4) is the Euler equation condition (6.3) after substituting for c_t and c_t^*. Equation (6.5) is the intertemporal budget constraint after substituting for the real interest rate in home goods. Equation (6.6) is the relative price that equilibrates the good market from (2.3). These conditions give us four non-linear equations in D_1, D_2, ρ_1, ρ_2.

The two curves in Figure 2.1 represent the pairs D_1 and D_2 that satisfy, respectively, condition (6.4) and (6.5), after substituting for ρ_t using (6.6).

Inequality (2.5) in the text is derived as follows. Differentiating (6.6) yields

$$\left(1-\alpha\right)e_{F,t}\rho_t\frac{d\rho_t}{\rho_t}=-\left(e_{H,t}+D_t\right)d\tilde{\alpha}.$$

If $D_t>0$ we have

$$-e_{H,t}d\tilde{\alpha}<\left(1-\alpha\right)e_{F,t}d\rho_t<0$$

and

$$\rho_t>\frac{\left(1-\tilde{\alpha}\right)e_{H,t}}{\left(1-\alpha\right)e_{F,t}},$$

which, combined, yield

$$\frac{d\rho_t}{\rho_t}>-\frac{d\tilde{\alpha}}{1-\tilde{\alpha}}.$$

Similar steps apply to the case $D_t<0$.

6.2. **Derivations of the stationary equilibrium in Section 4.**

Proof of Proposition 1. Using $p^*c^* = p_H c_H^* + p_F c_F^*$ and substituting the bond demand by foreign consumers (which is analogous to (3.2)) in equation (4.2), gives

$$p^*c^* = p_F e_F^* - (i-g)\,a + (i_b - g)\,\psi^{\frac{1}{\gamma}}\,(i-i_b)^{-\frac{1}{\gamma}}\,p^*c^*.$$

Solving for p^*c^* gives

$$p^*c^* = \frac{1}{1+(g-i_b)\,\psi^{\frac{1}{\gamma}}\,(i-i_b)^{-\frac{1}{\gamma}}}\left[p_F e_F^* - (i-g)\,a\right].$$

Substituting in the bond demand by foreign consumers and using (4.1) we obtain

$$p_H c_H + p_F c_F = p_H e_H + (i-g)\,a + \frac{(g-i_b)\,\psi^{\frac{1}{\gamma}}\,(i-i_b)^{-\frac{1}{\gamma}}}{1+(g-i_b)\,\psi^{\frac{1}{\gamma}}\,(i-i_b)^{-\frac{1}{\gamma}}}\left[p_F e_F^* - (i-g)\,a\right].$$

Substituting $p_H c_H + p_F c_F$ and p^*c^* in the market clearing condition in the home good market, after some algebra, we get

$$\tilde{\alpha}\,(i-g)\,\frac{a}{p_F} + \frac{\tilde{\alpha}\,(g-i_b)\,\psi^{\frac{1}{\gamma}}\,(i-i_b)^{-\frac{1}{\gamma}} + 1 - \alpha}{1+(g-i_b)\,\psi^{\frac{1}{\gamma}}\,(i-i_b)^{-\frac{1}{\gamma}}}\left[e_F^* - (i-g)\,\frac{a}{p_F}\right] = (1-\tilde{\alpha})\,\frac{p_H}{p_F}e_H,$$

which yields (4.4). Notice that

$$pc = p_H c_H + (1+\tau)\,p_F c_F = \frac{1}{\alpha + \frac{1-\alpha}{1+\tau}}\,(p_H c_H + p_F c_F)\,,$$

so substituting the expressions above for p^*c^* and $p_H c_H + p_F c_F$ we can compute the supply of bonds needed to support the stationary equilibrium using

$$B = \left(\frac{i-i_b}{\psi}\right)^{-\frac{1}{\gamma}}\left(\frac{1}{\alpha + \frac{1-\alpha}{1+\tau}}\,(p_H c_H + p_F c_F) + p^*c^*\right).$$

Equilibrium in the foreign good market follows by Walras' law. It is easy to check that conditions (4.3) imply that both $p_H c_H + p_F c_F$ and p^*c^* are positive. □

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