Lecture 9: Reputational Bargaining II

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Overview

Last lecture:

- Abreu and Gul (2000): Introduce obstinate types to bargaining.
- Reputational bargaining in discrete time with frequent offers $\approx$ continuous-time war-of-attrition.
- When offers are frequent and players have a rich set of commitment types, each player's payoff $\approx$ his Rubinstein bargaining payoff.

This lecture:

1. Alternative formulation of the reputational bargaining problem.
2. What will happen when players have outside options?
3. Can we use this machinery to deliver sharp predictions in repeated games with two comparably patient players?

Kambe (1999)

- Time $t \in [0, +\infty)$. Two players with discount rates r_1 and r_2.
- Before time 0, players simultaneously announce their demands $\alpha_1^*, \alpha_2^* \in [0, 1]$.
- If $\alpha_1^* + \alpha_2^* \leq 1$, then the game ends at 0 where player i receives $\alpha_i^* + \frac{1}{2}(1 - \alpha_1^* - \alpha_2^*)$.
- If $\alpha_1^* + \alpha_2^* > 1$, then play enters a *war-of-attrition phase*.

  Player i becomes committed at time 0 with prob $\varepsilon_i > 0$ (is player i's private info and is independent of whether player $-i$ is committed).

  At every $t \in [0, +\infty)$, the flexible type of every player decides whether to concede.

  Player i chooses α_i^* in order to maximize their expected payoff.

Result

Theorem 1 in Kambe (1999)

When $\varepsilon_1, \varepsilon_2 \to 0$ while keeping $\frac{\varepsilon_1}{\varepsilon_2}$ fixed, every equilibrium converges to the following limit point.

- *Players' initial demands are their Rubinstein payoffs $\left(\frac{r_2}{r_1+r_2}, \frac{r_1}{r_1+r_2}\right)$.*
- *Players will reach a deal without any delay.*

Intuition: Player i secures payoff close to $\frac{r_{-i}}{r_i+r_{-i}}$ by demanding $\frac{r_{-i}}{r_i+r_{-i}}$.

- Player $-i$ has an incentive to make a compatible offer in order to avoid the loss from being committed.

Kambe (1999) also considers the case in which whether player i is committed is known to player i before choosing α_i^*.

- Results are less clean, require stronger refinement, and restrict attention to pure strategies when choosing demands.

Kambe (1999) *vs* Abreu and Gul (2000)

Advantages of Kambe's formulation.

- The commitment types' demands are endogenous.
- Avoid requirements on rich type spaces.
- Convenient in context with incomplete info about values/costs/quality, or when players can make complicated commitments.
- Examples: Wolitzky (2012), my recent work with Maren, and so on.

Disadvantages of Kambe's formulation:

- Why players do not know whether they are committed or not when choosing their initial demands? Stories?
- The signaling formulation is not tractable.

Compte and Jehiel (2002): Outside Options

Discrete time bargaining game with one commitment type on each side.

- $t = 0, \Delta, 2\Delta, \ldots$
- In even periods, P1 either takes the outside option (the game ends), or makes a new offer.

  P2 either accepts P1's offer and ends the game, or rejects the offer.
- In odd periods, P2 either takes the outside option or makes a new offer.

  P1 either accepts P2's offer or rejects.
- If a player takes the outside option, then payoffs are (β_1^*, β_2^*), satisfying

$$1 - \alpha_2^* < \beta_1^* < \frac{1 - e^{-r_2 \Delta}}{1 - e^{-(r_1 + r_2)\Delta}} \approx \frac{r_2}{r_1 + r_2},$$

$$1 - \alpha_1^* < \beta_2^* < \frac{1 - e^{-r_1 \Delta}}{1 - e^{-(r_1 + r_2)\Delta}} \approx \frac{r_1}{r_1 + r_2}.$$

  Outside option is better than conceding, but is worse than each player's Rubinstein bargaining payoff.

Benchmark: Game without Commitment Type

Theorem: Binmore, Shaked and Sutton (1987)

Suppose players' payoffs from the outside option are such that

$$\beta_1^* < \frac{r_2}{r_1 + r_2},$$

and

$$\beta_2^* < \frac{r_1}{r_1 + r_2},$$

then the unique subgame perfect equilibrium attains the same outcome as the Rubinstein bargaining game without any outside option.

Intuition: Since the outside option is inferior to the Rubinstein bargaining payoff, taking the outside option is not a credible threat.

Result: No Reputation Building

Theorem: Compte and Jehiel

In every PBE of the reputational bargaining game with outside options,

- *The rational type of player 1 demands* $\frac{1-e^{-r_2\Delta}}{1-e^{-(r_1+r_2)\Delta}}$ *at time 0 and the rational type player 2 accepts immediately.*
- *If player 1 demands* α_1^**, then the rational player 2 takes the outside option.*
- *If player 1 demands sth greater than* $\frac{1-e^{-r_2\Delta}}{1-e^{-(r_1+r_2)\Delta}}$ *but not* α_1^**, then player 2 rejects and offers* $\frac{1-e^{-r_1\Delta}}{1-e^{-(r_1+r_2)\Delta}}$.
- *If player 2 demands* α_2^* *in period* Δ*, then the rational type player 1 takes the outside option.*

- When a player imitates the commitment type, his opponent takes the outside option immediately.
- Otherwise, play proceeds as in the Rubinstein bargaining game.

Why no reputation building?

Rational players have no incentive to imitate the commitment type. Why?

- Outside option $>$ concession $\Rightarrow$ Rational type never concedes.
- If my opponent never concedes, then there is no benefit for me to imitate the commitment type.
- The reputational equilibrium in Abreu and Gul unravels.

  Board and Pycia (14): outside options unravel the Coase conjecture.

Comments:

- Contrasts to Binmore, Shaked and Sutton (87): Outside options do not matter if they lead to payoffs inferior to Rubinstein bargaining payoffs.
- What if there is a rich set of commitment types?
- Is the Rubinstein bargaining payoffs a robust prediction?
- How should we think about wars, strikes, and so on?

Motivation: Repeated Games with Contracts

In general, it is hard to make sharp predictions in repeated games with two equally patient players.

Abreu and Pearce (2007): Sharp predictions in repeated games when

- players can sign a binding contract,

  after which future play is pinned down by the terms of the contract.

Example:

-	L	R
T	$1,1$	$0,0$
B	$0,0$	$0,0$

Before agreeing on a contract, player 1 chooses $\alpha_{1,t} \in \Delta\{T, B\}$ and player 2 chooses $\alpha_{2,t} \in \Delta\{L, R\}$. A contract specifies what payoffs players receive in future periods, subject to feasibility constraints.

Model

Stage game: two-player finite game $\mathcal{G} = (I, A, U)$.

In each integer time $t = 0, 1, 2, \ldots$, player i chooses $\alpha_i \in \Delta(A_i)$ and offers a binding contract (v_1, v_2) to player j.

- After signing a contract, continuation values are (v_1, v_2).
- We focus on Pareto optimal contracts.

Players' mixed actions are perfectly monitored.

At every $t \in [0, +\infty]$, players can accept the other player's contract.

Player i's payoff if an agreement (v_1, v_2) is reached at τ:

$$r \int_0^{\tau} e^{-rt} u_i(\alpha_{1,t}, \alpha_{2,t}) dt + e^{-r\tau} v_i,$$

where $\alpha_{i,t}$ is player i's action at time $\lfloor t \rfloor$.

Commitment Types

Player $i \in \{1, 2\}$ is either rational (w.p. $1 - z_i$) or committed (w.p. z_i).

A finite set of commitment types Γ_i for player i.

- Every $\gamma_i \in \Gamma_i$ specifies $\alpha_i \in \Delta(A_i)$ and (v_1, v_2), s.t. commitment type γ_i takes action α_i until their contract (v_1, v_2) is accepted.

Conditional on committed, player i's type follows distribution $\pi_i \in \Delta(\Gamma_i)$.

Before the game starts, players simultaneously announce which commitment type they want to imitate.

- Every commitment type truthfully announces their type.
- Every rational type decides which commitment type to announce, or announces that they are rational.

Important: Once the game starts at time 0, each player's belief assigns positive prob to at most one commitment type.

How to solve this model?

Directly solving this model is hard.

- If there exists some particular commitment type for each player, then players' payoffs are pinned down regardless of other types.

Detour: Nash Bargaining (Nash 1950)

Convex bargaining set $\Pi \subset \mathbb{R}^2$, and disagreement point $(d_1, d_2) \in \Pi$.

- Let
$$\Pi(d_1, d_2) \equiv \left\{ (d_1', d_2') \in \Pi \middle| d_1' \geq d_1, d_2' \geq d_2 \right\}.$$
- Nash bargaining payoff:
$$u^N(d_1, d_2) \equiv \arg \max_{(u_1, u_2) \in \Pi(d_1, d_2)} \left\{ (u_1 - d_1)(u_2 - d_2) \right\}.$$

One can show that $u^N(d_1, d_2)$ is uniquely defined and is Pareto efficient.

Detour: Nash Bargaining with Threat (Nash 1953)

Normal-form game $\mathcal{G} \equiv (A, U)$, let Π be the convex hull of feasible payoffs.

1. Players simultaneously choose $\alpha_1 \in \Delta(A_1)$ and $\alpha_2 \in \Delta(A_2)$.
2. Players' payoffs are given by $u^N\Big(u_1(\alpha_1, \alpha_2), u_2(\alpha_1, \alpha_2)\Big)$,

   i.e., Nash bargaining payoff with threat point $\Big(u_1(\alpha_1, \alpha_2), u_2(\alpha_1, \alpha_2)\Big)$.

Theorem: Nash Bargaining with Threat

Suppose $\mathcal{G} \equiv (A, U)$ is finite, the game where players payoffs are $u^N\Big(u_1(\alpha_1, \alpha_2), u_2(\alpha_1, \alpha_2)\Big)$ admits at least one Nash equilibrium.

All Nash equilibria lead to the same payoff $u^*(\mathcal{G}) \in \mathbb{R}^2$.

Assumptions on the Set of Commitment Types

We assume that NBWT posture is adopted by at least one commitment type.

Assumption: NBWT Posture Exists

For every $i \in \{1,2\}$, *there exists* $\gamma_i^* \in \Gamma_i$ *such that* γ_i^* *offers contract* $u^*(\mathcal{G}) \equiv (u_1^*, u_2^*)$ *and plays his equilibrium strategy in the NBWT game* α_i^*.

We assume that after a player has a perfect reputation for being any commitment type, their opponent has a strict incentive to concede.

Assumption: NBWT Type Penalizes Rejection

For every $i \in \{1,2\}$ *and* $\gamma_i \equiv (\alpha_i^*, u_1^*, u_2^*) \in \Gamma_i$,

$$u_j^* > \max_{a_j \in A_j} u_j(a_j, \alpha_i^*).$$

Theorem: Repeated Games with Contracts

Theorem: Abreu and Pearce (2007)

Under the two assumptions on Γ_i. For every $\varepsilon, R > 0$, there exists $\bar{z} > 0$, such that if $\max\{z_1, z_2\} < \bar{z}$ and $\max\{\frac{z_1}{z_2}, \frac{z_2}{z_1}\} \leq R$, then players' payoffs in any PBE of the repeated game with contracts is within ε of $u^*(\mathcal{G})$.

Proof: Suppose P1 announces NBWT bargaining posture $\gamma_1^* \equiv (\alpha_1^*, u_1^*, u_2^*)$ and never accepts any contract that offers less than his NBWT payoff u_1^*.

- If P2 offers a contract that gives P1 $\geq u_1^*$, then P1's payoff $\geq u_1^*$.
- Next: If P2 takes action α_2 and offers contract (v_1^*, v_2^*) s.t. $v_1^* < u_1^*$ and $v_2^* > u_2^*$, we show that P1's concession rate is higher than P2's.
- Similar to Abreu and Gul, if a player's concession rate is higher, then his opponent concedes with prob close to 1 at time 0 when $z_1, z_2 \to 0$.

Concession Rates when $v_1^* < u_1^*$ and $v_2^* > u_2^*$

Recall that

- P1 offers NBWT payoffs (u_1^*, u_2^*) and takes NBWT action α_1^*.
- (v_1^*, v_2^*) is P2's offer with $v_1^* < u_1^*$ and $v_2^* > u_2^*$, and P2 commits to α_2.

Let λ_i be player i's concession rate.

P2 is indifferent between accepting P1's contract and waiting:

$$\lambda_1(v_2^* - u_2^*) = r(u_2^* - u_2(\alpha_1^*, \alpha_2)).$$

P1 is indifferent between accepting P2's contract and waiting:

$$\lambda_2(u_1^* - v_1^*) = r(v_1^* - u_1(\alpha_1^*, \alpha_2)).$$

P1 has an advantage iff $\lambda_1 > \lambda_2$, which is equivalent to

$$\frac{u_2^* - u_2(\alpha_1^*, \alpha_2)}{v_1^* - u_1(\alpha_1^*, \alpha_2)} > \frac{v_2^* - u_2^*}{u_1^* - v_1^*}.$$

Concession Rates when $v_1^* < u_1^*$ and $v_2^* > u_2^*$

We want to show that:

$$\frac{u_2^* - u_2(\alpha_1^*, \alpha_2)}{v_1^* - u_1(\alpha_1^*, \alpha_2)} > \frac{v_2^* - u_2^*}{u_1^* - v_1^*}.$$

Since (α_1^*, α_2^*) is an equilibrium of the NBWT game, P2's payoff in the NBWT game is weakly lower than u_2^* when the threat point is (α_1^*, α_2).

$$(w_1^*, w_2^*) \equiv \arg \max_{(w_1,w_2) \geq (u_1(\alpha_1^*,\alpha_2), u_2(\alpha_1^*,\alpha_2))} \left\{ (w_1 - u_1(\alpha_1^*, \alpha_2))(w_2 - u_2(\alpha_1^*, \alpha_2)) \right\}.$$

We must have $w_2^* \leq u_2^*$ and $w_1^* \geq u_1^*$, which means:

- Either

$$l \equiv \frac{u_2^* - w_2^*}{w_1^* - u_1^*} \underbrace{\geq \frac{v_2^* - u_2^*}{u_1^* - v_1^*}}_{\text{since the bargaining set is convex}}$$

- or $w_2^* = u_2^*$ and $w_1^* = u_1^*$

$$l \equiv \frac{v_2^* - u_2^*}{u_1^* - v_1^*} = \frac{v_2^* - w_2^*}{w_1^* - v_1^*}.$$

Concession Rates when $v_1^* < u_1^*$ and $v_2^* > u_2^*$

We need to show that

$$\frac{u_2^* - u_2(\alpha_1^*, \alpha_2)}{v_1^* - u_1(\alpha_1^*, \alpha_2)} > \frac{v_2^* - u_2^*}{u_1^* - v_1^*}.$$

Case 1: $l \equiv \frac{u_2^* - w_2^*}{w_1^* - u_1^*} \geq \frac{v_2^* - u_2^*}{u_1^* - v_1^*}$.

- Since (w_1^*, w_2^*) maximizes $(w_1 - u_1(\alpha_1^*, \alpha_2))(w_2 - u_2(\alpha_1^*, \alpha_2))$, and $(w_1^* - \Delta, w_2^* + l\Delta)$ belongs to the bargaining set for small Δ,

$$l(w_1^* - u_1(\alpha_1^*, \alpha_2)) - (w_2^* - u_2(\alpha_1^*, \alpha_2)) \leq 0 \quad \Rightarrow \quad l \leq \frac{w_2^* - u_2(\alpha_1^*, \alpha_2)}{w_1^* - u_1(\alpha_1^*, \alpha_2)}.$$

- Since $u_2^* > w_2^*$, $u_1^* < w_1^*$, and $v_1^* < u_1^*$, we have

$$l \leq \frac{w_2^* - u_2(\alpha_1^*, \alpha_2)}{w_1^* - u_1(\alpha_1^*, \alpha_2)} < \frac{u_2^* - u_2(\alpha_1^*, \alpha_2)}{u_1^* - u_1(\alpha_1^*, \alpha_2)} < \frac{u_2^* - u_2(\alpha_1^*, \alpha_2)}{v_1^* - u_1(\alpha_1^*, \alpha_2)}.$$

Concession Rates when $v_1^* < u_1^*$ and $v_2^* > u_2^*$

We need to show that

$$\frac{u_2^* - u_2(\alpha_1^*, \alpha_2)}{v_1^* - u_1(\alpha_1^*, \alpha_2)} > \frac{v_2^* - u_2^*}{u_1^* - v_1^*}.$$

Case 2: $w_2^* = u_2^*$, $w_1^* = u_1^*$, and $l \equiv \frac{v_2^* - u_2^*}{u_1^* - v_1^*}$

- Since (u_1^*, u_2^*) maximizes $(w_1 - u_1(\alpha_1^*, \alpha_2))(w_2 - u_2(\alpha_1^*, \alpha_2))$, and $(u_1^* - \Delta, u_2^* + l\Delta)$ belongs to the bargaining set for small Δ,

$$l(u_1^* - u_1(\alpha_1^*, \alpha_2)) - (u_2^* - u_2(\alpha_1^*, \alpha_2)) \leq 0 \quad \Rightarrow \quad l \leq \frac{u_2^* - u_2(\alpha_1^*, \alpha_2)}{u_1^* - u_1(\alpha_1^*, \alpha_2)}.$$

- Since $v_1^* < u_1^*$, we have

$$l \leq \frac{u_2^* - u_2(\alpha_1^*, \alpha_2)}{u_1^* - u_1(\alpha_1^*, \alpha_2)} < \frac{u_2^* - u_2(\alpha_1^*, \alpha_2)}{v_1^* - u_1(\alpha_1^*, \alpha_2)}.$$

Summary

Proof: Suppose P1 announces NBWT bargaining posture γ_1^* and never accepts anything that offers less than his NBWT payoff u_1^*.

- If P2 offers a contract that gives P1 $\geq u_1^*$, then P1's payoff $\geq u_1^*$.
- If P2 takes action α_2 and offers contract (v_1^*, v_2^*) s.t. $v_1^* < u_1^*$ and $v_2^* > u_2^*$, then P1's concession rate is higher than P2's.

P1 can guarantee payoff $\approx u_1^*$ by imitating their NBWT type.

Similarly, P2 can guarantee payoff $\approx u_2^*$ by imitating their NBWT type.

- Since (u_1^*, u_2^*) is Pareto optimal, players' payoffs must be close to (u_1^*, u_2^*) in every equilibrium.

Non-Stationary Bargaining Postures

Abreu and Pearce (2007) also consider non-stationary bargaining postures.

- Their payoff prediction remains robust.
- The announcement stage (or the transparent commitment type assumption) is very important.
- Wolitzky (2011) shows a folk theorem in repeated games with contracts without the announcement stage.

What will happen when players have different discount factors?

- How to adjust the formula for the Nash product to take into account this change?
- Treat this as a Pset question.