On the Welfare Costs of Perceptions Biases

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Abstract

Are households significantly harmed by deviating from rational expectations when forecasting inflation? If the benefits of accurate beliefs about inflation are low, households may find it optimal to allocate attention elsewhere. This paper analyzes two established inflation perceptions biases in French households and evaluates their welfare effects. The first bias is the frequency bias, where households overweight goods that they purchase frequently but are a small share of their consumption basket. I find that households in my sample fixate on bread prices. The second bias is that households consistently overestimate the current inflation rate, which I call the level bias. I estimate the magnitude of these biases using a confidential French household survey. In order to evaluate the welfare losses of the two biases, I develop a partial-equilibrium model where households save in a single nominal bond subject to inflation risk. The level bias significantly reduces welfare and asset formation, while the frequency bias has negligible effects.

JEL Codes: E31, E70, G51
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1 Introduction

To make spending and saving decisions, households must assess the current price level and form expectations about future prices. A large literature based on survey evidence shows that these expectations often deviate from full information rational beliefs due to a variety of possible causes, such as noisy information, asymmetric loss functions, and personal experiences.\(^1\) It remains unclear, however, whether deviating from rational inflation beliefs affects household welfare significantly. This paper studies this question.

This paper examines two established biases in household perceptions and constructs a model to evaluate the welfare implications. I focus on the effect of biases on inflation perceptions, which are the beliefs that households have about the current inflation rate. The first bias is the frequency bias, which describes the phenomenon in which households overweight the prices of goods that they purchase frequently when they think about inflation.\(^2\) For example, if food prices in France rise suddenly, households may raise their perceptions of the inflation rate significantly, even though food is a modest component of their consumption basket. The second bias is that households consistently overestimate the current inflation rate.\(^3\) I refer to this phenomenon as the level bias. These biases may seem surprising, given that the cost of obtaining accurate information about the current inflation rate is low.\(^4\) If these biases were harming households, it would be relatively simple to correct them.

I evaluate welfare losses in five steps. First, I use a confidential French household survey to establish that perceptions about the current inflation rate are correlated with expectations and savings behavior. Second, I provide causal evidence that French households suffer from the frequency bias by overweighting bread inflation when reporting their perceptions.\(^5\) Third, I analyze the level bias and find that the average French household overestimates the inflation rate by 2.5 percentage points. The size of this effect varies by income and education status.\(^6\) Fourth, I develop a partial equilibrium model in which households use their biased inflation perceptions to make savings decisions in order to self-insure against idiosyncratic income shocks. I find that the consum-

\(^1\)See Woodford (2003), Capistran and Timmermann (2009), and Malmendier and Nagel (2015), respectively.
\(^2\)Tversky and Kahneman (1973) originally postulated the frequency bias for general contexts, while Georganas et al. (2014) applied it to price inflation.
\(^3\)See Abildgren and Kuchler (2019) for a discussion of this behavior.
\(^4\)The national statistical agencies of many developed countries post inflation statistics publicly on their website, which means that the monetary cost of obtaining accurate information about current inflation is almost zero.
\(^5\)See Brachinger (2008), Shwayder (2012), Armantier et al. (2016a), D’Acunto et al. (2019b) for other papers measuring the frequency bias.
\(^6\)Papers such as Bryan and Venkatu (2001b), Bruine de Bruin et al. (2010), Ehrmann et al. (2017), and D’Acunto et al. (2019a) have also documented demographic differences in household inflation expectations.
tion equivalent welfare loss of the level bias is 0.17% of steady state consumption. However, the
consumption equivalent loss of the frequency bias is miniscule, less than 0.001% of steady state
consumption. Finally, I conduct counterfactual analyses and find that a) reducing the correlation
between bread and the overall inflation rate only slightly increases welfare loss, b) low-income
households suffer a greater welfare loss than high-income households, and c) if households have
biased inflation expectations but accurate perceptions of current inflation, their welfare loss in-
creases, which suggests that biased inflation perceptions can partly offset the effect of inaccurate
inflation expectations.

In my empirical work I use France’s Monthly Consumer Confidence survey (CAMME) to es-
timate the magnitude of the inflation perceptions biases. The survey is conducted on a monthly
basis from 2004 to 2017, during which each household is re-surveyed for up to three consecutive
months. I combine the households’ beliefs with nationally aggregated inflation statistics.

My three main empirical findings are as follows. First, perceptions of the current inflation rate
are linked to households’ expected inflation and their economic decisions. I find that inflation per-
ceptions are associated with at least 39% of the variance of inflation expectations at the household
level. To provide evidence that households are acting on their perceptions, I analyze the effect of
inflation perceptions on self-reported declarations about savings. In the survey, French households
that report higher inflation perceptions are less likely to believe that it is a good time to save. In
summary, biased inflation perceptions could lead to suboptimal economic decisions.

Second, I establish that French households overweight bread inflation when forming percep-
tions about the current inflation rate. To test this hypothesis, I regress household inflation per-
ceptions on the overall inflation rate and the bread inflation rate. Under the framework of perfect
information and rationality, households should assign no weight towards the bread inflation rate,
because any relevant information it contains has already been incorporated into the overall inflation
rate. However, I find that households put more weight on bread inflation than they do on the overall
inflation rate. If the bread inflation rate rises by 1 percentage point, the average French household
raises its inflation perception by 0.66 percentage points. In comparison, if the overall inflation rate
rises by 1 percentage point, the average French household only raises its inflation perception by
0.57 percentage points. The regression may suffer from an endogeneity problem. To resolve this
concern, I rerun my regression analysis with the lagged world wheat inflation rate as an instrument
for the bread inflation rate. I find that bread inflation remains a statistically significant predictor

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7 This finding is consistent with Jonung (1981).
8 Shwayder (2012) and Coibion and Gorodnichenko (2015) have used world oil prices as a proxy for gasoline
of inflation perceptions even with the world wheat instrument and that the coefficient on bread increases to 0.86. Bread inflation affects French inflation perceptions.

My third empirical contribution is to measure the level bias, document heterogeneity by demographics, and examine several possible causes. Households overestimate the actual inflation rate by 2.5 percentage points on average. This overestimation is relatively constant over my sample period from 2004 to 2017. I find that low-income households and low-education households overestimate the current inflation rate by more than high-income and high-education households. I propose and test several possible explanations for the level bias and the difference by income. I examine whether households asymmetrically weight price increases relative to price decreases, but find that this weighting does not explain the magnitude of the level bias. The income difference in the level bias cannot be explained by differences in consumption baskets by income.\(^9\)

Following my empirical work, I turn to the task of evaluating how these biases affect household welfare. I develop a partial equilibrium heterogeneous agent model. Households have a risky, idiosyncratic labor income that they partially smooth by purchasing a nominal bond, which is subject to inflation risk and a no-borrowing constraint. Households use their biased inflation perceptions to forecast future inflation, which can result in inaccurate inflation expectations. When households solve their savings problem each period, they act as if their biased perceptions and expectations are correct.\(^10\) However, from the perspective of an outside observer with rational expectations, these households make sub-optimal savings decisions.

According to my model, the welfare loss stemming from the level bias is several orders of magnitude larger than the welfare loss associated with the frequency bias. These welfare differences are driven by bond-holding behavior: households that suffer from only a level bias hold about three-fourths as many bonds in the steady state as households that have rational beliefs. In contrast, households with the frequency bias hold about the same number of bonds as those with rational beliefs. The intuition behind these results is that households with the level bias consistently believe that future inflation will be high, which reduces the real return to nominal bonds and discourages them from saving. Lower asset holdings means that households are less able to smooth out consumption in response to shocks in their labor income, reducing their overall welfare.

\(^9\)The frequency bias does not explain the level bias in my dataset. The average overall inflation rate in France during my sample is 1.6%, while the average bread inflation rate is 1.7%. Even if households only paid attention to the bread inflation rate they would still perceive the correct inflation rate on average.

\(^10\)If a household was aware that it had an level bias, it could correct for the bias and obtain rational expectations.
I conduct three extensions to the model in order to explore the intuition behind these results. 

First, the bread inflation rate in France is highly correlated with the overall French inflation rate in my dataset, which may explain why the welfare loss stemming from the frequency bias is relatively small. To test this explanation, I solve the model with an alternative parameterization where bread is a worse signal for overall inflation than in the data. Specifically, I set the correlation between bread inflation and overall inflation rate to be negative. The welfare loss associated with the frequency bias triples compared to the baseline parameters, but remains lower than the welfare loss from the level bias. Second, I consider using a set of parameters from the U.S. context, in which households overweight gasoline inflation when forming their perceptions. The welfare loss associated with the resulting parameterization is around twenty times larger than the benchmark parameters, although still less than the level bias. This result reflects the fact that gasoline prices are much more volatile than bread prices.

Second, I investigate how biased perceptions affect welfare. Biased perceptions influence households through two channels: biased expectations and biased wealth perceptions. Households with perception biases will have distorted expectations of the future inflation rate. Overestimating the future inflation rate is equivalent to underestimating the expected real return to savings, which will affect households’ incentive to save. Regarding perceptions, I assume that households know their nominal wealth at the start of each period, but not their purchasing power.\textsuperscript{11} For example, if a household erroneously believes that the current inflation rate is high, then it will believe its real wealth is lower than it actually is.

To disentangle these two channels, I consider a version of the model where households have biases when forecasting inflation, but know the current inflation rate perfectly.\textsuperscript{12} After solving this model, I find the welfare loss associated with the level bias rises compared to the version of the model where households have the level bias in perceptions and expectations. This result is driven by the fact that households with a level bias in perceptions are incentivized to purchase more bonds. If households in my model erroneously believe that the inflation rate was high, then they believe that the purchasing power of their nominal wealth has fallen. Because of the model’s no-borrowing constraint, households will increase their savings to build their buffer stock of savings back up. Because households with the level bias consistent overestimate the current inflation rate, this will result in a larger bond holdings in the steady state. Overall, biased inflation perceptions

\textsuperscript{11}Households in my model do not know the current inflation rate or price level.

\textsuperscript{12}If household only have the level bias for inflation expectations, then every period they believe that the future inflation rate will be 2.5 percentage points higher than they would if they had rational expectations. If households have the frequency bias when forecasting, then they correctly weight bread when estimating current inflation, but overweight it when forecasting inflation.
can partly offset inflation expectations by encouraging households to increase their savings.

Finally, I examine how variation in the level bias by household income affects welfare loss and aggregate bond-holding. High income households have a smaller level bias than low-income households, which means that the welfare loss for low-income households is significantly greater than high-income. However, it is unclear if aggregate bond-holdings is significantly affected. If high-income households hold a disproportionate share of bonds and suffer from a small level bias, then the total number of bonds held in steady state may only deviate slightly from rational expectations. In my model, I find that the aggregate bonds held is still lower than the rational expectations amount, because high-income households still suffer from a moderate level bias.

The paper is laid out as follows. Section 2 introduces the survey data and links inflation perceptions with expectations and savings. Section 3 measures the magnitude of the frequency bias in bread. Section 4 estimates the level bias. Section 5 introduces the model. Section 6 describes the strategy for parameterizing the economy. Section 7 presents the results of the model. Section 8 concludes.

**Related literature**

This paper is related to several fields of literature. First, it contributes to the subfield that measures the welfare effects of imperfect information or non-rational expectations. The rational inattention literature, which was pioneered by Sims (2003) and Sims (2010), studies whether households have a limited amount of attention to allocate towards different macroeconomic variables. Mackowiak and Wiederholt (2015) incorporates inflation misperceptions into a rational inattention business cycle model and finds that households optimally choose to pay little attention to inflation relative to other macroeconomic variables. Kamdar (2019) focuses more closely on households and finds that households choose to focus on real wages rather than prices. My paper differs from this rational inattention literature because I do not derive these perceptions biases from a household optimization problem, but from the survey data. The paper outside of the rational inattention sub-field that is the most similar to my work is Rozsybal and Schlafmann (2019), which analyzes how overestimating the persistence of income affects households’ savings decisions. Although my paper’s model follows a similar approach to theirs, I study biased inflation beliefs rather than biased personal income beliefs.

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13 See Kumar et al. (2015), Cavallo et al. (2017), and Coibion et al. (2018) for some examples of rational inattention in firms.
My paper builds off of several papers that study how households use inflation expectations to make savings decisions. A number of papers assert that inflation expectations affect readiness to spend or self-reported spending, but they disagree about the sign of the effect. Armantier et al. (2015), D’Acunto et al. (2016) and D’Acunto et al. (2019a) find that households increase spending if their inflation expectations rise, while Burke and Ozdagli (2014) and Bachmann et al. (2015) find the opposite effect. Vellekoop and Wiederholt (2017) links a household opinion survey to their actual financial data and finds that households do reduce their savings when their inflation expectations are higher, which supports the mechanism in my paper. Like these papers, my paper documents that households with higher inflation perceptions are less willing to save, but it goes further to evaluate the effects of this behavior.

The frequency bias has been extensively documented in other contexts and using different methods. Georganas et al. (2014) conducts a laboratory experiment and demonstrates that consumers overweight frequently encountered prices when evaluating inflation. Shwayder (2012) and Coibion and Gorodnichenko (2015) provide evidence that U.S. households overweight gasoline when forming expectations, although Binder (2018) suggests that gasoline prices do not affect inflation perceptions. Both Cavallo et al. (2017) and D’Acunto et al. (2019b) study the inflation perceptions of consumers in relation to their supermarket purchases and find that people react to price increases in goods they purchase. D’Acunto et al. (2019b) constructs a frequency-weighted consumer price index, which is similar to the approach of Brachinger (2008). My analysis contributes to this field by establishing the frequency bias for a single good (e.g. bread in France) and by evaluating the effect of the frequency bias on households’ welfare.

Lastly, my paper is related to a body of work that investigates how inflation beliefs vary by demographics, particularly by income or education. Arioli et al. (2017) and Abildgren and Kuchler (2019) provide a good overview of how low-income and low-education households consistently overestimate past inflation rates relative to high-income and high-education households, as well as different possible causes for this effect. The literature suggests a number of possible explanations for this phenomenon. First, households could be purchasing different consumption baskets and therefore face different inflation rates. However, Arion et al. (2018) constructs consumption

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14 Many surveys, such as the well-studied Michigan Survey of Consumers, include a question about households’ willingness to spend, which means that it is easy to study this question.

15 Armantier et al. (2016a) conducts an experimental survey where they provide households with accurate inflation statistics. They find that low-income and low-education households update their beliefs less in response to the new information. This result suggests that low-income households not only have less accurate beliefs, but more difficulty processing new information.

16 Johannsen (2014) uses Bureau of Labor Statistics Consumption Expenditures Surveys to examine how consumption baskets vary by demographic status and finds that the consumption baskets can explain the dispersion in
baskets by income group and find that the inflation rates do not diverge significantly.\textsuperscript{17} Another possible explanation is that low-income households have low financial literacy and are unfamiliar with concepts like inflation.\textsuperscript{18} A third possibility is that households asymmetrically view price increases compared to price decreases.\textsuperscript{19} I test several of these explanations. Ultimately, I am agnostic about the cause of the level bias. I contribute to this literature not only by evaluating the welfare effect of the level bias, but by studying how the welfare effect varies by household income.

2 Inflation Perceptions

In this section I describe the household survey methodology and establish the relevance of inflation perceptions. First I introduce the household survey. Next, I show that French households forecast inflation from inflation perceptions. I conclude by linking inflation perceptions with beliefs about saving.

2.1 Survey Data

I use data from the Monthly Consumer Confidence Survey (CAMME), which is collected by the National Institute of Statistics and Economic Studies (INSEE) in France. The CAMME is a monthly survey of 1,800 French households that inquires about consumer behavior and expectations. Although the survey was first collected in 1958, my sample will be limited to the period from 2004-2017 when INSEE asked detailed questions about inflation beliefs.\textsuperscript{20} CAMME includes a wide range of questions about economic opinions; however, my paper focuses on survey questions on inflation beliefs and willingness to save. Inflation beliefs refers to how households believe prices have changed over the previous 12 months and how they believe prices will change over the next 12 months. I also use demographic variables, such age, education, family status, gender, and income. Altogether the dataset includes 171,606 households that answer the question about quantitative inflation perceptions. Appendix A describes the survey wording in more detail.

The CAMME does not record the exact value of household income. Rather than reporting households income, INSEE records which quartile they fall in, i.e. between the 1st and 2nd income responses.

\textsuperscript{17}Diamondy et al. (2019) constructs household-specific inflation rates and conclude that differences in experienced inflation does not explain differences in expectations. Bryan and Venkatu (2001a) find that men and women have different beliefs about food and clothing inflation rates as well as the overall inflation rate.

\textsuperscript{18}See Lusardi and Mitchell (2008), Lusardi et al. (2010), and Lusardi et al. (2017).

\textsuperscript{19}See Capistran and Timmermann (2009), where forecasters form their expectation of inflation using an asymmetric loss function.

\textsuperscript{20}INSEE did not conduct the CAMME survey in the month of August until 2008. Aggregated statistics were reported as interpolations of the responses in July and September.
I define “low-income” to indicate households that report income below the 1st quartile, and “high-income” to indicate households that report income above the 3rd quartile. Each month INSEE takes all of the household respondents, orders them by household income, and divides the sample into fourths. I assume that these sample income quartiles more or less correspond to the national income quartiles when calculating the effect of biased inflation perceptions by households income.

When INSEE conducts the CAMME survey, they resurvey each household for up to three consecutive months, i.e. February, March, and April. This methodology is an example of a revolving panel survey, which possesses some of the attributes of a panel dataset. I can use the revolving panel to study how households update their perceptions in response to changes in inflation. CAMME does not possess unique household identification numbers. As part of this paper, I use internal survey variables to construct unique household identifiers, which I use to track households across all three waves that they respond. I can only track households across waves between 2004 and 2014. Approximately 40% of households that answer the inflation perceptions question are present for three consecutive months.

The CAMME survey allows me to examine the understudied topic of inflation perceptions. Most surveys of households beliefs ask about inflation expectations rather than inflation perceptions. For example, two of the most commonly used surveys on inflation beliefs are the Michigan Survey of Consumers and the Survey of Consumer Expectations in the United States. While both surveys have a rich set of questions overall, they restrict their questions to expectations about future inflation. By analyzing the CAMME microdata, I can study inflation perceptions and tests hypotheses about the information set of the households.

I complement my survey dataset with detailed French inflation statistics. These statistics serve two purposes: I can use them to calculate the inflation rates of specific goods categories, and I can construct inflation rates for specific household demographics. I obtain the pricing data from the

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21Even the CAMME microdata does not contain the actual monthly income of households in the sample, presumably for confidentiality.
22Before 2008, INSEE would skip August and resurvey some households in September. For example, a household may be surveyed in July, September and October.
23For example, the first time a household is surveyed is called its first wave, the second time is its second wave, etc.
24I specifically use the NUMFA variable in the CAMME. After repeated inquiries to INSEE, I believe that this is a semi-unique variable related to where households are stored in the internal database.
25INSEE cannot be certain that they contact the same person within a household each time they survey it. It is possible that they are surveying the spouses of the original respondents in subsequent waves.
26On January 2015 INSEE introduced a new variable to denote households, which I cannot link to pre-2015 observations. INSEE introduced yet another variable in January of 2016.
Harmonized Consumer Price Index (HICP), which I use to calculate the overall inflation rate. The HICP contains aggregated price indices for different categories of goods, such as food, gasoline, and rent. In order to obtain the consumption weights for households, I draw on the Household Budget Survey (HBS), which records the fraction of their income households spend on different goods categories. I combine the consumption weights with the pricing data in HICP to create inflation rates specific to household demographic groups, i.e. low-income.

2.2 Inflation Perceptions and Expectations

Although households may hold inaccurate perceptions of inflation, these biases may not be harmful unless they affect other household beliefs and result in sub-optimal household decisions. If a household believes that the inflation rate is 5% but accurately forecasts the future inflation rate at 2%, then biased perceptions have little economic significance. To address this issue, this subsection will link inflation perceptions to inflation expectations, and the subsequent subsection will address the effect of perceptions on savings.

I find that households use their perceptions of the current inflation rate to forecast future inflation. Figure 1 presents the average inflation perception and the average inflation expectation for households over the sample period. I also plot the actual inflation rate over the past 12 months as a comparison. This graph suggests two facts: households have inaccurate inflation perceptions, and aggregated inflation perceptions are correlated with aggregated inflation expectations. If households are aware of the true inflation rate, then the perceived inflation should be close to the actual inflation rate. As Figure 1 shows, the average household has inaccurate inflation perceptions. Average perceptions and expectations in Figure 1 co-move closely, particularly after 2007 when the actual inflation rate begins to fluctuate. The correlation between the two time series is 0.822, which indicates that expectations are linked to perceptions in aggregate.

I exploit the CAMME microdata to confirm that the relationship between perceptions and expectations holds on a household level. I regress inflation expectations on inflation perceptions using the CAMME survey microdata. I follow the standard approach of assuming that households believe that the inflation rate follows an auto-regressive process with one lag. Let the operator $E_{i,t}$

\footnote{I censor any responses that are larger than 20% in magnitude as outliers. Roughly 5% of observations in any given month lie outside of this window; incorporating them in the analysis does not meaningfully affect results. I choose the 20% threshold because in some months, particularly in 2008, a significant fraction of inflation perceptions are above 10%. Choosing a threshold of 10% or 15% would lead me to drop a large fraction of responses. My results are robust to the 15% cutoff.}

\footnote{Assuming that inflation follows a AR(1) process is fairly standard in the literature. See Evans (2001) and Mal-}
denote household $i$’s expectation of a variable at time $t$, where $t$ is defined on a monthly frequency. Household $i$’s perception of inflation over the past 12 months is $E_{i,t} \pi_{t'}$, and their expectation of inflation over the next twelve months is $E_{i,t} \pi_{t'+12}$. The variable $D_i$ denote demographic controls for individual $i$, including income, education, age, and sex. The regression specification is

$$E_{i,t} \pi_{t'+12} = \alpha + \beta E_{i,t} \pi_{t'} + D_i + \epsilon_{i,t}$$  \hspace{1cm} (1)$$

Table 1 presents results and shows a positive relationship between inflation expectations and perceptions. In all specifications, inflation perceptions are a statistically significant predictor of inflation expectations. In the baseline analysis of column (1), where I do not control for any demographic variables, inflation perceptions explain 39% of the variation in expectations. A 1 percentage point increase in inflation perceptions translates to a 0.57 percentage point increase in expectations.

The link between perceptions and expectations holds in robustness checks. I rerun the analysis with demographic controls and find that the relationship between perceptions and expectations remains unchanged. Inflation expectations cannot be explained solely by between-demographic group effects. Limiting the analysis to the period after the financial crisis in column (3), when the inflation rate is less volatile, does not weaken this relationship. Lastly, I limit my regression analysis to households with income below the 1st quartile in column (4) and above the 3rd quartile in column (5). The coefficient on inflation perceptions is similar for both analyses, which indicates that the relationship between perceptions and expectations is not driven by an unusual subsample of the population.

Household fixed effects may still play a large role in forming inflation perceptions and expectations. I use the revolving panel component of the survey to test this hypothesis. In column (6) I analyze how households update their expectations between their first wave and their third wave in response to updates to their inflation perceptions. I a household raises its perception of inflation, then presumably it should increase its expectation as well. The new regression specification is a differenced version of equation (1):

$$\Delta E_{i,t} \pi_{t'+12} = \alpha + \beta \Delta E_{i,t} \pi_{t'} + \epsilon_{i,t}$$  \hspace{1cm} (2)$$

The regression results in column (6) of Table 1 indicate that if a household raises its perceived inflation rate by one percentage point, then it also updates its expected future inflation by about

\footnote{mendier and Nagel (2015) for examples.}

\footnote{That is, inflation from period $t-12$ to period $t$.}
0.48 percentage points. The $R^2$ score of 0.28 is relatively high, given that this regression seeks to explain all variation in household updating of expectations. However, the true relationship between perceptions and expectations may be even stronger. In Appendix B I explore the possibility that some households may be incorrectly coded as having expectations of 0%. If so, then this analysis understates the true relationship between perceptions and expectations. In summary, I find that there is a strong link between inflation perceptions and inflation expectations, which is consistent across time and demographics.\textsuperscript{30}

### 2.3 Inflation Perceptions and Savings

Biased inflation perceptions can only generate welfare losses if they can induce households to take an action, such as increasing or reducing their savings. According to the household Euler equation, households should decrease their savings if they expect that the future inflation rate will be higher, because a higher price level in the future reduces the purchasing power of their nominal assets. Vellekoop and Wiederholt (2017) find that Dutch households with higher inflation expectations do reduce their savings in an economically appropriate fashion. I estimate the effect of inflation perceptions on attitudes about savings.

The CAMME survey includes several questions about savings beliefs and planned savings. I focus on the question that asks whether it is a good time for people to save instead of the question about whether households actually plan to save. I suspect that households’ plans to save are affected by variables other than expected inflation, such as households’ expected future income. In contrast, beliefs about saving in general should be more associated with expected inflation and the nominal interest rate. Households have multiple qualitative responses, as described in Appendix A. I create a binary response variable from this question that has two values: yes or no.

I conduct a logit analysis to study the effect of perceptions on beliefs about savings. A logit analysis regress the logarithm of the odds that a household believes that it is a good time to save against a linear combination of explanatory variables.\textsuperscript{31} The predicted result of the logit regression is that a higher inflation perception is associated with being less willing to save. However, the logit regression requires additional controls. Beliefs about savings may be correlated with other

\textsuperscript{30}The link between perceptions and expectations may be even stronger than I indicated though: see Appendix B for further details.

\textsuperscript{31}Another possible technique is an ordered logit, where the dependent variable is a categorical variable with more than two values that have some sort of ordering. The CAMME survey does allow multiple options for household beliefs. However, I found that reporting the results for the ordered logit was more complicated than a logit, so have included the latter for simplicity’s sake.
characteristics, such as household income or their beliefs about the general economic situation. I control for other demographic statuses \(D_t\), including education and income. The variable \(F_{i,t}\) is an indicator variable corresponding to the households’ beliefs about the future economic outcome.\(^3\) I also incorporate year fixed effects and cluster the errors by the monthly date to capture the effect of the macroeconomic variables.

The regression setup is

\[
\text{logit}(P(\text{It is a good time to save})_{i,t}) = \alpha + \beta E_{i,t} \pi_t + D_i + F_{i,t} + \epsilon_{i,t}
\]  

(3)

I present the results of the logit regression in Table 2. Households with higher inflation perceptions are less likely to believe that it is a good time to save. Incorporating demographic controls in column (2) and year fixed effects in column (3) does not affect the significance of inflation perceptions on willingness to save. Lastly, inflation perceptions remain statistically significant even after controlling for beliefs about the future economic conditions in column (4).

Although inflation perceptions are statistically significant, the economic significance is harder to discern. Logit regressions are non-linear, so a 1 percentage point increase does not result in the dependent variable changing by a constant \(x\%\). Instead, I calculate the marginal effect of inflation perceptions evaluated at the mean of the other covariates. Figure 2 shows how, as inflation perceptions rise from 0\% to 10\%, the probability of saying that it is a good time to save drops from 64\% to 56\%.

3 Frequency Bias

In this section I document the frequency bias in inflation perceptions. I begin by describing the frequency bias in more detail. I then propose that bread is an ideal candidate for the bias and conduct a simple ordinary least squares regression. Next, I consider that my results could be suffering from two empirical issues: endogeneity and serial correlation. I conduct an instrumental variable regression and run a differenced regression to address these problems.

3.1 Frequency Bias Bread

The frequency bias was originally proposed as a psychological phenomenon. Tversky and Kahne- 
man (1973) assert that people judge the probability of events using the availability of evidence

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\(^3\)See Appendix A for a description of the wording for this question and the possible answers.
that comes to mind. Ranyard et al. (2008) argues that when considering beliefs about inflation, availability should be seen as the frequency with which people purchase certain goods. Georganas et al. (2014) provides experimental evidence for the presence of a frequency bias by giving study participants a list of prices and asking them the about inflation rate. They find that people fixate on the prices of goods they encounter frequently. In summary, the frequency bias is not novel and has been rigorously supported.

Which consumption goods could result in a frequency bias? A good that creates a frequency bias in inflation perceptions would be some product that people purchase often and disproportionately focus on when evaluating changes in prices. This product should be something that households purchase regularly, but that represents a small component of the household’s budget. For example, gasoline may be a good candidate for the frequency bias in the U.S., because many people purchase gasoline on a weekly basis and view posted gasoline prices throughout the day. On the other hand, automobiles are not a good candidate, because people tend to buy a new automobile once every couple of years.

I propose that French households overweight bread prices when evaluating inflation rates. Bread can be purchased on a daily or weekly basis. Bread is also a small share of the annual household budget. According to the Household Budget Survey, the typical French household spends 0.4% of its income on bread. Although I do not observe how bread spending varies by income in France, bread does not vary as a share of consumption across income in Belgian version of the HBS. Finally, bread can be purchased from a specialized store, i.e. a bakery. Unlike a supermarket, where households may view dozens of prices over the course of one visit, people can view a small number of prices and buy only a handful of items at a bakery, making it easier for people to notice and keep track of prices.

Anecdotal evidence supports the assertion that French people purchase bread regularly. According to the industry group Observatoire du Pain, French people buy bread fairly often and consume it in meals.33 Their web brochure claims that twelve million households visit a bakery to purchase bread on any given day, which is roughly a fifth of the population of France. These numbers should be interpreted as a lower bound on the salience of bread in France, because the Bread Observatory organization represents bakeries only, and does not count supermarkets where people can purchase bread.

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33See Opinion Valley (2016).
I calculate the year-on-year inflation rate for bread using HICP data. Figure 3 plots the aggregated bread inflation rate against the overall inflation rate at a monthly frequency. Two relevant facts stand out. First, the average bread inflation rate over the sample is roughly the same as the average overall inflation rate, about 1.7%. The bread inflation rate is the same as the overall inflation rate. Second, the bread inflation rate has a high, but not perfect correlation with the overall inflation rate. Bread could serve as an excellent signal for overall inflation.

3.2 Frequency Bias Regression

As a first step, I study the effect of changes in the price of bread on how French households perceive overall inflation using a naive ordinary least squares regression. I regress household inflation perceptions on the actual overall inflation rate, with the addition of the actual bread inflation rate. Let $\pi_t$ be the overall inflation rate for the household consumption basket and $\pi_{t}^{Bread}$ be the inflation rate of aggregate bread prices over the past twelve months. The regression specification is:

$$E_{i,t}\pi_t = \alpha + \beta \pi_t + \gamma \pi_{t}^{Bread} + \epsilon_{i,t} \quad (4)$$

Full-information rational expectations, imperfect information, and the frequency bias all make different predictions about the result of this regression. Under full-information rational expectations, the coefficients should be $\beta = 1$ and $\gamma = 0$, because all relevant information contained in bread prices has already been incorporated in the overall inflation rate, $\pi_t$. Households should not have any perception errors for the average month, which results in $\alpha = 0$. If households suffer from noisy information, then $\beta < 1$ as they only view past prices through the distorted lens of noisy or sticky information. However, sign of $\gamma$ is a priori unclear.

The high correlation between bread and overall inflation poses a challenge. If the correlation between the two is high, then my analysis may suffer from multi-collinearity, as there is not enough variation in bread inflation that is orthogonal to the overall inflation rate. If this concern is true, then the standard errors on $\gamma$ should be large and the effect of bread inflation will be statistically insignificant. My results are not biased upwards by a high correlation.

Table 3 displays the results. Column (1) is the baseline regression of inflation perceptions on inflation and demonstrates that inflation affects perceptions. Column (2) compares the effect of noise.

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Optimally, I would obtain the bread prices on a store-level. For example, Berardi et al. (2017) tracks the prices of supermarket goods purchased online throughout France. Unfortunately, I do not have access to their confidential dataset and their data only goes as far back as 2014.
the actual overall inflation rate and the overall bread inflation rate. I find that these coefficients are almost identical, 0.66 compared to 0.57. The coefficient on bread inflation is statistically significantly larger than 0, which provides evidence against the households having rational expectations. I cannot reject the null hypothesis that the coefficient on bread inflation and overall inflation are the same with a t-test.\(^{35}\)

It’s possible that the households’ inflation perceptions are not thinking about bread, but overall food prices when considering inflation.\(^{36}\) If changes in bread prices are correlated with changes in the overall food price index, then the results in column (2) would be misleading. This effect would still be an example of the frequency bias, but households would fixate on overall food prices instead of bread specifically. Column (3) introduces a control for the overall food inflation rate. This specification does not substantially alter the results: food inflation is statistically insignificant and the bread inflation coefficient barely changes.

The regression results for the frequency bias may be driven by an unusual subsample, such as low income households. This may be economically significant for two reasons. First, if only the poorest households fixate on bread, then it is possible that this effect has limited effect on overall savings or pricing for the economy as a whole, as poor households have the least resources. Second, high-income households tend to have more accurate inflation beliefs. I rerun the analysis above on the high-income households, under the hypothesis that the effect of bread will be weaker. Column (4) displays the regression results for only the top quartile of income. I find that the coefficients are mostly unchanged from the overall sample of households: high-income households fixate on bread as much as households in general.

### 3.3 Endogeneity and Serial Correlation

In this subsection I address two challenges, endogeneity and serial correlation. First, bread inflation may not cause perceptions, but instead reflect endogeneity. Second, my analysis incorporates time series, which means that it may suffer from serial correlation.

#### 3.3.1 Endogeneity

The results from my ordinary least-squares regression may be spurious due to endogeneity. This endogeneity could take two forms. First, there may be a confounding variable that affects both

\(^{35}\)The vertical coefficient in the regression, which I will call “level bias,” will be explored in greater detail in the following section. I will postpone further discussion of this coefficient until then.

\(^{36}\)See Armantier et al. (2016b).
French household inflation perceptions and bread prices. Second, these results may reflect reverse causality. If French people believe that prices are rising quickly and substitute towards relatively cheap, low-quality bread, then firms may view this as a demand shock and respond by raising their own prices.

I use world commodities prices as an instrument for domestic bread inflation in France. Agricultural ingredients are key components for making bread and drive costs. I use the year-on-year percentage change in world wheat prices as an instrument for the year-on-year inflation rate for bread in France. I obtain the monthly world wheat prices from the Chicago Board of Mercantile Exchange. As the Figure 4 shows, there is a correlation of 0.48 between wheat and French bread prices, which indicates that wheat inflation is a leading cause bread inflation. The correlation between wheat inflation and French bread inflation is highest when I lag wheat inflation by six months. This commodities approach has been used when studying the effect of gasoline inflation. Crude oil prices are often used as a proxy, because the pass-through between the input of crude oil to gasoline is quite high (see Shwayder (2012)).

Using world wheat prices as an instrument should solve the endogeneity concern, particularly the reverse causation problem. It seems unlikely that French households’ demand for bread is large enough to affect world wheat prices. However, the world wheat prices could be affected by global business cycles, raising concerns that it could affect inflation perceptions through some real output channel as well, which would violate the exclusion restriction of instrumental variables. I will add unemployment and real GDP growth statistics that I have obtained from INSEE to control for these concerns.

I conduct a two-stage least-squares regression. Unemployment and real GDP growth are denoted by $M_t$. The regression specification is

$$E_{i,t} \pi_t = \alpha + \beta \pi_t + \gamma \hat{\pi}_{t}^{Bread} + M_t + \epsilon_{i,t}$$

(5)

$$\hat{\pi}_{t}^{Bread} = \alpha_1 + \beta_1 \pi_{t-6}^{Wheat}$$

(6)

Table 4 display the results of this instrumental variable regression. Columns (1) and (2) both run the instrumental variable approach. Bread inflation remains a significant determinant of French inflation inflation perceptions, even when I use world wheat prices as an instrument to partial out

\[\text{37}^\text{Perhaps transportation costs affect both bread and households beliefs through some other channel.}\]

\[\text{38}^\text{I specifically use the prices of hard red winter wheat, a variety of wheat.}\]
any confounding factors. The F-statistic for the first stage of the regression is large, with a p-value below 0.001. The coefficient on bread rises from the 0.66 in the ordinary least-squares regression to 0.85 in the instrumental variable approach. French households are actually reacting to changes in the bread inflation rate as opposed to some confounding factor; this relationship is more than a simple correlation.

The increase of the bread coefficient could reflect measurement error in the bread inflation rate. If aggregated bread prices are measured with an error when reported to the HICP dataset, then using an instrument of world commodities prices would help eliminate the error and gives more accurate regression results. The coefficient on bread in the instrumental variable analysis should be seen as more accurate than the result in the ordinary least squares analysis.

3.3.2 Serial Correlation

My analysis uses time series, which carry their own statistical issues, namely spurious correlation and serial correlation. For example, recent work suggests that inflation in developed countries follows a random-walk process. If I run an Augmented Dickey-Fuller test, as reported in Table 5, I cannot reject the null hypothesis that the overall French inflation rate is a random walk. Moreover, bread inflation also fails to reject the random walk null hypothesis. If one random walk time series is regressed on another, uncorrelated random walk time series, then regressing the former on the latter often results in erroneously significant regression coefficients.

Inflation perceptions may also suffer from serial correlation due to the way the survey data is collected. If one household faces a given year-on-year inflation rate $\pi_{2012m1}$, then a household in the next month views the year-on-year variable $\pi_{2012m2}$. These two inflation rates overlap for 11 months, specifically February 2011 until December 2012, so there is significant correlation between the two. If households view the month-on-month inflation rate with an error, then any perception error will show up over multiple months, which means the assumption of uncorrelated perception errors in the ordinary least squares regression is incorrect.

I resolve the serial correlation concern by differencing my regression. I use the revolving panel structure of the CAMME survey to study how households update their perceptions between their first and third wave responses. I regress this perceptions update on changes in the year-on-year inflation rate for overall goods and bread. In brief, if the inflation rate of bread rises over the course of two months, do we see households raising their inflation perceptions as well? If my previous results were driven by a serial or spurious correlation, then bread inflation should become statistically insignificant. The regression specification is
 Columns (3) and (4) of Table 4 report the regression results. Bread prices remain a statistically significant determinant of perceptions: an increase in the bread inflation rate by 1 percentage point results in perceptions rising by a bit less than 0.5 percentage points. Changes in the bread inflation rate appear to be a larger driver than the overall inflation rate of perceptions, but I cannot reject that null hypothesis that overall inflation and bread inflation have the same effect. Incorporating the macroeconomic controls slightly reduces the effect of overall and bread inflation, but does not eliminate it. Bread remains weakly more important than overall inflation, supporting the frequency bias hypothesis.

The explanatory power of my analysis is low, because there is considerable heterogeneity in how households update their perceptions. The R-squared values for these regressions are low, at an abysmal 0.005. Households may either form beliefs form their own idiosyncratic experiences (as in Madeira and Zafar (2015); Mankiw and Reis (2002)) or have other private sources of information I cannot capture. If I run the regression on levels, my R-squared is normally around 0.05, an order of magnitude larger.

In summary, I have established the presence of a frequency bias fixated on bread in this section. I have obtained a reasonable parameter estimate of 0.861 for the effect of actual bread inflation on inflation perceptions, which I can use to evaluate how the frequency bias distorts saving and spending decisions for households. I have also conducted several regressions to check robustness, which allow me to establish some causality from bread inflation to inflation perceptions. Next, I examine the level bias.

4 Level Bias

This section begins by measuring the size of the level bias. I evaluate how much low-income households overestimate the current inflation rate relative to high-income households. I then study three possible causes for the level bias and the income difference: different consumption baskets, asymmetric loss, and a skew in perception beliefs.
4.1 Level Bias Size

Figure 1 displays an obvious vertical gap between mean inflation perceptions and actual inflation. The purpose of this section is to estimate the size of this gap and study how it varies by income. One possible scenario is that low-income households dramatically overestimate past inflation, but that high-income households have an accurate understanding of the average inflation rate, and thus no vertical bias. Within the broader economy this would discourage low-income households from saving. However, as low-income households had barely any savings to begin with, a level bias that only affects low-income households would not do much to reduce aggregate savings (although it may hurt a substantial fraction of households).

Figure 5 depicts the average inflation perception for high-income households compared to low-income households over time, plotted against the actual overall inflation rate. The difference between low-income and high-income households is not constant. Prior to the financial crisis high-income and low-income households behave mostly the same. After 2009 they begin to diverge, especially after 2014. High-income inflation perceptions are falling and becoming more accurate, almost as if high-income households are gradually learning the true parameters of the economy. Unfortunately, my sample frame is not long enough to capture this low-frequency adaptation for high-income households.

I am also interested in how sensitive the different income classes are to the actual inflation rate. As noted in the preliminary section, the average household updates their inflation perception at a roughly one-for-one basis with changes in the actual inflation rate. A priori, I expect that high-income households are more aware of innovations to the past inflation, and are adapt more quickly to any shifts in the inflation rate. Low income households would be more insensitive to any shifts in the inflation rate.

In the literature, the level bias can be attributed to demographics beyond income status. Low-education respondents, women, and elderly respondents may consistently over-perceive current inflation or overpredict future inflation. I incorporate these demographic variables as well, to disentangle how much of the level bias can be attributable to solely income. The regression specification is:

39One plausible story is that the introduction of the Euro, which is controlled by the European Central Bank, operated as a regime shock or a sudden change to the monetary policy parameters. Since 2002, French households have had relearn the stochastic process of inflation under this new ECB regime.

40see Blanchflower and MacCoille (2009); Bruine de Bruin et al. (2010); Bryan and Venkatu (2001a); Johannsen (2014); Jonung and Laidler (1988)
\[ E_{i,t} \pi_t = \alpha_{\text{Income}} + \beta_{\text{Income}} \pi_t + D_i + \epsilon_{i,t} \]  

(8)

where the subscript \textit{Income} denotes the income class of individual \( i \), and \( D_i \) contains non-income demographic controls. The regression table below presents the results.

Column (1) of Table 6 reports the baseline regression without differentiation by income. Column (2) reports the regression specification above controlling for income without any additional demographic controls. Low-income households over-perceive the current inflation rate by roughly 1.2 percentage points more compared to high-income households. These differences are statistically significant according to the reported t-tests. High-income households are more sensitive than low-income households to past inflation. However, low-income households do respond to the current inflation rate. This parallels the graph above, where we can see that low-income households react to changes in the inflation rate, particularly when inflation spikes in 2008.

Column (3) shows the regression results if I control for the other demographic variables. The intercept for low-income households drops to 2.5%, shrinking the income differential. The reason for this is that low-education households and low-income households in general have a larger bias. However, households that are low-income tend to be low-education as well: the explanatory variables are correlated. Column (2) therefore reports the effect of income status on perceptions, combined with how education affects perceptions through the correlation with income.\(^{41}\)

4.2 Possible Explanations

In this subsection I will examine three possible explanations for the level bias. First, I will examine whether income differences in the level bias are due to different consumption baskets for high income households compared to low-income. Second, I will study whether households could be reacting more strongly to price increases than decreases. Third, I will examine the median perceptions.

4.2.1 Consumption Baskets

Demographic groups may actually face different inflation rates. Low-income households might purchase goods from noncompetitive stores with high markups and volatile costs relative to high-income households. If this hypothesis is true, then the level bias would not reflect different house-
hold biases, but rather the reality of different inflation. Arion et al. (2018) at INSEE investigate this hypothesis by constructing consumer price indices for different income deciles of households. They go about this by obtaining the consumption basket weights for different households using the Household Budget Survey (HBS), and then using the aggregated price data by consumption good type to construct two differently weighted consumer price indices. The INSEE researchers find that although there are differences in the inflation rates for a particular period, on average inflation rates are only 0.1 percentage points higher for the lowest income decile than the highest. I attempt to duplicate their approach and show the results in the first graph of Figure 6.\textsuperscript{42} The inflation rate for low-income households rarely deviate significantly from the inflation rate for high-income households.

The level bias could be driven by the dispersion of price changes for goods. If households overweight large price changes, then a rise in inflation dispersion could lead to a higher perceived inflation rate. The second graph of Figure 6 depicts the cross-good volatility of the low-income and high-income households,\textsuperscript{43} using the same weights derived from the HBS. Although the low-income households’ consumption basket displays a greater dispersion of inflation rates than high-income’s, the difference for most periods is negligible. Inflation dispersion cannot explain differences in the level bias.

### 4.2.2 Asymmetric Weighting Price Increases

Households are biased towards remember price increases more than price decreases. Psychologically, this would be an example of loss aversion, where the marginal disutility of a negative event is larger than the marginal utility of an equivalent positive event.\textsuperscript{44} Using the nationally aggregated data of pricing by consumption good category, I construct an extreme version of this loss-averse perceived inflation rate, where I define

\[
\pi_{loss} = \frac{1}{\sum_j 1[\pi_j > 0] \omega_j} \sum_j 1[\pi_j > 0] \omega_j \pi_j
\]  

(9)

\(\pi_{loss}\) is the inflation rate that households would obtain if they only considered prices that increased, and disregarded any prices that stayed the same or decreased. This is equivalent to saying

\textsuperscript{42}My expenditure data is coarser than the data used by the INSEE researchers. They have consumption data for households down to the 4-digit COICOP level, i.e. bread. I have prices at the 3-digit level, i.e. bread and cereals.

\textsuperscript{43}I define volatility across goods instead of across time. For example, if a household only purchases apples and bikes, and the year-on-year inflation rate for both goods is 2%, then the household’s cross-good inflation volatility is 0.

\textsuperscript{44}See Kahneman and Tversky (1979) for a description of loss aversion.
that price increases are infinitely more important to households than price decreases, in terms of relative weights. Figure 7 compares $\pi^{loss}$ to the actual inflation rate and the overall mean inflation perception. As we can see, although the loss-averse inflation rate is higher on average, it does not completely close the level bias.

### 4.2.3 Median Beliefs

The level bias could be driven by the distribution of the responses. Figure 8 plots the mean and median inflation perceptions over the sample period. The level bias is only half as large when evaluating median beliefs. To present a richer picture, I display two histograms in Figure 9 of responses.\footnote{I depict the responses for March 2010 and February 2015. There is no particular reason that I pick these two dates.} The distribution of inflation perceptions has a clear skew, with a thick, long right tail and a cluster at 0%.

In summary, households consistently overperceive the inflation over the past twelve months. This level bias varies by the income status of the respondent. Low-income households overperceive past inflation to a greater degree than high-income households, even when controlling for other demographic characteristics. The average size of this level bias is 2.45 for households overall. For low-income households the level bias is 3.15, while for high-income households it is 1.99. I use these results to estimate the welfare implications in following sections.

### 5 Model

In this section, I build a partial equilibrium household that can incorporate biased inflation perceptions. The purpose of the model is to evaluate the welfare loss of the two perceptions biases. The model answers three questions. First, do the frequency bias and the level bias have significant negative effects on households? Second, which is worse? Third, how sensitive are my results to different calibrations?

The model’s key feature is that households do not know the current inflation rate due to their perceptions biases. Households also use their biased perceptions to forecast future inflation, which results in non-rational inflation expectations. Biased households choose their level of savings as if their perception of inflation are correct, but are subject to the actual inflation rate each period.
This section begins by describing the basic components of the household problem. I then set up the households’ nominal and real budget constraints. A necessary component of this is introducing how biased beliefs about inflation affect the household’s problem. Next, I briefly describe how the inflation and income evolve. I then write down the recursive formulation for households with rational expectations and for households with biased inflation perceptions. I conclude by discussing how households with the frequency bias or the level bias actually view inflation.

5.1 Model Setup

I use a partial equilibrium model with a continuum of infinitely-lived households. Households receive an idiosyncratic stochastic endowment $Y$, which can be viewed as a risky labor real income. Households maximize expected utility, which is time-separable and discounted at a constant rate $\beta$. The households have a per-period utility function

$$U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$$

Households purchase consumption goods from two sectors, $C_{1,t}$ and $C_{2,t}$. Households aggregate consumption goods using a Cobb-Douglas function to obtain total consumption,

$$C_t = C_{1,t}^{\gamma} C_{2,t}^{1-\gamma}$$

The household problem can be viewed as a two-step problem. First, households decide how much to spend on consumption. Then households allocate consumption across the two sectors. Prices for the consumption goods in both sectors are exogenous to households. The household consumption allocation problem results in a general price index, $P_t = P_{1,t}^{\gamma} P_{2,t}^{1-\gamma}$, which is the cost of purchasing one unit of composite consumption $C_t$. The inflation rate between period $t - 1$ and period $t$ is

$$\pi = \pi_{1,t}^{\gamma} \pi_{2,t}^{1-\gamma}$$

5.2 Asset Decision

At the start of each period households know their nominal income and their nominal assets from the previous period perfectly. Households choose how much of their nominal wealth to allocate towards purchasing consumption goods and how much to save. Households can purchase nominal

\footnote{This two sector approach is used to model the frequency bias. In the context of my paper, one can think of them as the “bread” sector and the “not-bread” sector.}
bonds $A_t^N$ each period, which pay a constant exogenous nominal interest rate $(1 + i)$ at the start of the next period. I assume that agents cannot issue their own nominal bonds, so that they are subject to a no-borrowing constraint. Let $C_t^N$ denote the household’s total expenditure on consumption goods and let $Y_t^N$ denote the household’s nominal idiosyncratic income. The household’s budget constraint in nominal terms at time $t$ is

$$C_t^N + A_t^N = (1 + i)A_{t-1}^N + Y_t^N$$
$$A_t^N \geq 0$$

(10)

Households obtain utility from their real consumption goods $C_t$, but their budget constraint and choice variables are both nominal. Households choose how much to spend on consumption goods $C_t^N$, even though they may not know $P_t$ accurately. Using the general price index $P_t$, I rewrite the nominal budget constraint in real terms. I define the variable $A_t = \frac{A_t^N}{P_t}$ as the household’s purchase of price deflated bonds. The real return to price deflated bonds that mature in period $t$ is $(1 + \frac{i}{\pi_t})$. The real consumption of households is $C_t = \frac{C_t^N}{P_t}$

The expectations operator for households is $E_t^*$. I assume that all households have the same biases and that there is no heterogeneity in beliefs. For example, the term $E_t^*[\pi_t]$ denotes the households’ belief about the current inflation rate at time $t$. If households have rational expectations,$^{47}$ then $E_t^*$ corresponds to the objective expectations operator $E_t$. I postpone a description of how the perceptions biases affect $E_t^*$ until below.

A household at time $t$ perceives its real budget constraint as

$$C_t + A_t = \frac{(1 + i)A_{t-1}}{E_t^*[\pi_t]} + Y_t$$
$$A_t \geq 0$$

(11)

This budget constraint depends on the perception of inflation $E_t^*[\pi_t]$. It may not be the same as the households’ actual budget constraint. For example, a household believes that prices have fallen by half over the past period, when in fact the inflation rate has been 0%. In this case, $E_t^*[\pi_t] = 0.5$, but $\pi_t = 1$. If the household had saved a large amount of assets $A_{t-1}$, then it would incorrectly

$^{47}$Here, I use rational expectations to refer to households that correctly perceive the current inflation rates and forecast future inflation using rational expectations.
believe that its financial wealth could purchase more real consumption goods $C_t$ than it actually could. The actual real budget constraint for households each period is

$$C_t + A_t = \frac{(1+i)A_{t-1}}{\pi_t} + Y_t$$

$$A_t \geq 0$$

I refer the households perceived budget constraint as its “subjective” budget constraint. In contrast, I refer to the actual real budget constraint as its “objective” budget constraint. The objective budget constraint is the budget constraint that an outside econometrician or a household with rational expectations would observe. This terminology will carry over to the households’ policy and value functions. Note that when households purchase consumption goods they are governed by the objective price index, which means that they receive utility from their objective consumption $C_t = \tilde{C}_t$ rather than their subjective consumption $\tilde{C}_t = \tilde{C}_t$. For the remainder of the paper, I will denote any subjective variables with a tilde.

The derivations above depend on several subtle assumptions about households, particularly regarding how they view uncertainty. Appendix D contains a more complete description of how to derive the subjective real budget constraint (11) from the nominal budget constraint (10).

Let the set $S = (\pi_1, \pi_2, Y)$ denote the exogenous states that affect the economy, where $(\pi_1, \pi_2, Y)$ follow a stochastic process to be described later. Then from the household’s perspective and beliefs $E^*_t$, the recursive formulation of its problem is

$$\tilde{V}(S,A) = \max_{A', C} u(C) + \beta E[\tilde{V}(S', A') | S]$$

$$s.t. C + A' = \frac{(1+i)}{E^*[\pi]} A + Y$$

$\tilde{V}(S,A)$ denotes the subjective value function. This is what the households believe their value function is in state $S$ with price-deflated bonds $A$, according to their system of expectations $E^*$. This is the value function that the households would obtain if their beliefs $E^*$ were correct.

### 5.3 Inflation and Income Processes

In this subsection I describe how the inflation rates and the idiosyncratic income processes objectively change. The aggregate inflation rate is
\[ \pi_t = \pi_{1,t}^{\gamma} \pi_{2,t}^{1-\gamma} = \frac{P_{1,t}^{\gamma} P_{2,t}^{1-\gamma}}{P_{1,t-1}^{\gamma} P_{2,t-1}^{1-\gamma}} \]  

where \( \pi_{i,t} \) is the inflation rate for goods sector \( i \) in period \( t \). \( \pi_1, \pi_2 \) follow a joint vector autoregressive process with one lag. Let \( \vec{\pi} = (\pi_1, \pi_2) \). Then

\[ \vec{\pi}_t = \alpha + A \vec{\pi}_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, V) \]  

The variance-covariance matrix \( V \) is not necessarily diagonal. I rewrite the time series process for inflation rates as terms of deviations from means

\[ (\vec{\pi}_t - \mu) = A(\vec{\pi}_{t-1} - \mu) + \varepsilon_t \]  

I assume that households are aware of the vector autoregressive structure of the inflation rates and use it to make forecasts of future inflation. For example, if a household believes that the inflation rates for the two sectors at time \( t \) is \( E^*_i[\vec{\pi}_t] \), then its forecast for the two inflation rates for time \( t+1 \) is

\[ (E^*_i[\vec{\pi}_{t+1}] - \mu) = A(E^*_i[\vec{\pi}_t] - \mu) \]  

The idiosyncratic income process is straightforward in contrast. The logarithm of households’ labor income follows an auto-regressive process with one lag.

\[ \log(Y_{i,t}) = \rho_Y \log(Y_{i,t-1}) + \varepsilon_{i,t} \]  

The exogenous state variables for household \( i \) at time \( t \) are \( S = (\pi_1, \pi_2, Y) \).

5.4 Recursive Formulation

I write the recursive formulation households’ problem under two specifications. First, I describe the recursive problem if the households have rational expectations and accurate inflation perceptions. I then describe the problem under a generic inflation perception bias \( E^*_i \).

5.4.1 Rational Expectations Setup

Consider the households’ problem with rational expectations. In a given period, agents know the true overall inflation rate from the previous period and have rational expectations about future inflation. All of the households subjective variables, i.e. their planned real consumption, are the same as the objective variables. Agents know the purchasing power of their nominal wealth each
The households’ problem is

\[ V^{RE}(S, A) = \max_{A', C} u(C) + \beta E[V^{RE}(S', A')|S] \]

\[ s.t. C + A' = Y + \frac{(1 + i)}{\pi} A, A' \geq 0 \]

\[ \pi = \gamma_1 \pi_1 - \gamma_2 \]

Inflation only enters the households’ optimization problem to the extent that it affects the expected ex-ante real return to savings next period, \( E_t \left[ \frac{(1+i)}{\pi_{t+1}} \right] \). I define the policy function for households’ with rational expectations as their optimal level of savings,

\[ g_{RE}(S, A) \equiv \arg\max_{A'} u(C) + \beta E[V^{RE}(S', A')|S] \]

such that the budget constraint holds.

5.4.2 Biased Belief Setup

Households with biased perceptions optimize a misspecified model of the economy. The households do not realize that they have incorrect perceptions about inflation, so they act as if their beliefs are correct.\(^{48}\) For a generic belief operator \( E^* \),\(^{49}\) the biased households’ subjective problem is

\[ \tilde{V}(S, A) = \max_{\tilde{A}', \tilde{C}} u(\tilde{C}) + \beta E^*[\tilde{V}(S', \tilde{A}')|S] \]

\[ s.t. \tilde{C} + \tilde{A}' = Y + \frac{(1+i)}{E^*[\pi]} A, \tilde{A}' \geq 0 \]

The households planned consumption \( \tilde{C} \) may not equal the households actual consumption, \( C \). If a household underestimates the current inflation rate, then the household believe that it will receive \( \tilde{C} \) consumption goods, but in fact objectively receive fewer goods. Its assessment of its value function is also subjective. For instance, a household that erroneously believes that the inflation rate is 10% each period will believe that its value function is much lower than a value function for

\(^{48}\)If households were aware that they had biased beliefs, then presumably they could correct for the biases and return to rational expectations.

\(^{49}\)I discuss the frequency bias and level bias in the next subsection.
a households with rational expectations.

The biased households solve their subjective optimization problem to obtain a policy function for optimal bonds savings $A'$. The policy function is

$$\tilde{g}(S,A) = \arg\max_{\tilde{A}'} u(\tilde{C}) + \beta E^*[\tilde{V}(S',\tilde{A}')|S], \text{ s.t. B.C. hold} \tag{21}$$

I will now describe how to obtain the objective consumption of a biased household. Recall that households choose their nominal expenditures. Once a household decides how to allocate its nominal wealth between consumption goods $C^N$ and $A^N$, it goes to the consumption market and purchase goods using the true, objective price level $P$. If the household underestimated the price level, then it receives fewer consumption goods then it planned and $C < \tilde{C}$. In general, the expression relating subjective, planned consumption to objective, actual consumption is

$$C = \frac{E^*[\pi]}{\pi} \tilde{C} \tag{22}$$

This intuition also applies towards purchases of price deflated bonds $A$. If a household’s inflation perceptions are biased, then $A' \neq \tilde{A}'$. The relationship between the two is

$$A' = \frac{E^*[\pi]}{\pi} \tilde{A}'$$

In terms of the biased household’s policy function, the relationship between the objective policy function and the subjective policy function is

$$g(S,A) = \frac{E^*[\pi]}{\pi} \tilde{g}(S,A) \tag{23}$$

The purpose of this formulation is to obtain the objective and subjective policy functions $g(S,A)$ and $\tilde{g}(S,A)$, respectively.

### 5.5 Perception Biases

This framework is general enough to allow comparisons of different belief systems, specifically rational expectations, the frequency bias, and the level bias. The subsections below discuss the setup of the perceptions biases.

#### 5.5.1 Frequency Bias Beliefs

Households combine the two types of consumption goods with the function $C_t = C_{1,t}^\gamma C_{2,t}^{1-\gamma}$. The actual inflation rate from this is $\pi_t = \pi_{1,t}^\gamma \pi_{2,t}^{1-\gamma}$. Let sector 1 be the sector that household fixate on
with the frequency bias, so that $\gamma$ is objectively small. If households have rational expectations, then they pay little attention to the inflation rate $\pi_1$ when perceiving or forecasting inflation.

I assume that household suffering from the frequency bias correctly measure $\pi_1$ and $\pi_2$. However, they place too much weight on the first sector and mis-aggregate these inflation rates according to the process

$$E_i^{FR}[\pi_t] = \bar{\pi}_t = \pi_1^{\gamma}, \pi_2^{1-\gamma}, \bar{\gamma} > \gamma$$

(24)

If inflation in the frequency biased sector is higher (lower), i.e. $\pi_1 > \pi_2 (\pi_1 < \pi_2)$, then $\bar{\pi} > \pi (\bar{\pi} < \pi)$. Although the frequency bias distorts the belief of the inflation rate between periods, households still solve their intra-period consumption allocation sub-problem optimally. \[50\]

Let $\bar{V}^{FR}(S,A)$ be the subjective value function for households with the frequency bias, i.e. the value function they would have if their beliefs were correct. \[51\]

Households with the frequency bias choose an optimal savings policy to solve

$$\bar{g}^{FR}(S,A) = \arg\max_A u(\bar{C}) + \beta E^{FR}[\bar{V}^{FR}(S',\bar{A}')|S]$$

\[
\bar{C} + \bar{A}' = Y + \frac{(1+i)\bar{A}}{\bar{\pi}}, \bar{A}' \geq 0
\]

(25)

5.5.2 Level Bias Beliefs

Households suffering from the level bias mis-perceive the inflation rates of both sectors. They both overestimate the current inflation rate by some constant $c$ and believe that the average, long-run inflation rate $\mu$ is larger than it is by $c$. The households understand the other parameters of the vector autoregressive process governing inflation. Households with the level bias but not the frequency bias aggregate the inflation rates of the two sectors using the correct parameter $\gamma$. They believe that the current inflation rates are

$$E_i^{LB}[\pi_{i,t}] = \pi_{i,t} + c$$

(26)

The state variable $S = (\pi_1, \pi_2, Y)$ is defined the same as in rational expectations or the frequency bias and follows the same transition dynamics. Households with the level bias choose a subjectively optimal savings policy to solve

\[50\]Households suffering from the frequency bias do not consume vast quantities of bread because they erroneously believe that it’s a huge component of their consumption basket.

\[51\]V^{FR} is a special case of the value function $V^*$ defined above.
\[ \bar{g}^{LB}(S,A) = \arg\max_{\tilde{A}} u(\tilde{C}) + \beta E^{LB} [\tilde{\gamma}^{LB}(S', \tilde{A}) | S] \]

\[ \tilde{C} + \tilde{A}' = Y + \frac{(1+i)A}{E^{LB}[\pi]}, \tilde{A}' \geq 0 \]

6 Model Parameterization

6.1 Preferences

The model period is quarterly. As is standard in the risk-sharing literature, I set \( \sigma = 2 \). I set \( \beta = 0.96 \), which I obtain from Wong (2016). I set \((1+i) = 1.01\) to match the average bank deposit rates for households over the 2007-2017 time period according to Banque Du France. The Household Budget Survey suggests that the typical French household spends about 0.4% of their income on bread, so I set \( \gamma = 0.01 \).

The idiosyncratic labor income follows an AR(1) process \( Y_t = \rho Y_{t-1} + \epsilon_{Y,t}, \epsilon_{Y,t} \sim N(0, \sigma^2_Y) \). I set the \( \sigma_Y = 0.34 \) and \( \rho = 0.96 \), which I take from Rozsypal and Schlafmann (2019).

6.2 Inflation and Biases

I assume that the idiosyncratic labor income is uncorrelated with the inflation rates \( \pi_1 \) and \( \pi_2 \). I estimate the inflation process described in subsection 4.2 using quarterly inflation data from France for bread inflation \( \pi^{Bread} \) and overall inflation \( \pi \) at a quarterly frequency. From this, I obtain the correlation of the shocks to the inflation rates.

I derive the parameters for the frequency bias and level bias from my empirical work above. I set \( \gamma = 0.676 \), which is the size of the coefficient of bread inflation relative overall inflation in my instrumental variable analysis. I set \( c = 2.5 \), which is the level bias I measure in the regressions.

7 Results

Using the model setup of section 5 and the parameterization of section 6, I find the policy functions \( g(S,A) \), \( g^{FR}(S,A) \), and \( g^{LB}(S,A) \). In this next subsection, I discuss how to derive the value functions of following these policies, and talk about estimating the welfare loss of following an

---

52I do not have access to French household wealth data, so I take the income parameters from elsewhere in the literature.
objectively suboptimal policy. I present the main welfare results for the paper. I conclude by presenting three extensions. First, I weaken the correlation between bread and overall inflation. Second, I study how households behave if they have perfect inflation perceptions but biased expectations. Third, I rerun the level bias for low-income and high-income households separately.

7.1 Welfare Loss Calculation

Households with perception biases believe that their value function for a given state is \( \tilde{V}(S, A) \), as depicted in equation (13). However, the objective value function for the households is actually

\[
V(S, A) = u\left(\frac{(1+i)A}{\pi} + Y - \frac{E[\pi]}{\pi}\tilde{g}(S, A)\right) + \beta E[V^*(S', \frac{E[\pi]}{\pi}\tilde{g}(S, A))|S]
\]  

(28)

From the perspective of an outside observer or someone with rational expectations, these biased households are implementing objectively suboptimal saving policies \( E[\pi] \tilde{g}(S, A) \) instead of \( g^{RE}(S, A) \). For any state and asset combination \((S, A)\),

\[
V^{RE}(S, A) > V(S, A)
\]

Because value functions and utilities are ordinal, I cannot simply divide one value function by another to obtain a reliable measure of welfare loss. I use the consumption equivalence method of Lucas (1987), which uses the CRRA utility function to measure what fraction of consumption households would pay each period to avoid business cycles. In my setup, this approach looks like

\[
E\sum_{t=0}^{\infty} \beta^t \frac{C^*_t^{1-\sigma}}{1-\sigma} = E\sum_{t=0}^{\infty} \beta^t \left(\frac{C^{RE}_t}{1-\lambda}\right)^{1-\sigma}
\]

where the consumption \( C^*_t \) on the left-hand side is the expected future discounted consumption under a perception bias and the consumption \( C^{RE}_t \) is the consumption under rational expectations. For a given combination \((S, A)\) of exogenous states and asset holdings, the fraction of consumption that households would be willing to pay to have rational expectations is

\[
\lambda = 1 - \left[\frac{V^*(S, A)}{V^{RE}(S, A)}\right]^{\frac{1}{\sigma}}
\]

(29)

where \( V^*(S, A) \) corresponds to the objective value function for the frequency bias or the level bias.

The stationary distribution of assets held by households depends on their inflation perceptions. In the long-run biased inflation perceptions cause households to accumulate the incorrect level of assets: if agents persistently overestimate the level of inflation, then in the long-run the stationary
distribution of asset holdings will be first-order stochastically dominated by the asset distribution under rational expectations. In order to facilitate a neat comparison, I will analyze the welfare loss at the rational expectations steady state. In brief, I ask “if agents wake up one day and transition from rational expectations to a bias (either frequency, level bias, or both), how much consumption would they be willing to sacrifice to avert this?”

The stationary distribution of assets under rational expectations can be calculated by setting an initial distribution of assets $A_0$, and then iterating forward using the policy rule $g(S,A)$ until convergence towards a distribution $\vec{A}_{SS}$. Because agents are subject to idiosyncratic income shocks each period, this distribution is nondegenerate.

In addition to welfare loss, I will estimate two additional metrics to provide intuition for the results. First, I sum up the distribution to find the total assets held in the economy. This analysis will allow me to study how these biases affect other macroeconomic variables besides households welfare. Second, I will find the mean squared error of using each of these two biased forecasting methods compares to rational expectations. This measure will indicate how strong the results of incorrect inflation expectations are.

7.2 Main Result

Table 6 displays the primary results of this paper. The top row displays the total assets held if agents possess rational expectations and the mean-squared error for using rational expectations.

The welfare loss for the frequency bias is relatively small, roughly 0.0007% of consumption. For comparison, Lucas (1987) estimate that the welfare loss attributable to business cycles is 0.001% of consumption. The magnitude of the welfare loss indicates that the frequency bias is not very damaging to households in the economy, and that overweighting bread can serve as a decent second-best approach. In contrast, the welfare loss of the level bias is an order of magnitude larger, almost 0.2% of consumption. The cause of this difference becomes apparent if we evaluate the amount of assets held. Agents with rational expectations and the frequency bias hold roughly the same level of assets in the steady state, despite the fact that they are subject to different subjective inflation stochastic processes. This is likely because the overall volatility of the inflation rate is similar for both types of agents, and they both seek to hedge against the same labor income risk. Another reason that the welfare loss is so small is that the frequency bias is really damaging if the bread inflation rate substantially differs from the overall inflation rate. Because the correlation is
so high, this is unlikely to happen.

Households with the level bias save three-fourths as many assets as households with rational expectations or the frequency bias. A higher inflation rate reduces the ex-post real return to saving each period, because nominal bonds purchase less when they mature. This effect discourages savings ex-ante. Because agents with the level bias persistently estimate that inflation will be 2.5 percentage points higher than rational expectations agents, they save dramatically less. Due to the no-borrowing constraint and the risky labor income, households with the level bias still wish to smooth out consumption volatility and accumulate precautionary savings against a large, persistent, negative shock to labor income. Because they sub-optimally undersave, the welfare effects of the level bias are large.

The third column evaluates the effect of biased inflation perceptions on the forecasts of future inflation. Rational expectations have the most accurate forecasts, because they incorporate all available information efficiently. The frequency bias is slightly more inaccurate, because the forecast is inaccurate whenever the current inflation rate of bread deviates significantly from the overall inflation rate. This event rarely happens, because bread has a high correlation with overall inflation. The level bias results in the largest forecast error, because households consistently overpredict the inflation rate relative to rational expectations.

The welfare loss of for households with both biases is less than the sum of the welfare losses of each bias individually. Households suffering from the frequency bias save slightly more in the steady state than agents with rational expectations, which reduces the asset underaccumulation of the level bias slightly.

7.3 Extensions

I conduct three extensions of the model in order to find how sensitive the results are to the calibration. First, I rerun the frequency bias under different calibrations of the bread inflation rate, either by making it less correlated or more volatile. Second, I examine a version of the model where households have accurate inflation perceptions, but biased inflation expectations. Third, I examine how the welfare effects of the level bias vary by income.

Rather than derive the analytical expressions for the mean-squared error, I run a Monte Carlo simulation of each expectations method assuming that inflation does follow the mixed VAR(1) process described in the Model section.
7.3.1 Frequency Bias Extension

One reason why the welfare loss for the frequency bias could be small is that bread is an excellent signal for the overall inflation rate in France. Not only is the correlation between bread and the overall inflation rate high, the standard deviation of bread inflation is also quite small. As a result, the probability that bread inflation will be substantially higher than the overall inflation rate is low. I recalibrate my model with the same vector autoregressive process for the two inflation rates, but I impose a negative correlation between the reduced form shocks for bread and the overall inflation rate.

A second exercise is to consider another example of the frequency bias in the real world that is more dramatic and damaging. In the United States gasoline prices affect people’s beliefs about the future inflation rate. Unlike bread prices in France, gasoline prices are extremely volatile and only weakly correlated with the overall inflation rate in the U.S., as Figure 10 shows. I run a vector auto-regression on the U.S. gasoline inflation rate and the U.S. overall inflation rate and insert that calibration into my model.

I use the Kumar et al. (2015) to obtain estimates for the size of the frequency bias on gasoline. I assume that gasoline is only 5% of a household’s consumption basket, but they perceive it to be much more, about 20% of their budget. I use these estimates to set \( \gamma_{US} = 0.05 \) and \( \tilde{\gamma}_{US} = 0.2 \), where sector 1 in my model for the US represents gasoline.

Table 7 below displays the results for all of my frequency bias counterfactuals, compared to rational expectations and the baseline frequency bias. For each of these exercises, I am also supposing that the economy behaves differently in real life, and that households with rational expectations recognize this. Flipping the correlation between bread and overall inflation triples the welfare loss, and slightly reduces the assets held. However, compared to the level bias, the frequency bias is still an order of magnitude less important. This is surprising, because in this setup bread is not just a bad signal for the overall inflation rate. A bad signal would have a correlation of 0. Instead, bread is such a terrible signal that it is anti-correlated and agents are still tracking it. If even a negatively correlated bread sector does not produce substantial welfare losses, then this suggests that the frequency bias is actually unimportant.

Binder (2018) suggests that despite the appearance of gasoline overweighting, U.S. households actually suffer from no frequency bias and use the negative serial correlation of gas inflation in their beliefs. For the exercise with U.S. households, I compare U.S. households suffering from the frequency bias to U.S. households with rational expectations. At no point am I comparing French households to American households. This does mean that the comparisons of “assets held” are weaker.
The frequency bias in with gasoline in the U.S. generates a much larger welfare loss than the frequency bias fixated on bread in France. I believe that two factors drive this result. First, the average gasoline inflation rate is higher than the average overall inflation rate in the United States. Households that overweight gasoline will, on average, overestimate the current inflation rate. Second, the U.S. gasoline inflation rate is much more volatile than bread in France, which means that it is more likely that the gasoline inflation rate will significantly diverge from the overall inflation rate, even if gasoline is correlated with the overall inflation rate. Informally, I find that if I increase the weight that households with the frequency bias put on gasoline to $\tilde{\gamma}_{US} = 0.5$, the magnitude of the welfare loss is similar to that associated with the French level bias.

### 7.3.2 Effect of Perceptions Bias

Biased inflation perceptions affect savings through two channels: a misperception of the real value of the households’ wealth and by biased expectations affecting the expected return to savings. To disentangle these effects, I set up a version of the model where households have perfect inflation perceptions in the current period, but have biased expectations going forward. Households may misjudge the real value of their wealth, even if they know their nominal wealth.

Households with the frequency bias now obtain their subjective policy function by solving:

$$
\hat{g}^{FR}(S, A) = \arg\max_{A'} u(Y + \frac{(1 + i)A}{\pi} - A') + \beta E^{FR}[\tilde{V}^{FR}(S', A') | S] \tag{30}
$$

For households with the level bias, they correctly assess the current inflation rate, but overestimate the inflation rate for the next period using the level bias. They obtain their subjective policy function by solving:

$$
\hat{g}^{LB}(S, A) = \arg\max_{A'} u(Y + \frac{(1 + i)A}{\pi} - A') + \beta E^{LB}[\tilde{V}^{LB}(S', A') | S] \tag{31}
$$

I take these subjective policy functions and plug them into the objective budget constraint as above. I obtain two objective value functions, $V^{FR}(S, A)$ and $V^{LB}(S, A)$, where households follow these biased rules. Using the same consumption equivalence method as above, I find how much consumption households would be willing to sacrifice to avoid these expectations-only biases. Table 9 displays the results.

Under biased inflation expectations, the frequency bias becomes even more insignificant. This result suggests that the frequency bias operates primarily through biased inflation perceptions and has limited effect on expectations. If households are forecasting future inflation expectation us-
ing a vector auto-regression, then any error in the current information set $E^t_\pi \pi_t$ has a diminished effect on expected inflation for the next period, depending on the persistence of inflation. For example, if the inflation rates follow a white noise process, then any errors in assessing the current inflation rate has no impact on inflation expectations. The frequency bias affects households primarily by making them misassess their real wealth, and either feel richer or poorer than they are.

In contrast, the welfare loss of the level bias grows if I eliminate the perceptions bias. If households consistently overperceive the inflation rate, then each period they believe that their real wealth is lower than it really is. If households cannot issue nominal bonds, then their level bias makes them believe that they are closer to their no-borrowing constraint than they actually are. As a result, households engage in some precautionary savings to avoid hitting the no-borrowing constraint. This precautionary savings partially offsets the biased belief that the subjective ex-ante real return to savings is low. Eliminating the level perceptions bias reduces the incentive to save, reduces total steady state savings, and therefore reduces welfare.

I solve the model for different magnitudes of the level bias and graph the results in Figure 11. I find that as Akerlof and Yellen (1985) predicts, a small level bias in the neighborhood of 0% generates negligible, second-order welfare losses, even though it generates a first-order reduction in assets held. As the level bias increases, households hold fewer and fewer assets in steady state, and the welfare loss grows. Regardless of the magnitude of the level bias, Figure 11 indicates that the biased perceptions partly offset the effect of the biased expectations by increasing savings and reducing the welfare loss.

7.3.3 Level Bias Extension

The level bias is more significant than the frequency bias in my model. However, unlike the frequency bias, the level bias is more heterogeneous by household. In the empirical work on the frequency bias, I found that high-income as well as low-income households overweight bread inflation. In contrast, low-income households have a worse level bias than the high-income households: they over-perceive past inflation a full percentage point more. This raises the question of whether the level bias matters, given that households with high income aren’t affected as much. If high-income households own most of the assets in the economy, then the macroeconomic implications of the level bias might be muted.\textsuperscript{56} In an extreme scenario, low-income households are consistently liquidity constrained and unable to increase borrowing. If they over-perceive the inflation rate, to the extent that their expected real-return to savings is lower, then they are unable

\textsuperscript{56}This line of thinking assumes that households income has a highly persistent component, which is supported in the literature.
to act on this belief by decreasing savings. Even though the level bias shows up in the household surveys, the bias has no effect on the total assets in the economy.

To study this effect, I rerun the model two different types of households, differing by their permanent income and the level bias. Let $y \equiv \log(Y)$, and let the permanent component of a household’s income be $\bar{y}_i$. I assume that households never transition from one permanent income status to the other, i.e. the permanent incomes are fixed. This simplifies the analysis by assuming that households have the same belief process over all horizons, and they do not transition from one form of level bias to another.\footnote{If the households’ permanent income can fluctuate between two states, then I can rewrite the problem with two value functions ($v_{low}$ and $v_{high}$), two policy functions, and solve the problem. However, in this framework low-income households are aware that they have a level bias and are aware that it will be weaker if they become high-income households, yet they don’t correct their level bias. If the setup was that high-income households had more information or better signals then this approach would be plausible.} The income process for households is:

$$y_{i,t} = \bar{y}_i + \rho Y y_{i,t-1} + \epsilon_{i,t}$$

(32)

I refer to the French Household Budget Survey to obtain the average income for households in the bottom and top quartile, which I use to pin down $\bar{y}_{low}$ and $\bar{y}_{high}$, the average incomes of low-income and high-income households respectively. In the model, both types of household have the same type of budget constraint, can purchase the same nominal bond, and are otherwise the same. Because the model is partial-equilibrium, the nominal interest rate is exogenous, and does not change to clear a market.

Table 8 reports the results of the welfare loss for low-income and high-income separately, and then for the aggregated economy. Because low-income households have a larger level bias, their welfare loss is higher and asset holdings lower than their high-income counterparts. In general, we would expect that the larger the level bias, the more significant the effect.

The last row reports the aggregate asset holdings of the income-decomposed model. The asset holdings are 3.260, which is larger than the baseline level bias result of 3.191. This makes sense, because the average level bias is 2.5, whereas the high-income level bias is 2.0. However, high-income households are so still biased enough that the aggregate asset holdings are much lower than in the rational expectations scenario. Even though decomposing the level bias reduces the aggregate effect, it does not eliminate it. Even high-income households are biased.
8 Conclusion

This paper has documented and evaluated two inflation perceptions biases, a frequency bias and a level bias. French households overweight bread prices when they consider inflation and consistently overestimate the inflation rate. Although the magnitude of these biases varies by household income, no demographic group has rational inflation perceptions. Simply by looking at the survey data, a researcher might conclude that the frequency bias and the level bias are both large and equally relevant for households.

In the second half of this paper I found that this statement is false. The level bias significantly reduces households’ savings incentives, which reduces their bond holdings and their ability to smooth out consumption. The welfare loss of the level bias is large. In contrast, despite the significance of the frequency bias in forming household perceptions, it has miniscule effects on household saving or welfare. The small welfare effect of the frequency bias is not due to the fact that bread is an adequate signal for overall inflation due to its high correlation. Weakening the correlation or increasing the volatility of bread inflation increases welfare loss, but never to the same extent as the level bias.

Going forward, this paper suggests that there are two types of biases in belief formation, reflecting the level bias and frequency bias. First order biases affect beliefs are about the averages of variables: the average inflation rate, the average income, etc. These beliefs cause significant distortions in the decisions of agents and result in large welfare losses. Second order beliefs affect how well the household reacts to changes in the variable. In the context of imperfect information, these relate to the noise of the signals that households receive. Second order beliefs may be able to explain a significant portion of household beliefs, but they ultimately have a marginal effect on household welfare. In summary, even though a bias may appear significant in the survey data, it may be insignificant when it comes to household welfare.
References


9 Figures

Figure 1: Inflation Expectations and Perceptions

Notes: The figure depicts the mean inflation perception and the mean inflation expectation for households in the CAMME survey over time. The actual inflation rate is calculated as the year-on-year percentage change in the HICP. Households with inflation perceptions above 20% are dropped when calculating the mean beliefs.
Figure 2: Savings Response

Notes: This figure depicts the probability that a household agrees with the statement that “it is a good time to save,” conditional on their inflation perception. The figure only displays results for integer responses between 0% and 10%. Equation (3) in section 2 describes the setup of the logit regression. Other explanatory variables, including demographic variables and monthly date, are evaluated at their means.
Notes: This figure displays the actual overall inflation rate and the bread inflation rate. The overall inflation rate refers to the inflation rate over the entire consumption basket. These inflation rates are calculated using consumer price index data taken from HICP. Bread refers to the CP0113 category of goods. Inflation rates are calculated as the year-on-year percentage change of the price index. The inflation perception time series is the mean inflation perception for French households in the CAMME survey. Inflation perceptions above 20% are dropped when calculating the mean.
Notes: This figure displays inflation rate for bread in France and the inflation rate for world wheat prices. See the notes of Figure 3 for a description of how bread inflation is calculated. Wheat prices are defined as the price of Hard Red Winter Wheat on the Chicago Mercantile Exchange. Inflation rates are calculated as the year-on-year percentage change in the price index.
Figure 5: Level Bias by Income

Notes: This figure displays the mean inflation perception for low-income households and high-income households in the CAMME survey. Low-income households are defined as those reporting a monthly household income below the bottom quartile of the sample. High-income households are those reporting an income above the top quartile. The overall inflation rate is the year-on-year percentage change in the consumer price index.
Figure 6: Inflation by Income Quartile

Inflation Rate by Income Quartile

Within-Consumption Basket Volatility

Notes: This figure displays the inflation rate and inflation volatility by household income. All calculations use the same inflation rate data from the HICP. Consumption weights on different goods categories vary by household income. Inflation volatility is the cross-sectional volatility of inflation rates across consumption goods categories.
Figure 7: Asymmetric Inflation Rate

Notes: This figure displays the overall inflation rate for French consumers and a weighted, asymmetric inflation rate. The asymmetric inflation rate is the average inflation rate for all goods categories where prices increase, weighted by household consumption. Household consumption weights come from the HBS and the inflation rates of different goods categories come from the HICP.
Figure 8: Median Perceptions

Notes: The figure depicts the mean inflation perception and the median perception expectation for households in the CAMME survey at a monthly frequency. The actual inflation rate is calculated as the year-on-year percentage change in the HICP consumer price index. Inflation perceptions above 20% are dropped when calculating the mean and median.
Figure 9: Histogram of Responses

Notes: The figure depicts two histograms of perceived inflation responses from the CAMME survey for two dates, March 2010 and February 2015. The bins have a width of 2 percentage points and the endpoints are even integers. The density of all bins sum to 1 in each histogram. All observations larger than 20% are dropped.
Figure 10: U.S. Gasoline Inflation

Notes: This figure displays overall inflation rate in the United States and the inflation rate of gasoline. Both rates are defined as year-on-year percentage increases in the price indices. Inflation data is taken from the Bureau of Labor Statistics. The frequency of the statistics is quarterly.
Figure 11: Effect by Size of Level Bias

Notes: The left panel shows the welfare loss that households face under different magnitudes of the level bias. The right panel depicts the steady state asset holdings of households. The full model refers to the version where households have biased inflation perceptions and expectations. The expectations model refers to the model where households have perfect inflation perceptions, but biased inflation expectations. See the text for more details.
### Table 1: Inflation Expectations

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<td>23589</td>
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</tbody>
</table>

**Notes:** This table shows the relationship between inflation expectations and inflation perceptions on a household level. Columns (1), (2), (4), and (5) are run the full sample from 2004-2017. Column (6) is run on the subsample of households between 2004-2014 that I am able to link across survey waves. High income households refer to households reporting an income above the top quartile of the sample. Standard errors are in parentheses. Errors are clustered by the month of the survey. ***,**, and * denote the 1, 5 and 10 % significance levels, respectively. See the text and equation (1) for more details.
Table 2: Savings Responses

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good Time To Save</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation Perceptions</td>
<td>-0.014***</td>
<td>-0.014***</td>
<td>-0.023***</td>
<td>-0.018***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.440***</td>
<td>0.170***</td>
<td>0.213***</td>
<td>0.244***</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.026)</td>
<td>(0.031)</td>
<td>(0.065)</td>
</tr>
<tr>
<td>Demographic Controls</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Date Controls</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Economic Opinion</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Pseudo $R^2$</td>
<td>0.001</td>
<td>0.015</td>
<td>0.024</td>
<td>0.027</td>
</tr>
<tr>
<td>Observations</td>
<td>158381</td>
<td>138960</td>
<td>138960</td>
<td>138960</td>
</tr>
</tbody>
</table>

Notes: This table shows the logarithm of the odds-probability of a household stating that “now is a good time to save” conditional on inflation perceptions and other variables. Standard errors are in parentheses. Errors are clustered by date. ***, **, and * denote the 1, 5 and 10 % significance levels, respectively. See subsection 2.3 and equation (3) for more details.

Table 3: Frequency Bias Regressions

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall Inflation</td>
<td>1.271***</td>
<td>0.572***</td>
<td>0.553***</td>
<td>0.640***</td>
</tr>
<tr>
<td></td>
<td>(0.073)</td>
<td>(0.073)</td>
<td>(0.077)</td>
<td>(0.085)</td>
</tr>
<tr>
<td>Bread Inflation</td>
<td>0.659***</td>
<td>0.646***</td>
<td>0.699***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.046)</td>
<td>(0.053)</td>
<td></td>
</tr>
<tr>
<td>Food Inflation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.031</td>
<td>-0.023</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.041)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>2.601***</td>
<td>2.591***</td>
<td>2.599***</td>
<td>1.958***</td>
</tr>
<tr>
<td></td>
<td>(0.111)</td>
<td>(0.078)</td>
<td>(0.079)</td>
<td>(0.085)</td>
</tr>
<tr>
<td>Sample</td>
<td>Full</td>
<td>Full</td>
<td>Full</td>
<td>High Income</td>
</tr>
<tr>
<td>t-test: Bread Infl. =</td>
<td>0.437</td>
<td>0.408</td>
<td>0.094</td>
<td>0.640</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.061</td>
<td>0.074</td>
<td>0.074</td>
<td>0.098</td>
</tr>
<tr>
<td>Observations</td>
<td>162243</td>
<td>162243</td>
<td>162243</td>
<td>49944</td>
</tr>
</tbody>
</table>

Notes: This table shows the results of regressing household inflation perceptions on the overall inflation rate, the bread inflation rate, and other variables. The sample is French households in the CAMME survey from 2004-2017. High income households refers to households reporting an income above the top quartile of the sample. Standard errors are in parentheses. Errors are clustered by date. ***, **, and * denote the 1, 5 and 10 % significance levels, respectively. See the text for more details.
### Table 4: Endogeneity and Serial Correlation Regressions

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_t$</td>
<td>$E_t$</td>
<td>$E_t$</td>
<td>$\Delta E_t$</td>
</tr>
<tr>
<td>Bread Infl.</td>
<td>0.852***</td>
<td>0.861***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.126)</td>
<td>(0.158)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall Infl.</td>
<td>0.366*</td>
<td>0.413***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.145)</td>
<td>(0.095)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chg. in Infl.</td>
<td></td>
<td></td>
<td>0.405***</td>
<td>0.332***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.103)</td>
<td>(0.091)</td>
</tr>
<tr>
<td>Chg. in Bread Inf.</td>
<td></td>
<td></td>
<td>0.495***</td>
<td>0.418***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.087)</td>
<td>(0.100)</td>
</tr>
<tr>
<td>Constant</td>
<td>2.588***</td>
<td>1.784</td>
<td>-0.718***</td>
<td>-0.732***</td>
</tr>
<tr>
<td></td>
<td>(0.079)</td>
<td>(1.220)</td>
<td>(0.043)</td>
<td>(0.054)</td>
</tr>
<tr>
<td>Macroeconomic Controls</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Regression Type</td>
<td>IV</td>
<td>IV</td>
<td>OLS</td>
<td>OLS</td>
</tr>
<tr>
<td>First Stage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-statistic</td>
<td>31.32</td>
<td>23.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.073</td>
<td>0.075</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>Observation</td>
<td>162243</td>
<td>162243</td>
<td>29969</td>
<td>29969</td>
</tr>
</tbody>
</table>

**Notes:** Errors are clustered by date. Standard errors are in parentheses. ***,**, and * denote the 1, 5 and 10% significance levels, respectively. See section 3.3 for details.

### Table 5: Dickey-Fuller Test

<table>
<thead>
<tr>
<th>Inflation Rate</th>
<th>Frequency</th>
<th>Number of Obs</th>
<th>Test Statistic</th>
<th>1% Critical Value</th>
<th>5% Critical Value</th>
<th>10% Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bread</td>
<td>Monthly</td>
<td>204</td>
<td>-1.567</td>
<td>-3.475</td>
<td>-2.883</td>
<td>-2.573</td>
</tr>
<tr>
<td>Overall</td>
<td>Monthly</td>
<td>205</td>
<td>-2.178</td>
<td>-3.475</td>
<td>-2.883</td>
<td>-2.573</td>
</tr>
<tr>
<td>Bread</td>
<td>Quarterly</td>
<td>68</td>
<td>-1.812</td>
<td>-3.555</td>
<td>-2.916</td>
<td>-2.593</td>
</tr>
<tr>
<td>Overall</td>
<td>Quarterly</td>
<td>68</td>
<td>-2.282</td>
<td>-3.555</td>
<td>-2.916</td>
<td>-2.593</td>
</tr>
</tbody>
</table>

**Notes:** This table shows the results of running a Dickey-Fuller test with a trend and one lag on the overall inflation rate and bread inflation rate separately. The Dickey-Fuller test is run on a monthly frequency and a quarterly frequency for comparison. The critical values for the test are displayed in the right-most three columns. See section 3 for details.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>1.251***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low-Income</td>
<td>3.152***</td>
<td>3.994***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.092)</td>
<td></td>
</tr>
<tr>
<td>High-Income</td>
<td>1.988***</td>
<td>3.294***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.081)</td>
<td></td>
</tr>
<tr>
<td>Low-Income X Infl.</td>
<td>1.111***</td>
<td>1.057***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.030)</td>
<td></td>
</tr>
<tr>
<td>High-Income X Infl.</td>
<td>1.359***</td>
<td>1.212***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.024)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>2.452***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demographic Controls</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>T-test:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poor constant = Rich constant</td>
<td>0.001&gt;</td>
<td>0.001&gt;</td>
<td></td>
</tr>
<tr>
<td>Observation</td>
<td>80409</td>
<td>80409</td>
<td>70369</td>
</tr>
</tbody>
</table>

Notes: This table shows the results of regressing household inflation perceptions on the overall inflation rate. The regressions include fixed effects for income quartile and allow the coefficient on the overall inflation rate to vary by income. High income and low-income households refers to households reporting an income above the top quartile and below the bottom quartile of the sample, respectively. Standard errors are in parentheses. Errors are clustered by date. ***,**, and * denote the 1, 5 and 10 % significance levels, respectively. See section 4 for details.
Table 7: Perception Biases Welfare Results

<table>
<thead>
<tr>
<th></th>
<th>Welfare Loss</th>
<th>Assets Held</th>
<th>Mean-Squared Forecast Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational Expectations</td>
<td>3.869</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency Bias</td>
<td>0.0007%</td>
<td>3.872</td>
<td>0.0057</td>
</tr>
<tr>
<td>Level Bias</td>
<td>0.17%</td>
<td>3.191</td>
<td>0.0250</td>
</tr>
<tr>
<td>Both Biases</td>
<td>0.17%</td>
<td>2.193</td>
<td>0.0307</td>
</tr>
</tbody>
</table>

Notes: This table shows the welfare effects for households suffering from the frequency bias only, the level bias only, or both biases together. Assets held refers to the total assets held by households in the steady state. The mean-squared forecast error is the forecast error for households in the model that are suffering from none or some of the biases. See the text for more details.

Table 8: Frequency Bias Extensions

<table>
<thead>
<tr>
<th></th>
<th>Welfare Loss</th>
<th>Assets Held</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational Expectations</td>
<td>3.869</td>
<td></td>
</tr>
<tr>
<td>Baseline Frequency Bias</td>
<td>0.0007%</td>
<td>3.872</td>
</tr>
<tr>
<td>Level Bias</td>
<td>0.17%</td>
<td>3.191</td>
</tr>
<tr>
<td>Negative Correlation</td>
<td>0.0014%</td>
<td>3.855</td>
</tr>
<tr>
<td>U.S. Gasoline</td>
<td>0.015%</td>
<td>3.338</td>
</tr>
</tbody>
</table>

Notes: This table shows the welfare effects for households suffering from different calibrations of the frequency bias. The negative correlation row refers to the calibration where bread inflation has a negative correlation with the overall inflation rate. The U.S. gasoline row refers to the calibration where the good with the frequency bias has the volatility and correlation with overall inflation of gasoline in the United States. The first three rows are reprinted from table 7 to facilitate comparison. See the text for more details.

Table 9: Level Bias by Income

<table>
<thead>
<tr>
<th></th>
<th>Welfare Loss</th>
<th>Assets Held</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational Expectations</td>
<td>3.869</td>
<td></td>
</tr>
<tr>
<td>Unweighted Level Bias</td>
<td>0.17%</td>
<td>3.191</td>
</tr>
<tr>
<td>Low-Income Level Bias</td>
<td>0.25%</td>
<td>3.055</td>
</tr>
<tr>
<td>High-Income Level Bias</td>
<td>0.11%</td>
<td>3.316</td>
</tr>
<tr>
<td>Income-weighted Average</td>
<td>0.18%</td>
<td>3.260</td>
</tr>
</tbody>
</table>

Notes: This table shows the welfare effects for different calibrations of the level bias. The low-income level bias row refers to the welfare loss for households with the level bias of low-income households in the sample. The high-income level bias row refers to the welfare loss for households with the level bias of high-income households. The income-weighted average row combines these results: the welfare loss is the unweighted average of low-income and high-income households, while the assets held is the average weighted by household income. See the text for more details.
Table 10: Expectations Channel

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th></th>
<th>Expectations</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Welfare Loss</td>
<td>Assets Held</td>
<td>Welfare Loss</td>
<td>Assets Held</td>
</tr>
<tr>
<td>Rational Expectations</td>
<td>3.869</td>
<td>3.869</td>
<td>3.869</td>
<td></td>
</tr>
<tr>
<td>Frequency Bias</td>
<td>0.0007%</td>
<td>3.872</td>
<td>0.00007%</td>
<td>3.872</td>
</tr>
<tr>
<td>Level Bias</td>
<td>0.17%</td>
<td>3.191</td>
<td>0.196%</td>
<td>3.115</td>
</tr>
</tbody>
</table>

Notes: This table shows the welfare effects for households suffering from the frequency bias only or the level bias only in the baseline model and the model with only biased expectations. The baseline model refers to the model where households have biased inflation perceptions and biased inflation expectations. The expectations model refers to the model where households have accurate inflation perceptions and biased inflation expectations. See the text for more details.
A Survey Methodology

The CAMME survey questionnaire follows the guidelines set out by the EU Harmonized Consumer Survey. The wording of the survey is standardized to facilitate cross-country comparisons, although the method by which statistical institutes conduct the survey varies. The inflation perceptions and expectations questions are slightly more complicated than the other survey questions. For example, the interviewers ask households about perceived inflation as a two part question. First, the interviewers ask respondents if they believe that prices have gone up, stayed the same, or gone down. If the respondents give any answer other than stayed the same, the interviewers follow up by asking for a precise number for inflation perceptions. If the respondent states they believe that prices stayed the same, they are automatically coded as having an inflation perception of 0%, and the interviewer skips the quantitative question. The inflation expectations component follows a similar two part structure.

The relevant questions and possible answers from the EU Guidelines are displayed below:

**Inflation Perceptions:**

<table>
<thead>
<tr>
<th>Q5:</th>
<th>How do you think that consumer prices have developed over the last 12 months? They have…</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ +</td>
<td>risen a lot</td>
</tr>
<tr>
<td>+</td>
<td>risen moderately</td>
</tr>
<tr>
<td>=</td>
<td>risen slightly</td>
</tr>
<tr>
<td>−</td>
<td>stayed about the same</td>
</tr>
<tr>
<td>− −</td>
<td>fallen</td>
</tr>
<tr>
<td>N</td>
<td>don’t know.</td>
</tr>
</tbody>
</table>

| Q51: | If question 5 was answered by 1, 2, 3 or 5: By how many per cent do you think that consumer prices have gone up/down over the past 12 months? (Please give a single figure estimate). Consumer prices have increased by , % / decreased by , %. |

**Inflation Expectations:**
**Q6:** By comparison with the past 12 months, how do you expect that consumer prices will develop in the next 12 months? They will...
+ + increase more rapidly
+ increase at the same rate
= increase at a slower rate
− stay about the same
− − fall
N don’t know.

**Q61:** If question 6 was answered by 1, 2, 3 or 5: By how many per cent do you expect consumer prices to go up/down change in the next 12 months? (Please give a single figure estimate).
Consumer prices will increase by , % / decrease by , %.

**Saving:**

**Q10:** In view of the general economic situation, do you think that now is...?
+ + a very good moment to save
+ a fairly good moment to save
− not a good moment to save
− − a very bad moment to save
N don’t know.

**Economic Situation**

**Q4:** How do you expect the general economic situation in this country to develop over the next 12 months? It will...
+ + get a lot better
+ get a little better
= stay the same
− get a little worse
− − get a lot worse
N don’t know.
B Inflation Expectations Correction

According to the user’s guide for the EU Harmonized Consumer Survey, the survey interviewer asks about the expected inflation rate in a two step process. First, the interviewer asks a qualitative question about whether the respondent believes that prices will go up, stay the same, or go down. If the respondent states that they believe that prices will go up or down, then the interviewer follows up with the quantitative question. If the respondent states that they expect prices to “stay the same,” then the interviewer skips the quantitative portion and codes a response of 0%. In a typical month, about 30% of respondents state that they expect “prices to stay the same” and are assigned a 0% response in this fashion. However, in September 2009 INSEE ran a short followup survey for these 0% households about survey wording. They found that about half of the households that stated that they expected prices to “stay the same” really meant “the inflation rate will stay the same.” The households were erroneously coded as 0%, when in fact their actual inflation expectations were higher.

I can not identify whether a specific household that states that they believe prices will “stay the same” actual hold that belief. As an imperfect solution, I create an indicator variable to identify any household that believes that prices have risen by more than 5% over the past twelve months, but that prices will “stay the same” over the next 12 months. The rationalie is that households that believe that the current inflation rate is high would be unlikely to actually expect inflation to drop to 0% in the future. For the rest of this appendix, I refer to these expectations as dubious 0% expectations.

I drop any households that give dubious 0% responses as defined above, and I rerun my analysis linking inflation expectations to inflation perceptions. Figure 12 displays the mean inflation perceptions and corrected inflation expectations time series. The discrepancy between inflation perceptions and expectations is small, particularly after 2011. Households appear to believe that inflation over the next 12 months will be closer to how it behaves over the past 12 months.

The link between inflation perceptions and inflation expectations is much higher on the household level, as Table 12 shows. The $R^2$ has jumped up to 0.6, which indicates that inflation perceptions explain most of the variation in inflation expectations. The large coefficient on inflation perceptions and the low constant term indicate that the best-fit line is close to 45 degrees and anchored near the origin. The dubious 0% expectations were dampening the effect of perceptions in

---

\[58\] Half of the respondents in the September 2009 supplemental survey actually did believe that the inflation rate would be 0%, so assigning them a non-zero belief would be incorrect.
my original analysis. Rerunning my regression with demographic variables, on subsamples, or on differenced expectations does not significantly affect the results.

Figure 12: Modified Expectations
Table 11: Expectations on Savings without Miscoding

<table>
<thead>
<tr>
<th></th>
<th>(1) $E_t \pi_{t+12}$</th>
<th>(2) $E_t \pi_{t+12}$</th>
<th>(3) $E_t \pi_{t+12}$</th>
<th>(4) $E_t \pi_{t+12}$</th>
<th>(5) $E_t \pi_{t+12}$</th>
<th>(6) $\Delta E_t \pi_{t+12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Perceptions</td>
<td>0.774***</td>
<td>0.769***</td>
<td>0.813***</td>
<td>0.805***</td>
<td>0.748***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.006)</td>
<td>(0.005)</td>
<td>(0.010)</td>
<td>(0.008)</td>
<td></td>
</tr>
<tr>
<td>Change in Inflation</td>
<td>0.200***</td>
<td>0.345***</td>
<td>1.026***</td>
<td>0.104**</td>
<td>0.241***</td>
<td>0.363***</td>
</tr>
<tr>
<td>Perceptions</td>
<td>(0.018)</td>
<td>(0.082)</td>
<td>(0.088)</td>
<td>(0.038)</td>
<td>(0.020)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.200***</td>
<td>0.345***</td>
<td>1.026***</td>
<td>0.104**</td>
<td>0.241***</td>
<td>0.363***</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.082)</td>
<td>(0.088)</td>
<td>(0.038)</td>
<td>(0.020)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>Demographic Controls</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Sample</td>
<td>Full</td>
<td>Full</td>
<td>06/2010-05/2011</td>
<td>High-Income</td>
<td>Low-Income</td>
<td>Full</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.599</td>
<td>0.639</td>
<td>0.576</td>
<td>0.609</td>
<td>0.305</td>
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<tr>
<td>Observation</td>
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<td>112661</td>
<td>56018</td>
<td>23109</td>
<td>41214</td>
<td>22515</td>
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</table>
C Solution Methodology

The model is solved using the endogenous grid point method of Carroll (2006). For the rational expectations version of the model, I set up the problem in real terms. I initialize a guess for the utility of consumption for each gridpoint in the combination of assets and exogenous states and iterate backwards using the Euler equation and budget constraint until I find the optimal policy solution. This technique returns the optimal policy function under rational expectations.

For households with biased perceptions, I apply the same technique as above. However, I set the exogenous states and transition matrix as if the households were correct. I use the endogenous gridpoints method to find the subjectively optimal policy functions by iterating until the solution converges. Next, I evaluate the objective value function under suboptimal policy by starting at each asset gridpoint and exogenous state and iterating forward with the policy function.

The last type of policy function to solve for is for households with accurate inflation perceptions, but biased inflation expectations. First, I set the exogenous states and transition matrix as if the households’ biased expectations were correct. I iterate backward with the endogenous grid point method until the policy functions converge. Next, I the policy function backwards one more time, using the objectively correct exogenous state and transition matrix. This grants me the policy function for households with accurate perceptions, but biased expectations. Finally, find the value function that a household would get by applying this suboptimal policy function.

I discretize the exogenous states using a multi-variate Tauchen method. This method involves setting up the correlated VAR process for \( \pi_1, \pi_2, \) and \( Y \), taking an orthogonal transformation of the variables to three orthogonal variables, and then discretizing that process. The orthogonal transformation of the exogenous state variable, \( \pi_1, \pi_2, \) and \( Y \), has 5 states, for an total of \( 5^3 = 125 \) states. Because \( \pi_1 \) and \( \pi_2 \) are correlated, the state for \( \pi_1, \pi_2, \) and \( Y \) are not evenly distributed in a cube.
D Nominal Budget Constraint

In this appendix I will set up the sequential markets version of the households’ problem in nominal terms, where households consider the price level $P_t$. I will then describe the steps and assumptions that are required to rewrite the households’ problem in a stationary, recursive form. Because households do not necessarily know the price level $P_t$ each period and $P_t$ may not be stationary, this derivation is not obvious.

I assume that households know their nominal income and nominal bond wealth at the start of the period, but may not know the current price level $P_t$ perfectly. The households choose how much of their nominal wealth to allocate towards consumption expenditures and how much to save, given their beliefs about the current price level. The households also form expectations about the real value of their future income and the inflation rate next period. The household’s problem at time $t$ is:

$$\max_{\{C^N_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$$

s.t. $C^N_t + A^N_t = (1 + i)A^N_{t-1} + Y^N_t$  \hspace{1cm} (34)

$$C_t = \frac{C^N_t}{P_t}$$ \hspace{1cm} (35)

At time $t$, households need to decide how much to spend on consumption given that they might not know the price of composite goods $P_t$ and decide how much to save given that they do not know the real rate of return $\frac{(1+i)}{\pi_{t+1}}$. I make two assumptions to simplify the analysis. First, I assume that at time $t$ households know $P_{t-1}$, the price level last period. This analysis allows me to divide the nominal budget constraint by $P_{t-1}$ to express the households budget constraint in real, stationary terms.\(^{59}\) Let $A_t$ denote quantity of nominal bonds purchased deflated by the price index, $A_t = \frac{A^N_t}{P_t}$. Then the household budget constraint at time $t$ is

$$\underbrace{\pi_t}_{Unknown} (C_t + A_t) = (1 + i)A_{t-1} + \underbrace{\pi Y_t}_{Known}$$ \hspace{1cm} (36)

Households know what their nominal income and wealth is each period, but they don’t know what the current inflation rate $\pi_t$ is. My second assumption is that households have certainty equivalence with regards to the inflation rate. Households only care about their average percep-

\(^{59}\)Making this assumption also prevents incorrect inflation perceptions from compounding over multiple periods. If households overestimated the inflation rate by 2 percentage points each period and never learns the true price leve, then after ten periods they will overestimate the price level by 20 percentage points.
tions of the current inflation rates, rather than the full distribution of $\pi_t$. I make this assumption for two reasons. First, under full information rational expectations households know $\pi_t$ exactly, so they have a degenerate distribution of subjective beliefs with all the mass put on $\pi_t$. To facilitate comparisons between full information rational expectations and biased perceptions, I assume that households with biased perceptions also believe that the inflation rate was $E^*\left[\pi_t\right]$ with certainty. Second, certainty equivalence is more analytically tractable in my setup. If households have certainty equivalence over current inflation $E^*\left[\pi_t\right]$, then I can set up the computationally problem to find the optimal policy for each subjective $E^*\left[\pi_t\right]$. In contrast, if households have a non-degenerate distribution of beliefs of $\pi_t$, then I would have to evaluate their optimal policy over each possible subjective distribution of $\pi_t$.

Dividing both sides of equation (36) by $\pi_t$ and imposing certainty equivalence results in what households believe is their real budget constraint

$$\tilde{C}_t + \tilde{A}_t = \left(1 + i\right)A_{t-1} - \frac{E^*\left[\pi_t\right]}{E^*\left[\pi_t\right]} + Y_t$$

(37)

Households choose their subjective bonds deflated by the price index $\tilde{A}_t$ and their subjective consumption $\tilde{C}$. The full recursive problem for the households becomes equation (20).

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60Technically if the coefficient of risk aversion $\sigma = 2$, the expression $E[\pi]$ shows up in the the households’ first order equations. I choose to make the certainty equivalence assumption more explicit.