Validation of x-ray line ratios for electron temperature determination in tokamak plasmas

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Abstract. X-ray imaging crystal spectroscopy (XICS) has been implemented on magnetic confinement fusion devices as a novel means of measuring local plasma temperature, impurity density, and flow profiles. At Alcator C-Mod, XICS allows for spatially-resolved, high spectral resolution measurements between 0.3 nm and 0.4 nm, enabling detailed analysis of He-like argon x-ray emission. Electron temperatures in the range of $0.5 \text{ keV} \leq T_e \leq 3.0 \text{ keV}$ are determined from He-like argon emissivity ratios of the $n = 3$ dielectronic satellites to the $w$-line and its surrounding $n \geq 3$ satellites, specifically the wavelength range of $3.9440 \text{ Å} \leq \lambda \leq 3.9607 \text{ Å}$. These data are validated against existing measurements of $T_e$ from electron cyclotron emission and Thomson scattering. Line ratio data are analyzed via a tomographic inversion procedure, overcoming the traditional issue of spectra being averaged over the plasma cross-section. The implications of utilizing x-ray line ratios as a valid local temperature diagnostic are not limited to Alcator C-Mod; properties of plasma in future experiments as well as in astrophysical settings can also be investigated. The results of this experiment confirm that x-ray line ratios can be used as an accurate electron temperature diagnostic. The electron temperature can be determined from the relation to the line ratio, $x$, as \[ T_e [\text{keV}] = 0.1552 x^{-0.7781} \] with $0.0223 < x < 0.2449$.
1. Introduction

1.1. Line-Ratio Based $T_e$ Measurements in Tokamak Plasmas

The detailed features of He-like emission from mid-$Z$ ions have been known for some time to contain a wealth of information on the local conditions (see reviews in [1, 2]) for both laboratory and astrophysical plasmas. The electron temperature, $T_e$, is a key quantity that can be deduced from spectra which resolve fine-structure features of the $n=2$ or $K_{\alpha}$ emission from He-like ions. The most prominent are four lines which are referred to as the resonance line $w_1 (1s^2 2p^1 P_1^1 - 1s^2 1S_0^0)$, the intercombination lines $x_1 (1s^2 2p^3 P_2^1 - 1s^2 1S_0^0)$ and $y_1 (1s^2 2p^3 P_1^1 - 1s^2 1S_0^0)$, and the forbidden line $z_1 (1s^2 2p^3 S_1^1 - 1s^2 1S_0^0)$. Between the $w$ and $z$ lines are weaker lines populated by dielectronic recombination of He-like ions or inner-shell ionization of Li-like ions. The dielectronic recombination lines in this region are emitted from doubly-excited ions with three bound electrons (Li-like ions) via $1s^2 2p(n\ell') - 1s^2 (n\ell')$, where the $n\ell'$ electron is referred to as the spectator electron. For $n'=2$, these transitions can be resolvable from the $w$, $x$, $y$, and $z$ lines, and have alphabetized labels discussed in [3], while for $n=3$, they begin to form bands of emission that merge with the resonance and intercombination lines.

Tokamak laboratory plasmas typically feature thermal electron distributions, allowing Maxwellian-averaged x-ray emission rates to be empirically validated. The first detailed comparison of line ratios involving dielectronic recombination with independent measures of $T_e$ was done using Ohmically-heated PLT plasmas where He-like iron (Fe XXV) was measured. Here the ratio of the $j$ satellite $(1s^2 2p^2 D_{5/2} - 1s^2 2p 2P_{3/2})$ to the $w$-line was measured along a single chord in plasmas with core $T_e$ from 1.65 keV to 2.30 keV, measured using electron cyclotron emission (ECE) [4]. It was shown that the $n=3$ satellites, unresolvable from the $w$-line, needed to be included in the modeling to improve agreement, but line ratios systematically predicted a lower $T_e$ by approximately 10%.

On JET, a database of $\sim$3000 He-like nickel (Ni XXVII) spectra from $\sim$100 pulses were analyzed, including a mix of Ohmic, ion cyclotron, and neutral beam heated plasmas. Ratios of the $t$ $(1s^2 2p^3 P)2s^2 P_{1/2} - 1s^2 2s^2 S_{1/2})$ satellite to $w$-line, $n \geq 3$ satellites to $w$-line, and $x$-line to $w$-line were compared to electron temperature from ECE for $3.0 \text{ keV} < T_e < 12.0 \text{ keV}$ [5]. This temperature range exceeds that which would be expected to localize the peak of the emissivity on-axis, estimated to be 8 keV, so modeling of the volumetric emission was done using a core density and temperature and a single parameter describing the peaking of the emissivity profile. Single chord, line-averaged data were used to constrain the on-axis $T_e$ for a range of assumed profile shapes. Both line ratios demonstrated the expected trend of the wide temperature range, but systematically higher $T_e$ was observed for ECE by approximately 10%, while line ratio results were in better agreement with LIDAR-based Thomson scattering (TS).

On TFTR, a database of He-like nickel spectra from 50 Ohmically-heated shots were analyzed where $T_e$ was a free parameter used in a least-squares fitting routine to model the intensity of $n=2$ satellite lines relative to that of the $w$-line. No modeling
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of the emission volume was done, and $T_e$ derived from the line ratios was compared to measurements of core $T_e$ from TS for $2.5 \text{ keV} < T_e < 5.0 \text{ keV}$ [6]. The trend of increasing $T_e$ as $n_e$ decreased over the 50 pulses was well reproduced, although TS observed a systemically higher temperature by approximately 10%. Line ratio based $T_e$ agreed better with that computed from a chord-averaged soft x-ray pulse height analyzer spectra which had been corrected for profile effects using the TS.

For He-like argon, measurements from a spatially scanable von Hamos spectrometer on Alcator C were used to measure profiles of the $k \left( 1s2p^2 \frac{3}{2} D_{3/2} - 1s2p^2 \frac{1}{2} P_{1/2} \right)$ to $\omega$-line ratio [7]. While multiple lines of sight with poloidal impact parameter covering $r/a < 0.5$ were used, no tomographic inversion was applied. Results showed reasonable agreement between temperatures deduced from the $k/\omega$ ratio and those measured by ECE for $0.5 \text{ keV} < T_e < 1.3 \text{ keV}$. This work was replicated on Alcator C-Mod using a similar multi-chord x-ray spectrometer system for $T_e < 1.5 \text{ keV}$ [8]. Experiments at the Berlin EBIT computed $T_e$-dependent $k/\omega$ ratios from mono-energetic electron impact measurements which were shown to agree well with HULLAC predictions for $T_e < 2.5 \text{ keV}$ [9]. More recently, all features of single, chord-averaged spectra were fit with a single $T_e$ and compared to ECE and TS on TEXTOR and NSTX, respectively. For TEXTOR, synthetic spectra were formed using electron density profiles from interferometry and the shape of the $T_e$ profile from ECE, with the core temperature a free parameter in the fit. Measured core $T_e$ from ECE over $0.8-2.2 \text{ keV}$ agreed to that found from the spectral fit to within approximately 10%, with AUTOSTRUCTURE used to compute data for the $n = 3$ satellite lines [10]. This is consistent with work done previously on TEXTOR in Ohmic and neutral beam heated plasmas over a similar temperature range [11]. On NSTX, emission rates computed using the Flexible Atomic Code (FAC) [12] were shown to be necessary to fit all features of the spectrum compared to earlier results using the MZ code [13]. Line ratio data were shown to agree to approximately 10% between $0.5 \text{ keV} < T_e < 1.2 \text{ keV}$ on Ohmically-heated plasmas [14].

A brief attempt to validate the use of x-ray imaging crystal spectroscopy to determine electron temperatures was previously made at Alcator C-Mod [15]; however, it is important to note that the data from this study did not account for the vignetting of the spectrum due to the structure of the spectrometer at Alcator C-Mod. To address this issue, data from the He-like argon spectra were only used where there was no vignetting in this present study. While the $k/\omega$ ratio would be preferred, as the lines are more easily distinguished, the vignetting forces line-ratio measurements to be from adjacent lines, and here the $n \geq 3$ dielectronic satellites will be used.

1.2. Background

Traditional methods of using line ratios to measure the electron temperature in tokamak plasmas rely on an averaging over the plasma cross-section. Some approaches attempt to correct this averaging by estimating the slope of the volumetric emission. The spatially resolved emission associated with x-ray imaging crystal spectroscopy (XICS)
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Figure 1. Measured He-like argon spectrum with wn3 region from 3.9440 Å to 3.9607 Å (shot: 1100325012, channel: 24, time: 1.1985 s).

enables local $T_e$ measurements to be produced from emission-line ratios by utilizing a tomographic inversion procedure [16]. Using XICS at Alcator C-Mod, the temperature dependencies of He-like argon spectral data can be analyzed to produce local line ratio measurements, which, for the first time, can be experimentally validated against independently measured $T_e$ profiles provided by TS and ECE. The resulting data can also be used to validate computational atomic physics theory in the form of argon $K$-shell population modeling from an *ab initio* atomic physics code called the Flexible Atomic Code (FAC) [12]. This spectral analysis was performed using the high resolution x-ray spectrometer with spatial resolution (HIREXSR) [15] at Alcator C-Mod [17] in conjunction with the HIREXSR analysis code (THACO) [18].

The results of this study intend to gauge the efficacy and accuracy of XICS as a means of measuring $T_e$. If there is a robust relationship between x-ray line ratios and $T_e$, XICS can be used to measure local electron temperatures in laboratory plasmas in which other means of making $T_e$ measurements – such as through TS or ECE – are not available. Moreover, the results can be applied to astrophysical plasmas in which direct $T_e$ readings would be impossible to obtain, but spectral data may be readily available.

1.3. Dielectronic Recombination

The region of the He-like argon spectrum under investigation contains the $w$-line and its neighboring $n \geq 3$ satellites. This will collectively be referred to as the wn3 region. More specifically, the wn3 region is defined here as spanning the wavelength range of 3.9440 Å to 3.9607 Å and is shown in Figure 1.

The satellites to the $w$-line are produced from dielectronic recombination from the He-like argon into doubly-excited states of Li-like argon. The first step of a dielectronic recombination process involves a positive ion capturing a surrounding electron. Unlike traditional radiative recombination, the energy released from the electron capture excites
a separately bound electron to create a doubly-excited state instead of producing continuum radiation \[19\]. Afterwards, an excited electron returns to its lower energy level and releases an x-ray photon that is detected by the spectrometer. The other excited electron is typically referred to as a spectator electron.

When discussing the \(n = 3\) satellites formed from dielectronic recombination, the value of \(n\) refers to the energy level where the spectator electron is located. Prior to dielectronic recombination, an \(\text{Ar}^{16+}\) ion will have two electrons in the ground state. However, after a dielectronic recombination process that produces an \(n = 3\) satellite to the \(w\)-line, the \(\text{Ar}^{15+}\) ion will have an electron configuration of \(1s^23\ell\), where \(\ell\) is an arbitrary angular momentum state. The decay that produces a photon observed by HIREXSR, as shown in Figure 1, contributes to the satellites near the \(w\)-line and results in a \(1s^23\ell\) configuration. The \(n = 3\) satellites are the sum of approximately 50 lines with 3 to 4 bright lines \[20, 21\]. The two brightest lines are \(1s^2p^3p^2D_5/2 - 1s^2^3p^2P_3/2\) at 3956.3 mÅ and \(1s^2p^3d^2F_{7/2} - 1s^2^3d^2D_5/2\) at 3952.0 mÅ \[22\].

The aforementioned \(w\)-line is primarily produced from electron impact excitation, which is a threshold process, while dielectronic recombination is a resonance process. Therefore, the rate coefficient for dielectronic recombination is different as a function of \(T_e\) for a Maxwellian electron distribution. As such, an emissivity ratio of the \(n = 3\) dielectronic satellites to the entire \(wn3\) region can provide a \(T_e\) measurement \[19\].

Emissivity, \(\varepsilon\), is a volumetric emission rate, or photon emission rate per unit volume, that is a function of density and temperature. It can be described by

\[
\varepsilon = n_en_qf(T_e),
\]

where \(n_e\) is electron density, \(n_q\) is the impurity charge state density, and \(f\) \((T_e)\) indicates a function of electron temperature. Motivation for taking a ratio of these spectral regions is due to the cancellation of the density dependencies in (1), causing the temperature-dependent excitation rates to be the main parameter \[23\]:

\[
\frac{\varepsilon_1}{\varepsilon_2} = \frac{f_1(T_e)}{f_2(T_e)} = f(T_e).
\]

As discussed in \[2\], the emissivity of the \(n = 3\) satellites has the form,

\[
\varepsilon_{\text{sat}} = n_en_qf_{\text{sat}}(T_e) \approx n_en_q\sum_i F_i \exp \left(-\frac{h\nu_i}{T_e}\right) \frac{1}{T_e^{3/2}},
\]

where \(F_i\) is the line factor, while the emissivity of the \(w\)-line is,

\[
\varepsilon_w = n_en_qf_{\text{exc}}(T_e) \approx n_en_qg \frac{\exp \left(-\frac{h\nu}{T_e}\right)}{\sqrt{T_e}},
\]

where \(f_{\text{exc}}(T_e)\) is for excitation and \(g\) is the Gaunt factor. Taking a ratio of (3) to (4) will yield

\[
\frac{\varepsilon_{\text{sat}}}{\varepsilon_w} \sim k \frac{1}{T_e}.
\]

One aspect of the He-like argon spectrum that has the potential to influence the production of valid \(T_e\) measurements at low temperatures is the degree of recombination.
populating the upper level for the $w$-line, or the ratio of $n_{q+1}$ to $n_q$. For the case of He-like argon, this is the $\text{Ar}^{17+}$ to $\text{Ar}^{16+}$ ratio, which can be specified in the theoretical computations of the argon $K$-shell rates. In the presence of recombination,

$$\varepsilon_w = n_e n_q f_{\text{exc}}(T_e) + n_e n_{q+1} f_{\text{rec}}(T_e),$$

(6)

and

$$\frac{\varepsilon_{\text{sat}}}{\varepsilon_w} = \frac{f_{\text{sat}}(T_e)}{f_{\text{exc}}(T_e) + \frac{n_{q+1}}{n_q} f_{\text{rec}}(T_e)}.$$  

(7)

In this experiment, electron temperatures were produced from a ratio of the $n = 3$ satellites to the $wn3$ region, which is qualitatively related to (7).

2. Experimental Methods

2.1. Target Plasma Conditions

In validating the use of x-ray line ratios for electron temperature measurements, certain plasma parameters were controlled to ensure that a wide range of temperatures was analyzed. One criterion was that the plasma was Ohmically heated. Furthermore, there were shot-to-shot plasma current and toroidal field scans with density adjusted during the discharge, which collectively allowed for a large $T_e$ range to be considered. Based on these parameters, six shots were available to analyze: 1100325010, 1100325012, 1100325016, 1100325023, 1100325025, and 1100325029. Time histories of these shots are shown in Figure 2.

X-ray spectra are measured from $0.512 \, \text{s} \leq t \leq 1.52 \, \text{s}$ in intervals of $\Delta t = 0.04 \, \text{s}$. The radial profiles for each electron temperature are averaged over this time interval prior to comparison. Since the temperature is a function of normalized minor radius, $\rho$, it allows for a direct comparison between emissivity ratio and electron temperature, as the tomographic inversion procedure computes emissivity as a function of $\rho$ as well. Argon $K$-shell population modeling from FAC uses experimental electron temperature data.

2.2. Obtaining an Emissivity Ratio

Measured 2D images from HIREXSR were discretized to form an averaging of the spectral brightness data (AVESPEC) as described in [18], and the $wn3$ region was isolated from the remainder of the He-like argon spectrum. Multiple Gaussians were fit to the AVESPEC, which were later integrated over wavelength to produce a given brightness. The brightness profile as a function of channel number can be passed through a tomographic inversion procedure that computes the emissivity as a function of $\rho$.

This process was performed for the $n = 3$ satellite group and for the entire $wn3$ region, producing an emissivity for both. Taking a ratio of these emissivities then allows for direct comparison with independent measures of $T_e$ as a function of $\rho$ and is interpolated as necessary. The TS and ECE data were already recorded as functions of
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While it is possible to take a ratio of the $n = 3$ satellite line to the $w$-line, the starting and ending wavelengths of the $w$-line are naturally ambiguous due to the neighboring satellites that blend into the end of the $w$-line. Therefore, taking the emissivity ratio of the $n = 3$ satellite line to the $wn3$ spectrum will produce similar results while consistently taking the ratio with respect to a defined wavelength range.

2.3. The Fitting Procedure

In order to minimize the error propagated through the tomographic inversion procedure, the multi-Gaussian fitting code to the AVESPEC had to be optimized. In the fitting code, there is a variety of parameters that can be changed. This includes the number of Gaussians to fit to the specified spectral region, the width and center of the Gaussians, and the tying of the Gaussians. This tying connects all of the satellites together as a set of features that will change relative to the resonance line. These parameters are passed to the Interactive Data Language (IDL) MPFITFUN function [25] that relies on constraints referred to as PARINFO. An optimized fit was obtained when the reduced $\chi^2$ [24]. Consequently, a plot of emissivity ratio as a function of temperature was created once this procedure was run for all 26 time points per shot.

Figure 2. Time history of analyzed shots (black: 1100325010, green: 1100325012, blue: 1100325016, purple: 1100325023, pink: 1100325025, red: 100325029).
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Table 1. Location of $n = 3$ satellites Gaussian.

<table>
<thead>
<tr>
<th>Center Wavelength (Å)</th>
<th>Reduced Chi-Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.9556</td>
<td>7.20</td>
</tr>
<tr>
<td>3.9557</td>
<td>6.78</td>
</tr>
<tr>
<td>3.9558</td>
<td>6.60</td>
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<tr>
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<td>6.78</td>
</tr>
<tr>
<td>3.9561</td>
<td>7.04</td>
</tr>
<tr>
<td>3.9562</td>
<td>7.37</td>
</tr>
<tr>
<td>3.9563</td>
<td>7.86</td>
</tr>
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</table>

Table 2. Location of fifth Gaussian.

<table>
<thead>
<tr>
<th>Center Wavelength (Å)</th>
<th>Reduced Chi-Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/A (4 Gaussians)</td>
<td>6.60</td>
</tr>
<tr>
<td>3.945</td>
<td>5.06</td>
</tr>
<tr>
<td>3.946</td>
<td>6.12</td>
</tr>
<tr>
<td>3.947</td>
<td>4.45</td>
</tr>
<tr>
<td>3.948</td>
<td>6.49</td>
</tr>
</tbody>
</table>

Chi-squared was minimized without over-constraining the data.

The location of a Gaussian modeling the $n = 3$ satellites was initially set to 3.9563 Å, resulting in an average reduced chi-squared value of 7.86 across all shots, channels, and time points. The changes made to the location of the $n = 3$ Gaussian are shown in Table 1, which resulted in an average reduced chi-squared value of 6.60. The fitting procedure initially started with four Gaussians with central wavelengths of 3.94912 Å, 3.9508 Å, 3.9519 Å, and 3.9558 Å, but two additional Gaussians increased the quality of the fit to the spectral data. A fifth Gaussian with a central wavelength of 3.9470 Å decreased the average reduced chi-squared to 4.45 and is shown in Table 2. While not as significant an improvement as the change in the location of the $n = 3$ Gaussian and the added 3.947 Å Gaussian, a sixth Gaussian with a central wavelength of 3.945 Å lowered the reduced chi-squared further to 4.22. It should be noted that the He-like argon $w$-line has a very minor non-Gaussian shape due to natural line broadening, so the additional fifth and sixth Gaussians account for this small residual at the lower wavelengths.

Since the line-ratio approach compares the $n = 3$ satellites to the $wn3$ region, it is important to accurately describe the $n = 3$ emission. Gaussians modeling the $n = 5$ and $n = 4$ satellite groups at 3.9508 Å and 3.9519 Å, respectively, were both tied to the $n = 3$ satellites with a Gaussian center of 3.9558 Å. Furthermore, the width of the $n = 4$ Gaussian was tied to the width of the $n = 3$ Gaussian. This tying allows for the $n = 3$ features to be obtained without the $n = 4$ Gaussian blending into the $n = 3$ region. The widths of the Gaussians used in the fitting procedure had a lower bound of 50% of the $w$-line width and an upper bound of 150% of the $w$-line width. The baseline of
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Figure 3. $wn3$ multi-Gaussian fitting procedure performed on a sample He-like argon AVESPEC (shot: 1100325012, channel: 24, time: 1.1985 s).

3. Results

The spectral brightnesses were integrated over wavelength and then passed through a tomographic inversion procedure to obtain emissivity values. Examples of this integration are shown in Figure 4 for the $n = 3$ satellite group and in Figure 5 for the $wn3$ spectral region. The emission is assumed not to vary within a flux surface, and a linear regularization technique was used to derive a smooth profile. Small scale structure in the residual is due to systematic error in detector flat-field calibration. By observing both the top (channel $>32$) and bottom (channel $<32$) of the plasma, the impact becomes negligible.

Since the data will not be accurate over the entire range of $\rho$, notably at the edge of
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Figure 4. $n = 3$ satellites brightness (shot: 1100325012, time: 0.7585 s). The black points shown are the experimental brightness values while the green line represents the brightness computed from the derived emissivity.

Figure 5. wn3 brightness (shot: 1100325012, time: 0.7585 s). The black points shown are the experimental brightness values while the green line represents the brightness computed from the derived emissivity.

the plasma where recombination becomes a significant factor, it is important to consider a valid domain of $\rho$ values to analyze. Figure 6 shows the emissivity obtained from the tomographic inversion of the line-integrated brightness for the $n = 3$ satellites and the entire wn3 region as a function of $\rho$. As is evident from the emissivity ratio vs. $\rho$ plot in Figure 7, the uncertainty in the emissivity ratio significantly increases beyond $\rho \approx 0.7$, and the monotonically increasing trend in emissivity ratio as a function of $\rho$ no longer exists. Therefore, the following electron temperature comparisons are plotted for $0 \leq \rho \leq 0.7$; nevertheless, the region of $0.7 \leq \rho \leq 0.9$ carries information about the level of recombination in the plasma and is addressed as well in Figure 13.

The comparison of emissivity ratio with the TS data ($\rho \leq 0.7$) is shown in Figure 8. Any points that had $x$-error bars greater than or equal to 0.3 keV or $y$-error bars
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Figure 6. Emissivity of the $n = 3$ satellites (black asterisks) and wn3 spectrum (red x’s) as functions of $\rho$. Note that the emissivity for the $n = 3$ satellites has been multiplied by a factor of ten in this plot for comparison purposes (shot: 1100325012, time: 1.1985 s, core $T_e \approx 2$ keV).

Figure 7. Emissivity ratio as a function of $\rho$ (shot: 1100325012, time: 1.1985 s, core $T_e \approx 2$ keV).

greater than or equal to 0.05 were removed in order to maintain a meaningful electron temperature trend that could be used to obtain a temperature from a given emissivity ratio. As a result, 1629 data points were plotted out of an original set of 1831 points (89.0% retained). The curves in colour with different line-styles that are plotted over the TS data are from the FAC [12]. Each of the five FAC curves has a unique Ar$^{17+}$ to Ar$^{16+}$ ratio: 1.0, 0.1, 0.01, 0.001, and 0.0001 from the bottom curve to the top curve. The same FAC curves are used in the following plots.

The comparison of emissivity ratio with the ECE data ($\rho \leq 0.7$) is shown in Figure 9. Any points that had $y$-error bars greater than or equal to 0.05 were removed. As a result, 950 data points were plotted out of an original set of 956 points (99.4% retained). The $x$-error bars were set as 10% of the temperature in keV at each point, as is typically
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Figure 8. Thomson scattering $T_e$ for $\rho \leq 0.7$.

Figure 9. Electron cyclotron emission $T_e$ with $\rho \leq 0.7$.

assumed with Alcator C-Mod electron cyclotron emission measurements.

A binned $T_e$ trend with 100 eV bins using the mean value of the data is shown in Figure 10 where $\rho \leq 0.7$. The $y$-errors bars were obtained from taking the standard deviation of the $y$-error within each 100 eV bin, and the $x$-error bars were set to $\pm 50$ eV.

Electron temperature as a function of emissivity ratio – with the $x$ and $y$ axes switched – and a power regression is shown in Figure 11. This fitted curve has an equation of $T_e[\text{keV}] = 0.1552x^{-0.7781}$, has a coefficient of determination of $r^2 = 0.971$, and is experimentally valid over the emissivity ratio range of $x = 0.0223$ to $x = 0.2449$ (refer to the supplementary information for a table of values of the experimental data points). Figure 12 compares experimental results with FAC and theoretical results from AUTOSTRUCTURE [10] with no recombination population. While the measured emissivity ratios are marginally less than FAC predictions from 1 keV to 2 keV, AUTOSTRUCTURE shows better agreement. Nevertheless, FAC predicts an approximately 10% higher ratio for all $T_e$. 
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Figure 10. Binned $T_e$ trend using 100 eV bins and the mean value based on Thomson scattering and electron cyclotron emission with error bars shown where $\rho \leq 0.7$.

Figure 11. $T_e$ as a function of emissivity ratio based on 100 eV bins and the mean values of Thomson scattering and electron cyclotron emission data with error bars is shown. The green line is an experimental fit of $T_e [\text{keV}] = 0.1552 x^{-0.7781}$ in the emissivity ratio range of $x = 0.0223$ to $x = 0.2449$ with data plotted for $\rho \leq 0.7$.

Figure 12. Comparison of experimental results with the Flexible Atomic Code and AUTOSTRUCTURE.
4. Discussion

The emissivity ratio trend with $T_e$ from TS and ECE (Figure 10) demonstrates, as anticipated, that x-ray line ratios from XICS can be used to obtain a reliable $T_e$ at the 10% level. Both the TS (Figure 8) and ECE (Figure 9) results reveal nearly identical trends between emissivity ratio and electron temperature, emphasized in the 100 eV
binned $T_e$ plot (Figure 10). The binned profile also indicates that the ratio of Ar$^{17+}$ to Ar$^{16+}$ is between 1.0 and 0.1 for $0.7 \text{ keV} < T_e < 2 \text{ keV}$.

Even though electron temperatures were produced in the range of $0.5 \text{ keV} \leq T_e \leq 3.0 \text{ keV}$, the accuracy of obtaining a $T_e$ value directly from an emissivity ratio is limited at high temperatures as the slope approaches zero. In this region, a given emissivity ratio with finite uncertainty can ambiguously correspond to a wide range of electron temperatures. While this may be sufficient for astrophysical plasmas, laboratory experiments interested in gyrokinetic model validation, where turbulence is sensitive to $\nabla (T_e)/T_e$, may not be able to use XICS line ratios. When the slope approaches zero, a different element can be used, but this requires significant diagnostic modification.

At electron temperatures greater than approximately 2 keV, the degree of recombination does not significantly impact the $T_e$ (Figure 13). In contrast, at electron temperatures less than approximately 0.8 keV, it becomes evident that the ratio of Ar$^{17+}$ to Ar$^{16+}$ plays an important role. This is especially clear at lower electron temperatures where the turnover when $\rho > 0.7$ data are included. It may be possible to account for this effect by including data from the $z$-line, which has a stronger population from recombination than the $w$-line.

While the use of x-ray line ratios from XICS is a valid means of measuring $T_e$, it is worth analyzing the use of an emissivity ratio compared to a brightness ratio. If these two ratios produced identical $T_e$ trends, the data points of an emissivity ratio vs. brightness ratio plot would fall along a line with a slope of 1, which is not the case as shown in Figure 14. In this plot, the emissivity ratio, a local quantity, at a given minor radius in the plasma is compared against the integrated brightness that has a tangency radius (e.g. impact radius, radius of closest approach, etc.) at the same minor radius. Since utilizing emissivity ratios requires the use of a tomographic inversion procedure, it has the advantage of producing local quantities. Even though the emissivity ratio and brightness ratio may be nearly equal at low $\rho$ values, as $\rho$ increases so does the difference between emissivity ratio and brightness ratio. Additionally, the use of a brightness ratio would indicate higher electron temperatures compared to the equivalent emissivity ratio. As the electron temperature decreases at high $\rho$ values, the emissivity ratio and brightness ratio profiles diverge further, which narrows the range of temperatures obtained from comparing line-integrated brightness to $T_e$ directly. Offsetting systematic errors are possible. For example, if AUTOSTRUCTURE were used for $T_e > 2 \text{ keV}$, where measured ratios are 5 to 10% higher than predicted, and brightness ratios used in place of emissivity ratios the effects would cancel. This stresses the importance of spatially-resolved XICS data to validate $T_e$-dependent line ratios.

It has been shown that a $T_e$ vs. emissivity ratio curve produced from experimental data can be used to determine an electron temperature based on a measured emissivity ratio independently from all theory. Provided that a similar spectral analysis approach is implemented, as discussed in Section 2.3, this can have applications for accurately measuring $T_e$ in certain astrophysical plasmas based on line ratio data. However, no
spatial imaging is used for analyzing astrophysical plasmas, as all observations are based on brightness ratios. Even though the He-like system is well understood, the same procedure could be used on more complex ions.

5. Conclusion

For the first time, He-like x-ray emissivity ratios from XICS have been validated against conventional electron temperature measurements in tokamak plasmas. Not limited to Alcator C-Mod, these results are applicable for other magnetic confinement fusion plasmas that have XICS measurements available. The use of local argon line ratios, specifically in the region containing the \( n \geq 3 \) satellites and the \( w \)-line, for electron temperatures in the range of \( 0.5 \text{ keV} \leq T_e \leq 3.0 \text{ keV} \) was successfully validated against existing measurements from Thomson scattering and electron cyclotron emission. An experimental power fit of \( T_e [\text{keV}] = 0.1552x^{-0.7781}, \) where \( x \) is an emissivity ratio value in the range of \( 0.0223 \leq x \leq 0.2449 \) for argon, accurately describes the electron temperature of the plasma. Despite this equation’s validity for \( \rho \leq 0.7 \), at higher \( \rho \) values, recombination into the \( w \)-line and thus the ratio of \( \text{Ar}^{17+} \) to \( \text{Ar}^{16+} \) becomes an influencing factor on the \( T_e \) values that needs to be considered.

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References

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