Demand Cycles and Heterogeneous Conformity Preferences

Leonie Baumann* and Wojciech Olszewski†‡

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Abstract

We explore the dynamics of demand for \( n \) designs of a good when agents have preferences for (anti-)conformity. Agents differ in their social status and each agent seeks to imitate those of higher status and to distinguish herself from those of lower status, relative to her own status. In each period, every agent chooses a design given each agent’s demand in the previous period. We show that demand dynamics resemble fashion cycles: Total demand for each design over time is repetitively bell-shaped, and, when positively demanded, a design trickles from high- to low-status individuals. At least for \( n = 3 \), the demand dynamics converge to a unique limit cycle. We obtain a similar (though weaker) convergence result for \( n = 4 \), and simulations suggest that the result holds for \( n = 4 \) and 5.

Keywords: fashion cycle, demand cycle, preference for conformity

JEL Codes: C73, D11, D91, E21, E32, E71, Z13

*Department of Economics, McGill University. Email: leonie.baumann@mcgill.ca.
†Department of Economics, Northwestern University. Email: wo@northwestern.edu.
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1 Introduction

Many choices made by consumers exhibit trends and fashions. Old fashions are replaced by new ones in everlasting repetitions. At the beginning of a typical fashion pattern, the demand for a product is low. Then, the demand grows until it reaches its peak, and declines until the fashion for the product ends. These demand dynamics are the life cycle of a fashion. When one fashion disappears, the life cycle of the next fashion takes off (Kotler, 1997, p. 533). In this sense, we can speak of repetitive fashion cycles. Clothing, food and nutrition, sports, neighborhoods, holiday destinations, topics in academia are only some choice categories among many which feature fashion cycles. In this paper, we show that such fashion cycles can arise when individuals have heterogeneous preferences for individuality and conformity.

Anecdotal evidence for heterogeneous preferences for individuality and conformity is manifold. Statements like “you cannot wear these glasses anymore, everybody is wearing them now” or “I do not like this band anymore, they have become so popular that they even charge for their concerts” express the wish to distinguish oneself from the majority. On the other hand, mottos like “do as your neighbors do” or “all my friends have x, so I also want x” express the wish to conform. Heterogeneous (anti-)conformity preferences have been confirmed by sociological and psychological research. In a well-known experiment, Asch (1951) found different tendencies among subjects to conform to the majority. Some subjects conformed to the majority in order not to differ, others “withdrew” from the majority because of the desire of “being an individual”. Snyder and Fromkin (1980) introduced the uniqueness theory according to which an individual’s need for uniqueness determines how similar or dissimilar she wants to be to others. Lynn and Harris (1997), Ariely and Levav (2000) and Timmor and Katz-Navon (2008) provided empirical evidence that an individual’s distinctive behavior in various environments is driven by her need for uniqueness. Berger and Heath (2008) adduce an individual’s desire to distinguish herself from disliked others as a further argument for anti-conformity.
Traditionally, fashion cycles have been a topic of sociological research. Veblen (1912), Simmel (1957) and Bourdieu (1984, 1993) proposed the trickle-down theory as an explanation of fashion cycles. According to the trickle-down theory, fashion goods serve upper classes to distinguish themselves from lower classes who in turn emulate the upper classes. Thus fashions move from the upper classes down to the lower classes. In criticism of the trickle-down theory, Blumer (1969), Sproles (1981), McCracken (1988), King (1965), Field (1970), and Vejlgaard (2008) separate fashion leadership from the upper class and argue that fashion innovators and leaders can come from any class, also lower classes and subcultures. They do not see the primary purpose of fashion in class distinction. Groups in the fashion diffusion process are roughly divided into “innovators, leaders, followers, and participants” (Blumer, 1969), whereby fashions trickle from innovators, over leaders and followers to participants. The underlying, not explicitly stated driving forces of these fashion cycle theories are heterogeneous conformity and anti-conformity preferences. Park (1998), Cholachatpinyo et al. (2002) and Workman and Kidd (2000) explicitly link the fashion process to heterogeneous (anti-)conformity preferences. They provide empirical evidence that fashion innovators are driven by the need to distinguish themselves and to be unique, whereas followers have stronger preferences for being similar to others and to conform.

This paper contains a simple formal economic model, which shows – with a minimal structure on preferences and behavior – how demand for goods waxes and wanes in cycles. Specifically, we propose a discrete time model in which the population is heterogeneous with respect to conformity preferences. Each agent’s conformity preference is defined by her location on the unit interval [0,1], with 0 representing the most individualistic preference, and 1 representing the most conformistic preference. In each period, every agent chooses which one of $n$ available designs of the same good to consume on the basis of the previous period distribution of demand across the population. In particular, an agent wants to imitate the others who are more individualistic than herself (perhaps of higher social status), and distinguish herself from the others who are more conformistic (perhaps of lower social
status). The agents’ choices divide them into subintervals of [0, 1] such that the agents from the same subinterval choose the same design, and the agents from different subintervals choose different designs. These partitions evolve over time.

The model generates typical fashion life cycles, which are the bell-shaped demands for each design over time, and the “trickling” process through heterogeneous (anti-)conformity preferences. The demand for a design moves from the most-left subinterval of [0, 1] to more-right subintervals, that is, from the consumers with most individualistic preferences to the consumers with more and more conformistic preferences. These dynamics of demand repeat over time. In the terminology of sociological theories, the fashion item trickles down from upper classes to lower classes, or from innovators over leaders and followers to participants. We show that for \( n = 3 \), independently of the initial subinterval distribution of demand for the available designs, the dynamics of demand converge to an infinitely repeated unique finite-period cycle spanning eight time periods. This finding is partially confirmed by our analytical and numerical results for \( n = 4 \) and \( 5 \). Finally, we deliver some comparative statics results, which illustrate how fashion cycles are affected by alternative assumptions regarding (anti-)conformity preferences.

**Related Literature.** In addition to the papers that we have already discussed, demand cycles with reference to heterogeneous conformity preferences have been studied in several other articles. The main distinctive feature of our analysis is the focus on the properties of fashion cycles in the presence of numerous designs, such as: trickling through the society, convergence, or the possibility that the individuals most to the left in the social hierarchy adapt – for the purpose of differentiating themselves – the designs chosen by the individuals most to the right.

Karni and Schmeidler (1990) consider an economy in which agents are divided into two social classes \( \alpha \) and \( \beta \). The members of the former class want to imitate other \( \alpha \) members and distinguish themselves from \( \beta \) members, and the members of the latter class want to conform to both \( \alpha \) and other \( \beta \) members. Consumers live in 9 periods, and every third period, they choose
a color of some good. Karni and Schmeidler describe an equilibrium of their model in which the demand for each color of the good displays a life cycle of six periods. Their analysis is most closely related to, and was most inspiring for our research. Their paper, however, contains only a carefully constructed example of a fashion cycle, which corresponds to our 3-designs limit cycle. In contrast, we explore the dynamics in a more general setting, and provide convergence and comparative statics results.

In Corneo and Jeanne (1999), agents are divided into two groups: “natives” and “tourists”. Agents decide whether to visit location 1 or 2. Tourists like to visit locations where natives go, but natives prefer to visit where there are fewer tourists. Location 2 is only known to some agents and information about it spreads via a random matching process between agents. If location 2 is better known among natives initially, then fashion cycles arise in which the natives act as fashion leaders and the tourists as followers. This model cannot explain fashion cycles in markets without information asymmetries. Our model also allows for studying a richer variety of fashion cycles, and some novel comparative statics result.

The following papers are less closely related to our analysis, but also explore heterogeneous conformity preferences as a driver for fashion cycles. Matsuyama (1991) analyzes a random matching game with nonconformists and conformists. Individuals choose between two consumption options before they are matched. Conformists obtain a high payoff if their match chose the same option, and nonconformists obtain a high payoff from miscoordination. If the intergroup matching is sufficiently likely and there is some choice inertia in the society, then the demand for the two options resembles fashion cycles with nonconformists acting as fashion leaders and conformists as followers. Berger et al. (2011) introduce a model in which an agent chooses from a number of goods while being concerned about her social image. This induces individuals to choose what others who she likes consume, and not to choose what the disliked others consume. The authors provide some results and intuition how their model might give rise to fashion cycles, but without exploring the dynamics rigorously. Granovetter and Soong (1983) model heterogeneous (anti-)conformity preferences by introducing agent-specific lower
and upper thresholds on the fraction of population above and below which the agent participates in a movement. If the lower threshold has not been reached, too few people participate in the movement for the agent to join, whereas if the upper threshold has been surpassed, too many people participate for the agent to join. The authors find that for certain distributions of lower and upper thresholds across the population, participation rates oscillate between different levels.

Another driver for fashion cycles, different from heterogeneous conformity preferences, has been suggested and explored in Pesendorfer (1995). The author shows that when the designs of consumed goods are a signaling device for an unobserved “quality” of consumers, then dynamic demand for different designs developed by a monopolist resembles fashion cycles. In his model, a new design serves as a signal for “high” types because of a high initial price which prevents “low” types from buying the design. Over time, the monopolist decreases the price such that more low types buy the design, and the signal value for high types is lost. At this point, the monopolist introduces a new design at a high price which serves as a new signal. This model cannot, however, explain fashion cycles in the categories of goods which have the same price, for example, as it is the case for given first names (Yoganarasimhan, 2017). Moreover, it does not account for the fact that trends are often started by individuals with a low budget and at low prices (Field, 1970; Sproles, 1981; Vejlgaard, 2008). For example, certain clothing and music styles were initiated in the youth and street culture, or lower income classes. Our model displays fashion cycles in the demand for goods with no price differences across designs, and accommodates fashions that are started by individuals with a low purchasing power.

**Roadmap.** The rest of the paper is organized as follows. In Section 2, we present our model. In Section 3, we study the special case of the model with \( n = 3 \) designs. We characterize the dynamics of demand in this case, and prove the convergence to a limit cycle. Section 4 contains the results in the more general setting of \( n \geq 3 \). We show that the demand dynamics exhibit periodic bell-shaped patterns, and that each design trickles from the
“top” towards the “bottom” of the population. We also present some partial convergence results in this more general setting. Finally, we discuss some alternative assumptions on preferences in Section 5, as well as the way in which the alternative assumptions affect the main results of the paper. We also provide some comparative statics results illustrating the effect of these new assumptions on the dynamics. We conclude in Section 6.

2 Model

A unit-mass population is continuously distributed according to a density function \( f \) on the \([0, 1]\)-interval. Every agent is indexed by \( i \in [0, 1] \) with \( i \) being equal to her location on the \([0, 1]\)-interval. An agent’s location is fixed and exogeneously given. The \([0, 1]\)-interval can be interpreted as a scale of social status with \( i = 0 \) being the highest status (maybe richest) agent and \( i = 1 \) being the lowest status (maybe poorest) agent.\(^1\)

Let \( a : [0, 1] \to (0, 1) \) be a continuous weight function where \( a(i) \) is the weight assigned to the agents at location \( i \). The weight function captures the idea that different agents might differ with respect to their “visibility” within the population. For example, agents in \([.1, .2]\) (say, celebrities) might be more visible in the population than agents in \([0, .1]\) (say, an “invisible” elite).

There are \( n \geq 3 \) available designs \( \{S_1, ..., S_n\} \) of some good. We assume that the price of each design is zero. This allows us to focus on the effects of heterogeneous conformity preferences.

The economy evolves in discrete time. Agents are assumed to be myopic. In each time period \( t > 0 \), each agent chooses to consume one design \( S_k \) that maximizes her utility given the distribution of demand in the previous period \( t-1 \). Let \( f_{t-1}(S_k, i) \) denote the mass of agents at location \( i \in [0, 1] \) that consume design \( S_k \) in period \( t-1 \). The distribution of demand in the previous period, \( f_{t-1} \), is common knowledge. We assume that every agent \( i \) wants to be similar to agents \( j < i \) (agents of higher status) and wants to distinguish

\(^1\)The social status may also refer to other attributes, for example, knowledge and “good taste” in the field of fashion, music or cuisine, or “coolness”.
herself from agents \(j > i\) (agents of lower status). This means as well that agents differ in how desirable they are to be imitated, in a manner that is inversely related to their own level of conformism: the more individualist an agent (the lower \(i\)), the larger the mass of agents \((j > i)\) who want to conform with \(i\)’s choice. Plausible channels for such a behavior might be a desire to signal high status or to appear fashionable where higher status agents have a superior fashion knowledge. We formalize these preferences in the following utility function. Agent \(i\)’s utility in period \(t\) from design \(S_k\) is given by

\[
u_i^t(S_k) = \int_0^i a(j) f_{t-1}(S_k, j) \, dj - \int_i^1 a(j) f_{t-1}(S_k, j) \, dj
\]

where the first and the second expression is the mass of agents \(j < i\) and \(j > i\), respectively, consuming \(S_k\) in \(t - 1\), with each agent \(j\) weighted according to her visibility.

The following tie-breaking rule for the case of indifference between two designs is assumed. If \(u_i^t(S_k) = u_i^t(S_l)\) with \(k < l\), then \(i\) strictly prefers \(S_k\) over \(S_l\).\(^2\) This ensures tractability of the dynamic distribution. Such lexicographic preferences arise for example if \(S_k\) gives a higher physical utility than \(S_l\), but agents primarily care about social utility and only secondarily about physical utility. Manzini and Mariotti (2007) show that such a sequential use of two rationales can account for observed cyclical choice behavior. Observe that the tie-breaking rule implies that in any period \(t > 0\), there is a unique option which is consumed by every agent at location \(i\). Thus \(f_{t-1}(S_k, i) \in \{0, f(i)\}\).

To gain intuition for agents’ preferences consider the following example with \(n = 3\), a uniform distribution of agents, and a constant weight function. Suppose that in period \(t - 1\) all agents \(i \in (0, 0.3)\) consume \(S_1\), all \(i \in (0.3, 0.5)\) consume \(S_2\), and all \(i \in (0.5, 1)\) consume \(S_3\). Figure 1 shows agent \(i\)’s utility in period \(t\) for each design given the distribution in period \(t - 1\) for all \(i \in [0, 1]\).

\(^2\)This assumption plays the role of a systematic tie-breaking. To see why suppose that design A is consumed by \([0, 1/3]\) and design B is consumed by \((1/3, 2/3)\). Then the agents from \((2/3, 1]\) would be indifferent between A and B, and, without systematic tie-breaking, they could partition themselves in many ways.
The utility in period \( t \) from design \( S_k \) is strictly increasing with a slope of 2 over the interval of agents who consumed \( S_k \) in period \( t - 1 \), everywhere else it is constant. From the figure it is obvious that the consumption choices in period \( t \) are such that all \( i \in (0, 0.3) \) choose \( S_1 \), all \( i \in (0.3, 0.5) \) choose \( S_2 \), and all \( i \in (0.5, 1) \) choose \( S_3 \).

In the analysis of the demand over time, we restrict to the following class of initial distributions. In period 0, the grand interval of types \([0, 1]\) is partitioned by cutpoints \( 0 = x_{0}^{0} < x_{1}^{0} < ... < x_{m(0)}^{0} = 1 \) so that the types from each interval \((x_{k-1}^{0}, x_{k}^{0})\), where \( k = 1, ..., m(0) \leq n \), choose the same design, and the types from different intervals \((x_{k-1}^{0}, x_{k}^{0})\) choose different designs. This implies that in each period \( t \geq 0 \), the grand interval of types is partitioned by cutpoints \( 0 = x_{0}^{t} < x_{1}^{t} < ... < x_{m(t)}^{t} = 1 \) so that the types from each interval \((x_{k-1}^{t}, x_{k}^{t})\), where \( k = 1, ..., m(t) \leq n \), choose the same design, and the types from different intervals \((x_{k-1}^{t}, x_{k}^{t})\) choose different designs. We disregard the choices of the finite sets of types \( \{x_{0}^{t}, x_{1}^{t}, ..., x_{m(t)}^{t}\} \). So, if \( m(t) < n \), we will say that there is no demand for some designs. We will call a design the design of the elite in period \( t \) if it is chosen by the types from interval \((x_{0}^{t}, x_{1}^{t})\). We will call a design the design of the bottom in period \( t \) if it is chosen by the types from interval \((x_{m(t)-1}^{t}, x_{m(t)}^{t})\).
3 Three Designs

We first characterize the dynamics of demand for \( n = 3 \) when the population is uniformly distributed (\( f \) is a constant) and all agents are weighted equally (\( a \) is a constant). We will see that the distribution of demand over time converges to a limit cycle. This implies that the demand for each option replicates the same cyclical pattern in the long run. In the limit cycle, the demand for each design is repeatedly bell-shaped and for each repetition, the design trickles from the top to the bottom of the population. The characterization for \( n = 3 \) with a uniform population also anticipates our results for the more general model.

**Proposition 1.** For \( n = 3 \) and any initial cutpoints (i.e., the cutpoints in period 0), the dynamics of demand converge to the following infinitely repeated cycle spanning 9 time periods:

\[
\begin{align*}
\text{Period}& & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\text{Cutpoints}& & 0 & \frac{1}{2} & \frac{3}{4} & 1 & \frac{1}{2} & \frac{3}{4} & 1 & \frac{1}{2} & \frac{3}{4} & 1 \\
\end{align*}
\]

The odd columns depict cutpoints (from the lowest to the highest), and the even columns depict the designs chosen by the types from the intervals between the cutpoints in the neighboring columns. In some rows, the last two entries are empty. This means that in the corresponding period, there is no demand for some design.

We first discuss Proposition 1 and then provide the proof.

Note the way in which the demand for each design evolves over the limit cycle, and so in sufficiently large periods.
Corollary 1. Suppose that $X$ becomes the design of the elite in period $t$ (but it was not the design of the elite in period $t - 1$). Then, either

(1) the demand for $X$ increases in period $t + 1$, increases again in period $t + 2$ when $X$ becomes the design of the bottom, decreases in period $t + 3$, and drops to 0 in period $t + 4$, and $X$ becomes the design of the elite again in period $t + 5$;

or

(2) the demand for $X$ increases in period $t + 1$, decreases in period $t + 2$ when $X$ becomes the design of the bottom, drops to 0 in period $t + 3$, and $X$ becomes the design of the elite again in period $t + 4$.

In addition, $X$ is chosen by every single type $i$ over each cycle.

The cyclical pattern of the dynamics of demand for each design is typical of fashions: Once a design is taken up by the elite, the growth path of the demand for the design is single-peaked. The design gradually moves through the entire type distribution from the elite to the bottom. Once having reached the bottom of the distribution, the process repeats with the elite picking up the design again. The bell-shaped demand for a design is consistent with research on product life cycles (for a discussion see Rink and Swan, 1979) which divides the evolution of demand for a product into four stages: the initial phase of introduction with low demand, the growth phase with rising demand, the maturity phase where demand has reached its peak, and finally a phase of decline where demand decreases until the product is out of the market. The reappearance of a design at the top of the market after the completion of a life cycle in our model is due to the limited set of designs. This can be interpreted as a revival of fashions commonly observed in reality.

The dynamic demand exhibits features that can be interpreted as an ever-going change in fashions: The demand for each design follows recurring life cycles, and the life cycles of different designs peak at different points in time. In Figure 2, we plot the demand for each design over 9 periods within the limit cycle described in Proposition 1. This illustrates continuously changing...
fashions. Let $t$ be a period in the limit cycle where designs $A$ and $B$ each receive $\frac{1}{2}$ of the total demand.

The rest of this section contains the proof of Proposition 1. The idea of the proof is that for any initial partition $0 = x^0_0 < ... < x^0_{m(0)} = 1$, the dynamics of the partition after a finite number of periods begin to follow the same pattern as that for the cycle described in Proposition 1 (see Lemma 3 for details).

**Lemma 1.** There is a $t$ such that $m(t) = 2$ and $x^t_1 \in (0, 1/2)$.

**Proof.** Case 1. If $m(0) = 1$, then $m(1) = 2$ and $x^1_1 = x^1_0/2 = 1/2$. So, $m(2) = 2$ and $x^2_1 = x^1_1/2 = 1/4$.

Case 2. If $m(0) = 2$ and $x^0_1 \in [1/2, 1)$, then $m(1) = 2$ and $x^1_1 = x^1_0/2 \in [1/4, 1/2)$.

Case 3. Suppose that $m(0) = 3$. We will show that $m(r) \leq 2$ for some $r$. We can then complete the proof by referring to Cases 1 and 2, in which 0 is replaced with $r$. Assume by contradiction that $m(r) = 3$ for all $r$. We show first that there exists $r$ such that the elite of period $r$ chooses in period $r+1$ the design of the bottom of period $r$. If $x^k_2 - x^k_1 \geq 1 - x^k_2$, then the elite must choose in period $k + 1$ the design of the bottom as otherwise $m(k + 1) \leq 2$. 

![Figure 2: Demand for each design within the limit cycle.](image-url)
If \( x_k^k - x_1^k < 1 - x_k^2 \), then the elite does not choose the design of the bottom in period \( k + 1 \). This implies that \( x_1^k < 1 - x_k^2 \) as otherwise \( m(k + 1) \leq 2 \). If the elite sticks with its design from period \( k \), then \( x_1^k \leq x_2^k - x_1^k \) and \( x_1^{k+1} = x_1^k + x_2^k/2 \) and \( x_2^{k+1} = x_2^k + (1 - x_1^k)/2 \). If the elite chooses the design of the “middle” from period \( k \), then \( x_1^k > x_2^k - x_1^k \) and \( x_1^{k+1} = x_1^k - x_2^k/2 \) and \( x_2^{k+1} = (x_2^k + x_1^k + 1)/2 \). Both cases imply that \( x_2^{k+1} - x_1^{k+1} \geq 1 - x_2^{k+1} \). Thus the elite must choose the design of the bottom in period \( k + 2 \) (otherwise \( m(k + 2) \leq 2 \)).

Suppose now that in period \( r + 1 \) the elite chooses the design of the bottom. Then it must be that \( 1 - x_2^r < x_1^r \) and \( 1 - x_2^r \leq x_2^r - x_1^r \). In addition, it must be that either \( x_1^r < x_2^r - x_1^r \) or \( x_1^r = x_2^r - x_1^r \) with the design of the middle in period \( r \) having a higher priority than the one of the elite in period \( r \), since otherwise we would have that \( m(r + 1) = 2 \). Thus,

\[
x_1^{r+1} = \frac{x_1^r + x_2^r - 1}{2} \quad \text{and} \quad x_2^{r+1} = \frac{x_1^r + x_2^r}{2},
\]

and this implies that \( x_2^{r+1} - x_1^{r+1} > 1 - x_2^{r+1} \) (since \( 1 - x_2^r < x_1^r \)) and \( x_2^{r+1} - x_1^{r+1} > x_1^{r+1} \). So, the elite chooses the design of the bottom in period \( r + 2 \) as otherwise \( m(r + 2) \leq 2 \).

Thus, after the elite chooses the design of the bottom in period \( r + 1 \), the elite chooses the design of the bottom in every period \( k > r + 1 \). This means that the size of the elite shrinks by more than a half from period \( k \) to period \( k + 1 \) for all \( k > r + 1 \). So, the size of the elite and the size of the bottom shrink to zero over time, \( x_k^k \rightarrow_{k=\infty} 0 \) and \( x_2^k \rightarrow_{k=\infty} 1 \), a contradiction to the second equation in (1).

\[\square\]

**Lemma 2.** There is a \( t \) such that \( m(t) = 2 \) and \( x_1^t \in [1/4, 1/3] \).

**Proof.** Let \( m(r) = 2 \) and \( x_1^r < 1/2 \). If \( x_1^r \in [1/4, 1/3] \), take \( t = r \). So, suppose that \( x_1^r \in (0, 1/4) \cup (1/3, 1/2) \). If \( x_1^r \in (0, 1/4) \), then \( m(r + 1) = 3 \) and \( x_1^{r+1} = x_1^r/2 \in (0, 1/8) \) and \( x_2^{r+1} = x_1^r + 1/2 \in (1/2, 3/4) \). This implies that \( m(r + 2) = 2 \) (since \( 1 - x_2^{r+1} < x_2^{r+1} - x_1^{r+1} \)), and \( x_1^{r+2} = x_1^r + 1/4 \in (1/4, 1/2) \). So, suppose that \( x_1^r \in (1/3, 1/2) \).
Define sequence \((z_n)_{n=1}^{\infty}\) by setting \(z_1 = 1/3\) and
\[
  z_{n+1} = \frac{4}{5} z_n + \frac{1}{10}, \text{ which is equivalent to } \frac{5}{4} z_{n+1} - \frac{1}{8} = z_n. \tag{2}
\]
Notice that this sequence is increasing and converges to \(1/2\). We will show by induction that if \(m(r) = 2\) and \(x^*_r \in (z_n, z_{n+1}]\), then there exists an \(s \geq r\) such that \(m(s) = 2\) and \(x^*_s \in (1/4, 1/3]\). Suppose that \(n = 1\) or that the statement is true for the numbers smaller than \(n\).

Since \((z_n, z_{n+1}] \subseteq (1/3, 1/2)\), we have that \(m(r+1) = 3\) and \(x^{r+1}_1 = x^r_1/2 \in (1/6, 1/4)\) and \(x^{r+1}_2 = x^r_1 + 1/2 \in (5/6, 1)\). This implies that the elite chooses in period \(r+2\) the design of the bottom from period \(r+1\). So, \(x^{r+2}_1 = (x^{r+1}_1 - 1 + x^{r+1}_2)/2 = 3x^r_1/4 - 1/4 \in (0, 1/8)\) and \(x^{r+2}_2 = x^{r+1}_1 + x^{r+1}_2/2 = x^r_1 + 1/4 \in (7/12, 3/4)\). Thus, \(m(r+3) = 2\) and \(x^{r+3}_1 = x^{r+2}_1 + x^{r+2}_2/2 = 5x^r_1/4 - 1/8 \in (z_{n-1}, z_n]\) by (2) for \(n > 1\), or \(x^{r+3}_1 \in (7/24, z_1] \subseteq (1/4, 1/3]\) for \(n = 1\).

Once a period \(t\) such that \(m(t) = 2\) and \(x^t_1 \in [1/4, 1/3]\) is reached, the dynamics begin to have a regular pattern. More specifically, the dynamics repeat a cycle spanning 9 periods, similar to that described in Proposition 1 except that the values of \(x^t_1\) and \(x^t_2\) vary over time.

**Lemma 3.** Let \(t\) be such that \(m(t) = 2\) and \(x^t_1 \in [1/4, 1/3]\). Then, the dynamics of demand repeat the following cycle spanning 9 time periods as described in Table 1:
\begin{tabular}{cccccc}
\hline
$r = t$ & 0 & $A$ & $x_1^t = x_1^t \in [1/4, 1/3]$ & $B$ & $x_2^t = x_1^t + 1/2 \in [3/4, 5/6]$ & $r = t + 1$ & 0 & $C$ & $x_1^{t+1} = x_1^t/2 \in [1/8, 1/6]$ & $A$ & $x_2^{t+1} = x_1^t + 1/2 \in [3/4, 5/6]$ & $B$ & 1 \\
$r = t + 2$ & 0 & $C$ & $x_1^{t+2} = x_1^t + 1/4 \in [1/2, 7/12]$ & $A$ & & 1 \\
$r = t + 3$ & 0 & $B$ & $x_1^{t+3} = x_1^{t+2}/2 \in [1/4, 7/24]$ & $C$ & & 1 \\
$r = t + 4$ & 0 & $A$ & $x_1^{t+4} = x_1^{t+3}/2 \in [1/8, 7/48]$ & $B$ & $x_2^{t+4} = x_1^{t+3} + 1/2 \in [3/4, 19/24]$ & $C$ & 1 \\
$r = t + 5$ & 0 & $A$ & $x_1^{t+5} = x_1^{t+3} + 1/4 \in [1/2, 13/24]$ & $B$ & & 1 \\
$r = t + 6$ & 0 & $C$ & $x_1^{t+6} = x_1^{t+5}/2 \in [1/4, 13/48]$ & $A$ & & 1 \\
$r = t + 7$ & 0 & $B$ & $x_1^{t+7} = x_1^{t+6}/2 \in [1/8, 13/96]$ & $C$ & $x_2^{t+7} = x_1^{t+6} + 1/2 \in [5/8, 74/96]$ & $A$ & 1 \\
$r = t + 8$ & 0 & $B$ & $x_1^{t+8} = x_1^{t+6} + 1/4 \in [1/2, 25/48]$ & $C$ & & 1 \\
\hline
\end{tabular}

Table 1
Lemma 3 follows immediately from the definition of preferences.

**Proof of Proposition 1.** In Lemma 3, \( x_{t+3}^t = x_{t+2}^t / 2 = x_t^t / 2 + 1/8 \). More generally,

\[
x_{t+3k}^t = x_{t+3(k-1)+2}^t / 2 = x_{t+3(k-1)}^t / 2 + 1/8
\]

for \( k = 1, 2, \ldots \). This implies that \( x_{t+3k}^t \to_{k=\infty} x \), where \( x = x/2 + 1/8 \). So \( x = 1/4 \). This obviously implies the convergence of the other cutpoints to the ones of the limit cycle described in Proposition 1.

\[\square\]

4 More than three Designs

In this section, we investigate the model for \( n \geq 3 \) again with the uniformly distributed and weighted population. In Section 4.1, we show that for all \( n \geq 3 \) the demand dynamics exhibit periodic bell-shaped patterns and that each design trickles from the top towards the bottom of the population. In Section 4.2, we present a specific cycle for \( n = 4 \). Numerical simulations suggest that this cycle is unique and that the dynamics converge to it.\(^3\) We prove formally that if the demand distribution happens to be in the proximity of the cycle, then it converges over time to the cycle.

4.1 Fashion Cycles

If the types from the interval \( (x_{k-1}^t, x_k^t) \) choose design \( X \), then we will call the length of the interval \( (x_{k-1}^t, x_k^t) \), the demand for design \( X \) in period \( t \), and we will denote this demand by \( d_t(X) \). If design \( X \) is not chosen by any interval of types in period \( t \), then there is no demand for design \( X \) in period \( t \) and we set \( d_t(X) = 0 \).

\(^3\)In the online appendix, we also present a specific cycle for \( n = 5 \). Again, simulations suggest that the cycle is unique and that the dynamics converge to it.
Theorem 1. The time path of the demand for each design consists of single-peaked cycles. More precisely, if $X$ is the design of the elite in some period $t$, but was not the design of the elite in period $t - 1$, then

$$d^t(X) \leq ... \leq d^*(X) \geq ... \geq d^t(X)$$

for some $\bar{t} \geq t^* \geq t$, such that $\bar{t} > t$. In addition, either (i) there is a positive demand for $X$ in period $\bar{t} - 1$ and no demand in period $\bar{t}$, or (ii) there is a positive demand for $X$ in period $\bar{t}$ but $X$ is not the design of the elite, and $X$ is the design of the elite in period $\bar{t} + 1$.

Proof. We will describe the way in which the demand for design $X$ changes from period $t$ to period $t + 1$. This depends on the position of the types who choose design $X$ in period $t$. To describe the changes, we consider the following two cases, numbered as I and II. Each of them will consist of several subcases, marked by letters A, B, etc., which in turn have subsubcases, numbered as 1, 2, etc.

I. Design $X$ is chosen by the interval $(0, x^t)$, that is, $X$ is the design of the elite in period $t$.

A. The demand for every design is positive in period $t$.

1. $\exists m > 1 x^t_1 \leq x^t_m - x^t_{m-1}$.

Then, the demand for design $X$ strictly increases in period $t + 1$. Indeed, the demand obviously increases when there is no $k > 1$ such that $x^t_1 > x^t_k - x^t_{k-1}$. In this case, all types from $(0, x^t_1)$ and some types from $(x^t_1, x^t_2)$ choose design $X$ in period $t + 1$. So, suppose that there is a $k > 1$ such that $x^t_1 > x^t_k - x^t_{k-1}$. Take the $k$ such that the design chosen by interval $(x^t_{k-1}, x^t_k)$ in period $t$ has the lowest demand among all designs, and this design has the highest priority among all designs with the lowest demand. Denote this design by $Y$. Let

$$x = x^t_{k-1} - x^t_k$$

(3)
and
\[ y = \frac{x^t_1 - (x^t_{k-1} - x^t_k)}{2}. \]  \hspace{1cm} (4)

Then, a measure \( y \) of types from interval \((0, x^t_1)\) chooses design \( Y \) in period \( t + 1 \) and the remaining measure from interval \((0, x^t_1)\) of \( x + y \) types chooses design \( X \). In addition, design \( X \) is chosen by a measure of at least \( x^t_1 = 2x + y \) types from interval \((x^t_{m-1}, x^t_m)\), where \( m > 1 \) is the lowest number such that \( x^t_1 \leq x^t_m - x^t_{m-1} \).

2. \( \forall m > 1 \ x^t_1 > x^t_m - x^t_{m-1} \).

Let \( k > 1 \) be such that the design chosen by the types from interval \((x^t_{k-1}, x^t_k)\) in period \( t \) has the lowest demand among all designs, and this design has the highest priority among all designs with the lowest demand. Denote this design by \( Y \). Define \( x \) and \( y \) by (3) and (4). In this case, a measure \( y \) of types from interval \((0, x^t_1)\) chooses design \( Y \) in period \( t + 1 \), and all remaining types from interval \((0, 1)\) choose design \( X \). Then, the demand for design \( X \) in period \( t + 1 \) may increase or decrease, depending on whether \( 1 - y > x + 2y \) or \( 1 - y < x + 2y \).

In addition, note that \( m(t+1) = 2 \), the types from interval \((0, x^{t+1}_1)\) choose design \( Y \), and the types from interval \((x^{t+1}_1, 1)\) choose design \( X \). Thus, the demand for \( X \) decreases in the following periods, until it becomes zero or the design becomes the design of the elite. In fact, the demand for \( X \) must decrease in period \( t + 2 \), because the elite in period \( t + 2 \) chooses a design for which the demand was zero in period \( t + 1 \).

B. The demand for at least one design is zero in period \( t \).

In this case, in period \( t + 1 \) a half of the types from interval \((0, x^t_1)\) chooses a design for which the demand was zero in period \( t \). Let this be design \( Y \). The remaining half chooses design \( X \).

1. \( \exists m > 1 \ x^t_1 \leq x^t_m - x^t_{m-1} \).

Then, the demand for design \( X \) strictly increases in period \( t + 1 \), because it is also chosen at least by a measure \( x^t_1 \) of types from interval \((x^t_{m-1}, x^t_m)\), where \( m > 1 \) is the lowest number such that \( x^t_1 \leq x^t_m - x^t_{m-1} \).

2. \( \forall m > 1 \ x^t_1 > x^t_m - x^t_{m-1} \). Note that this case also applies if \( x^t_1 = 1 \) and
there does not exist an \( x_m^t \) with \( m > 1 \).

Then, all types from \((x^t_1, 1]\) choose design \( X \). The demand for design \( X \) may increase or may decrease in period \( t+1 \), depending on whether \( x^t_1 < 1 - x^t_1/2 \) or \( x^t_1 > 1 - x^t_1/2 \).

The note made at the end of case I.A.2 also applies to this case. In particular, the demand decreases in the following periods, until it becomes zero or the design becomes the design of the elite.

The analysis of Case I completes the proof when design \( X \) begins as the design of the elite, and we are in subcase A.2 or B.2. When \( X \) begins as the design of the elite, and we are in subcase A.1 or B.1, the demand for \( X \) increases until we end up in subcase A.2 or B.2, or in Case II below. So, the proof will also be completed, if the single-peaked pattern is proved when we begin in Case II.

II. Design \( X \) is chosen by the interval \((x^t_{k-1}, x^t_k)\), that is, \( X \) is not the design of the elite in period \( t \).

A. \( \exists_{l<k} x^t_l - x^t_{l-1} > x^t_k - x^t_{k-1} \).

In this case, demand for design \( X \) in period \( t+1 \) can come only from the types in interval \((0, x^t_1)\). Moreover, this happens only when several conditions are satisfied. First, it must be that \( x^t_1 > x^t_k - x^t_{k-1} \). In addition, the demand for design \( X \) must be the lowest among the demands for all designs, and if there are other designs with the lowest demand, design \( X \) must have the highest priority among them. If these conditions are satisfied, then design \( X \) becomes the design of the elite in period \( t+1 \). Otherwise, the demand for design \( X \) drops to zero in period \( t+1 \).

B. \( \forall_{l<k} x^t_l - x^t_{l-1} < x^t_k - x^t_{k-1} \).

In this case, no type above \( x^t_k \) chooses in period \( t+1 \) a design chosen in period \( t \) by the types below \( x^t_{k-1} \). Of course, no type below \( x^t_{k-1} \) chooses design \( X \) in period \( t+1 \).

Let \( Y \) be the design chosen in period \( t \) by the types from interval \((x^t_{l-1}, x^t_l)\), where \( l < k \), whose length is the highest among such intervals, and if there are other such intervals with the highest length, then \( Y \) has the highest priority.
among the designs chosen by the types in these intervals with the highest length.

The types from interval \((x_{t-1}^k, x_t^k)\) are divided into those who choose design \(X\) and those who choose design \(Y\) in period \(t+1\). A measure \(x+y\) of them chooses \(Y\), and a measure \(x\) of them chooses \(X\), where

\[
y = x_t^l - x_t^{l-1}
\]

and

\[
x = \frac{(x_t^k - x_t^{k-1}) - (x_t^l - x_t^{l-1})}{2} > 0.
\]

1. \(\exists_{m>k} x_t^m - x_t^{m-1} \leq x_t^m - x_t^{m-1}\).

Let \(m\) be the lowest number with this property. Then, the demand for \(X\) strictly increases in period \(t+1\), because a measure \(x_t^m - x_t^{m-1}\) of types from interval \((x_{t-1}^m, x_t^m]\) chooses design \(X\).

2. \(\forall_{m>k} x_t^m - x_t^{m-1} > x_t^m - x_t^{m-1}\). Note that this case also applies if \(x_t^l = 1\) and there does not exist an \(x_t^m\) with \(m > k\).

Then, all types from interval \((x_t^l, 1]\) choose design \(X\) in period \(t+1\). So, the demand for design \(X\) may increase or may decrease in period \(t+1\), depending on whether

\[
\sum_{m>k} (x_t^m - x_t^{m-1}) > x + y \quad \text{or} \quad \sum_{m>k} (x_t^m - x_t^{m-1}) < x + y.
\]

To see why, notice that the demand for design \(X\) in period \(t\) is \(2x + y\), and a measure \(x\) of the types who choose \(X\) in period \(t\) also choose \(X\) in period \(t+1\).

In addition, the demand for \(X\) decreases in the following periods, until it becomes zero or the design becomes the design of the elite.

C. \(\exists_{l<k} x_t^l - x_t^{l-1} = x_t^l - x_t^{l-1}\) but \(\forall_{l<k} x_t^l - x_t^{l-1} \leq x_t^l - x_t^{l-1}\).

In this case, the demand for design \(X\) in period \(t+1\) can come only from the types in interval \((x_t^k, 1]\); for this to happen, it must be that the designs chosen in period \(t\) by intervals \((x_{t-1}^l, x_t^l]\), \(l < k\), such that \(x_t^l - x_t^{l-1} = x_t^l - x_t^{l-1}\) have lower priorities. If this condition is satisfied, then the analysis in this
case is analogous to cases B. 1-2. Otherwise the demand drops to zero.

Remark 1. Both (i) and (ii) in the hypothesis of Theorem 1 are possible. For example, in the limit cycle for $n = 3$, design $A$ is the design of the elite in period 1. The demand for it comes from the middle interval in period 2, and then it becomes the design of the bottom in period 3. After that the demand for $A$ drops to zero in period 4, and the design becomes the design of the elite again in period 5. In contrast, in the limit cycle for $n = 5$ (provided in the Online Appendix), it happens that a design is the design of the bottom in period $t$, and then becomes the design of the elite in period $t + 1$.

Theorem 1 describes the evolution of the demand for each design. One may also wonder about the evolution of the social ranking for each design. For any period $t$ in which design $X$ is chosen by some interval of agents, let $r^t(X)$ be the average type of agents that choose $X$. That is, if the types from interval $(x_{k-1}^t, x_k^t)$ choose design $X$, then $r^t(X) = (x_{k-1}^t + x_k^t) / 2$.

Corollary 2. The number $r^t(X)$ strictly increases in $t = t_1, ..., t - 1$.

Proof. Consider any $t = t_1, ..., t - 2$. If $X$ is consumed by $(0, x_k^t)$ where $x_k^t < 1$ in period $t$, then $X$ is consumed only by types higher than $x_k^t / 2$ in period $t + 1$.

If $X$ is consumed by $(x_{k-1}^t, x_k^t)$ where $0 < x_{k-1}^t < x_k^t < 1$ in period $t$, then $X$ is consumed only by types higher than $(x_{k-1}^t + x_k^t) / 2$ in period $t + 1$.

If $X$ is consumed by $(x_{k-1}^t, 1)$ in period $t$, then $X$ is consumed by $(x_{k-1}^t + 1)$ where $x_{k-1}^t + 1 \geq (1 + x_{k-1}^t) / 2$ in period $t + 1$.

Thus, in each life cycle of design $X$, the design moves from higher status agents to lower status agents through the population and the social ranking strictly deteriorates over the cycle.

Finally, we leave open some questions regarding the structure of the cycles:

(1) Suppose that $X$ is the design of the elite in period $t$. Does every type from $[0,1]$ (except possibly a finite number of them) choose $X$ in some period
Is this true at least for large enough values of $t$? The answer is positive for $n = 3$, in the limit cycles for $n = 4$ and 5, and, as we will see later, for initial partitions determined by cutpoints $0 = x_0^0 < x_1^0 < ... < x_{m(0)}^0 = 1$ which are sufficiently close to those in a limit cycle for $n = 4$.

(2) In a given period, is the demand coming from more-right intervals higher, except the most-right interval, i.e., is $x_1^t - x_0^t < x_2^t - x_1^t < ... < x_{m(t)}^t - x_{m(t)-2}^t$, for large enough values of $t$? The answer is again positive for $n = 3$, in the limit cycles for $n = 4$ and 5, and for initial partitions determined by cutpoints $0 = x_0^0 < x_1^0 < ... < x_{m(0)}^0 = 1$ which are sufficiently close to those in a limit cycle for $n = 4$.

Questions (1) and (2) are closely related. It is easy to see that the positive answer to Question (1) implies that $x_1^t - x_0^t \leq x_2^t - x_1^t \leq ... \leq x_{m(t)-1}^t - x_{m(t)-2}^t$ for large enough values of $t$, and, the other way around, a positive answer to Question (2) implies the positive answer to Question (1) for large enough values of $t$.

One essential difficulty in obtaining the answer to these questions is that even if the demand for $X$ is higher than the demand for $Y$ in period $t$ when $X$ is chosen by the types from $(x_{i-1}^t, x_i^t)$ and $Y$ is chosen by the types from $(x_{k-1}^t, x_k^t)$, where $k < l < m(t)$, it may no longer be so in period $t + 1$.

(3) It is also an open question if the demand for each of the $n$ available designs must be positive in an infinite number of periods. Again, the answer is positive in the special cases in which it was positive to questions (1) and (2).

4.2 Limit Cycles

We now describe a finite-period cycle for four available designs, and in Section C of the online appendix, we describe a finite-period cycle for five available designs.

The cycle for $n = 4$ has 28 periods. The following table describes the first seven periods. Again, the odd columns depict cutpoints (from the lowest to the highest), and the even columns depict the designs chosen by the types from the intervals between the cutpoints in the neighboring columns. In some
rows, the last two entries are empty. This means that in the corresponding period, there is no demand for some design. The full description of the finite-period cycle that is repeated can be obtained by replicating the columns with cutpoints four times, and shifting the design from the last column of the previous replica to the first column, the first column of the previous replica to the second column, etc.

\[
\begin{array}{cccccc}
 t & x_0^t & design & x_1^t & design & x_2^t \\
 1 & 0 & A & \frac{167}{4804} & B & \frac{1442}{4804} \\
 2 & 0 & A & \frac{222}{1201} & B & \frac{4804}{1201} \\
 3 & 0 & D & \frac{111}{1201} & A & \frac{4804}{1201} \\
 4 & 0 & C & \frac{111}{2402} & D & \frac{4804}{1201} \\
 5 & 0 & B & \frac{111}{4804} & C & \frac{4804}{2402} \\
 6 & 0 & B & \frac{167}{1201} & C & \frac{774}{1201} \\
 7 & 0 & A & \frac{167}{2402} & B & \frac{554}{1201} \\
\end{array}
\]

Simulations suggest that the finite-period cycle described above, and the finite period cycle described in the online appendix are unique finite-period cycles for \( n = 4 \) and 5, respectively. Simulations also suggest that for all initial cutpoints, the dynamics of demand converge to these cycles (see the online appendix for the MATLAB code). In particular, we obtained convergence for \( n = 4 \) and 5 in all cases we explored numerically. The online appendix contains two examples of simulations for \( n = 4 \) to illustrate the convergence.

We next show formally that if the initial cutpoints are close enough to the cutpoints in some period of the finite-period cycle for \( n = 4 \) described above, then the dynamics converge to this cycle. Note that our simulations also show convergence for initial cutpoints that are not close to the ones of the finite-period cycle.

We first provide a definition of closeness of cutpoints. Suppose that a cycle (for \( n \) available designs) has exactly \( T \) periods, and all designs have positive demand in period \( t \).
Definition 1. We will call cutpoints $x_t^0, x_t^1, ..., x_t^n$ congruent to the cutpoints of a finite-period cycle $x_0^*, x_1^*, ..., x_n^*$ (from period $t$) when the interval partitions determined by the two sets of cutpoints in each period $t, t+1, ..., t+T-1$ comprise the same number of intervals, and the design chosen by each interval of one partition coincides with the design chosen by the corresponding interval of the other partition.

It is straightforward to see that the cutpoints $0 = x_0, x_1, ..., x_n = 1$ in a sufficiently small neighborhood of the cutpoints $0 = x_0^*, x_1^*, ..., x_n^* = 1$ of any finite-period cycle are congruent, except when in some period of the finite-period cycle, two or more intervals of the partition have equal lengths. The inspection of cutpoints shows that no two intervals in any period of the limit cycle for $n = 4$ have equal lengths. (In contrast, some intervals have equal lengths in some periods of the limit cycle for $n = 3$.)

Let $x^*, y^*, z^*$ be the interior cutpoints of the limit cycle for $n = 4$ described above in period $t = 1$ in which all four designs have a positive demand.

Proposition 2. Suppose that some cutpoints $x, y, z$ in some period $t$ are in a sufficiently small neighborhood of the limit cycle cutpoints $0 < x^* < y^* < z^* < 1$ in period 1; in particular, $x, y, z$ are congruent to the cutpoints $x^*, y^*, z^*$. Then, the partitions induced by $x, y, z$ in the following periods converge to the partitions determined by $x^*, y^*, z^*$. In addition, the design chosen by each interval of the partition induced in each period $s \geq t$ by $x, y, z$ coincides with the design chosen by the corresponding interval of the partition induced in period $s$ by $x^*, y^*, z^*$.

Proof. The assumption that $x, y, z$ are congruent to $x^*, y^*, z^*$ implies that in periods $t+1, ..., t+28$ the cutoffs induced by $x, y, z$ belong to interval $(0, 1)$. By algebra, we obtain the formulas for cutpoints in periods $t+1, ..., t+7$:

\[
x_{1}^{t+1} = x + \frac{1}{2}y, \quad x_{2}^{t+1} = -\frac{1}{2}x + y + \frac{1}{2}z;
\]

\[
x_{1}^{t+2} = \frac{1}{2}x + \frac{1}{4}y, \quad x_{2}^{t+2} = \frac{3}{4}x + y + \frac{1}{4}z;
\]
\[ x_1^{t+3} = \frac{1}{4}x + \frac{1}{8}y, \quad x_2^{t+3} = \frac{7}{8}x + \frac{3}{4}y + \frac{1}{8}z; \]
\[ x_1^{t+4} = \frac{1}{8}x + \frac{1}{16}y, \quad x_2^{t+4} = \frac{11}{16}x + \frac{1}{2}y + \frac{1}{16}z, \quad x_3^{t+4} = \frac{3}{4}x + \frac{11}{16}y + \frac{1}{8}z + \frac{1}{2}; \]
\[ x_1^{t+5} = \frac{15}{32}x + \frac{5}{16}y + \frac{1}{32}z, \quad x_2^{t+5} = x + \frac{13}{16}y + \frac{1}{8}z + \frac{1}{4}; \]
\[ x_1^{t+6} = \frac{15}{64}x + \frac{5}{32}y + \frac{1}{64}z, \quad x_2^{t+6} = \frac{31}{32}x + \frac{23}{32}y + \frac{3}{32}z + \frac{1}{8}; \]
\[ x_1^{t+7} = \frac{15}{128}x + \frac{10}{128}y + \frac{1}{128}z, \quad x_2^{t+7} = \frac{92}{128}x + \frac{66}{128}y + \frac{8}{128}z + \frac{1}{16}, \quad x_3^{t+7} = \frac{109}{128}x + \frac{82}{128}y + \frac{11}{128}z + \frac{5}{8}. \]

Analogously, by plugging in the cutpoints \( x_1^{t+1} = x, y, z \) in the above calculation of cutpoints, we obtain the cutpoints in periods \( t + 8, \ldots, t + 13 \), and iterating this procedure in all remaining periods of the limit cycle spanning 28 periods. In period \( t + 28 \), they are:
\[ x_1^{t+28} = \frac{1262199}{33554432}x + \frac{449671}{16777216}y + \frac{107297}{33554432}z + \frac{47075}{2097152}, \]
\[ x_2^{t+28} = \frac{4197743}{16777216}x + \frac{2990979}{16777216}y + \frac{356843}{16777216}z + \frac{915405}{4194304}, \]
and
\[ x_3^{t+28} = \frac{10479977}{33554432}x + \frac{3733603}{16777216}y + \frac{890887}{33554432}z + \frac{864303}{1048576}. \]

Denote by \( x', y', z' \) the cutpoints in period \( t + 28 \), when the cutpoints in period \( t \) are \( x, y, z \). Our algebra shows that \( x', y', z' \) are all linear functions of \( x, y, z \) with positive coefficients that sum up to less than 1. (Of course, the cutpoints in each of the 28 periods are linear functions of \( x, y, z \) with positive coefficients, but the coefficients in some periods sum up to more than 1.)

This implies the convergence of partitions to the limit cycle as follows. By linearity with positive coefficients, we obtain that if the max-norm of \((x - x^*, y - y^*, z - z^*)\) is equal to \( \varepsilon \) for a sufficiently small \( \varepsilon \), then cutpoints \( x', y', z' \) are closer to cutpoints \( x^*, y^*, z^* \) than cutpoints \((x^* - \varepsilon)', (y^* - \varepsilon)', (z^* - \varepsilon)'\), and closer to cutpoints \( x^*, y^*, z^* \) than cutpoints \((x^* + \varepsilon)', (y^* + \varepsilon)', (z^* + \varepsilon)'\). The
linearity with positive coefficients that sum up to less than 1 implies that the distance between cutpoints \((x^* - \varepsilon)', (y^* - \varepsilon)', (z^* - \varepsilon)'\) and cutpoints \((x^* + \varepsilon)', (y^* + \varepsilon)', (z^* + \varepsilon)'\) to \(x^*, y^*, z^*\) is strictly smaller than \(\varepsilon\). So, the max-norm of \((x' - x^*, y' - y^*, z' - z^*)\) is smaller than \(\varepsilon\) times the sum of the coefficients.

5 Different Distributions, Weight Functions and Conformity Preferences

In the previous sections, we considered the uniform distribution \(f\) of the population and the constant weight function \(a\). This is without loss of generality, because in the general case we can change variables passing to percentiles. More precisely, the case with general \(f\) and \(a\) is equivalent to the uniform-constant case in which index \(i\) is replaced with index

\[
i' = \frac{\int_0^1 a(j)f(j)\,dj}{\int_0^1 a(j)f(j)\,dj}.
\]

In this section, we discuss some alternative assumptions on preferences, and the way in which they affect the main results of the paper. In particular, we provide some comparative statics results illustrating the effect of these new assumptions on the dynamics. Assume first that the payoff of agent \(i\) from choosing the same design as agent \(j\) is proportional to the social distance between the two agents. More precisely, let \(u_i(S_k)\) be given by

\[
\int_0^i c(i - j)f_{t-1}(S_k, j)\,dj - \int_i^1 c(j - i)f_{t-1}(S_k, j)\,dj = \int_0^1 c(i - j)f_{t-1}(S_k, j)\,dj
\]

for some \(c > 0\).

Observe first that any interval partition \(0 = y_0^t < y_1^t < ... < y_{n(t)}^t = 1\) of \([0, 1]\) in period \(t\), induces an interval partition of \([0, 1]\) in period \(t + 1\). This
follows from the fact that
\[ \frac{du_i^t(S_k)}{di} = c(y_k^t - y_{k-1}^t) \]
for all \( i \notin \{y_1^t, ..., y_{n(t)-1}^t\} \) and \( k = 1, ..., n(t) \). Hence, the utility of each design is increasing linearly over the grand interval of types. This obviously generates a partition structure in period \( t + 1 \).

Theorem 1 and Corollary 2 extend to this setting, and this easily follows from their proofs. This alternative weighting, proportional to the distance, creates a pressure for cutpoints to move to the right, that is, for the aggregate demand for any first \( k > 1 \) designs to become greater, compared to the uniform-constant weighting studied in the previous sections. Intuitively, the reason is that if under the uniform-constant weighting type \( i \) is indifferent between the designs chosen in period \( t \) by two different intervals, then under the proportional weighting she prefers the design chosen by the more-left interval.

Denote by \( 0 = y_t^0 < y_t^1 < ... < y_t^n(t) = 1 \) the cutpoints in the setting with the proportional weighting. Recall that the cutpoints in the setting with the uniform-constant weighting are denoted by \( 0 = x_t^0 < x_t^1 < ... < x_t^m(t) = 1 \).

**Proposition 3.** Suppose that \( y_k^t = x_k^t \) for \( k = 0, ..., m(t) \); in particular, \( n(t) = m(t) \). In addition, suppose that the elite in period \( t + 1 \) does not choose a design chosen by a non-elite interval of types in period \( t \). Then, \( x_{k+1}^t \leq y_{k+1}^t \) for \( k = 1, ..., n(t + 1) \); in particular, \( n(t + 1) \leq m(t + 1) \).

The condition that the elite in period \( t + 1 \) does not choose a design chosen by a non-elite interval of types in period \( t \) refers to the uniform-constant weighting, but this implies that the same condition is satisfied under the proportional weighting.

**Proof.** For \( k = 1 \), if \( m(t) < n \), then \( y_{k+1}^t = x_{k+1}^t = x_{k+1}^{t+1} \).

Thus, suppose that \( k = 1 \) and \( m(t) = n \), or \( k > 1 \). Consider the types from interval \((x_{k-1}^t, x_k^t)\). Note that none of these types chooses in period \( t + 1 \) a design chosen in period \( t \) by the types from \((x_{l-1}^t, x_l^t)\) for any \( l > k \). For \( k = 1 \), this follows from the assumption that the elite in period \( t + 1 \) does not
choose a design chosen by a non-elite interval of types in period $t$; and for $k > 1$, the types from interval $(x_{k-1}^t, x_k^t)$ prefer the designs chosen in period $t$ by the types from $(x_{l-1}^t, x_l^t)$ where $l < k$ to the designs chosen in period $t$ by the types from $(x_{l-1}^t, x_l^t)$ where $l > k$.

Call the $k$-th design chosen in period $t+1$ the design chosen by the types from the interval $(x_{k-1}^{t+1}, x_k^{t+1})$ and $(y_{k-1}^{t+1}, y_k^{t+1})$, respectively. Suppose that $X$ is the $k$-th design chosen in period $t+1$ under the uniform weighting, and $Y$ is the $k$-th design chosen in period $t+1$ under the proportional weighting. Suppose further that $X$ was chosen in period $t$ by the types from interval $(x_{l(X)}^t, x_{l(X)}^t)$, and that $Y$ was chosen in period $t$ by the types from interval $(x_{l(Y)}^t, x_{l(Y)}^t)$. We will show by induction that $l(Y) \geq l(X)$ and $x_{l(Y)}^t - x_{l(Y)-1}^t \geq x_{l(X)}^t - x_{l(X)-1}^t$.

For $k = 1$, this is true, because $l(X) = l(Y) = 1$.

So, suppose that $k > 1$. If the $(k-1)$-th design in period $t+1$ under the proportional weighting was chosen in period $t$ by the types from $(x_{l'-1}^t, x_{l'}^t)$, then

$$
\int_{x_{l'-1}^t}^{x_{l'}^t} (i-j) dj = \int_{x_{l(Y)-1}^t}^{x_{l(Y)}^t} (i-j) dj
$$

for some $i$, and the RHS of this equation is greater than the LHS for $i'$ higher than $i$. In particular, this means that $x_{l(Y)}^t - x_{l(Y)-1}^t > x_{l'}^t - x_{l'-1}^t$. By the inductive assumption, this implies that either: (a) one of the designs chosen in period $t$ by intervals $(x_{l'-1}^t, x_{l'}^t)$, where $l' < l(Y)$, is the $k$-th design under the uniform weighting and thus $l(X) < l(Y)$, or (b) the design $Y$ is also the $k$-th design under the uniform weighting and thus $l(X) = l(Y)$. In case (a), it must be that $x_{l(Y)}^t - x_{l(Y)-1}^t \geq x_{l(X)}^t - x_{l(X)-1}^t$. Otherwise the types that choose $Y$ in period $t+1$ under the proportional weighting would prefer (over $Y$) the design chosen in period $t$ by the types from $(x_{l(X)}^t, x_{l(X)}^t)$. And obviously $x_{l(Y)}^t - x_{l(Y)-1}^t = x_{l(X)}^t - x_{l(X)-1}^t$ in case (b).

We must finally show that $x_k^{t+1} \leq y_k^{t+1}$. If $l(Y) > l(X)$, then it follows because $x_k^{t+1} \leq x_{l(Y)}^t$ and $y_k^{t+1} \geq x_{l(Y)}^t$. And if $l(Y) = l(X)$, then note that $y_k^{t+1}$ is equal to the $i$ that solves equation (6), and this solution must be
greater than

\[ x_{k+1}^t = x_t^i - x_t^i - x_t^i + x_t^i - x_t^i. \]

**Remark 2.** Note that the proposition is false if the elite in period \( t + 1 \) chooses a design chosen by a non-elite interval of types in period \( t \). Indeed, if \( m(t) = n \), and \( \int_{x_{i-1}}^{x_i} (0 - j) dj > \int_0^{x_i} (0 - j) dj \), which implies that \( x_t^i < x_{i-1} \), for some \( l \), then types close to 0 choose in period \( t + 1 \) a design chosen in period \( t \) by the types from \((x_{i-1}^i, x_i^i)\) for some such \( l \). Denote this design by \( Y \). However, under the proportional weighting, design \( Y \) cannot be chosen by types higher than the \( i \) such that

\[ \int_{x_{i-1}}^{x_i} (i - j) dj = \int_0^{x_i} (i - j) dj, \]  

because these types prefer the design chosen in period \( t \) by the elite. Note now that the \( i \) that satisfies equation (7) is smaller than \( x_{i+1}^t = [x_t^i - (x_t^i - x_t^i - 1)]/2 \).

Next, we consider an alternative formulation of preferences in which an agent weights higher- and lower-status agents asymmetrically. Assume now that the benefit of agent \( i \) from choosing the same design as any agent \( j < i \) is higher than the loss from choosing the same design as any agent \( j > i \). More precisely, let

\[ u_i^t(S_k) = c \int_0^{x_i} f_{t-1}(S_k, j) dj - \int_1^{x_i} f_{t-1}(S_k, j) dj \]  

for some \( c > 1 \). (Of course, we would obtain the “opposite” results by assuming that \( c < 1 \).)

As with the original preference specification from Section 2, any interval partition \( 0 = z_0^i < z_1^i < ... < z_{l(t)}^i = 1 \) of \([0, 1]\) in period \( t \) induces an interval partition of \([0, 1]\) in period \( t + 1 \). Theorem 1 and Corollary 2 also extend...
to this setting, and this easily follows from their proof. The asymmetric weighting creates a pressure for cutpoints to move to the left, that is, for the aggregate demand for any first \( k > 1 \) designs to become smaller compared to the uniform-constant weighting. Intuitively, the reason is that if under the uniform-constant weighting, type \( i \) is indifferent between the designs chosen in period \( t \) by two different intervals (of different lengths), then under the asymmetric weighting, she prefers the more right interval. Denote the cutpoints in the setting with the asymmetric weighting by \( 0 = z_0^t < z_1^t < ... < z_{l(t)}^t = 1 \).

**Proposition 4.** Suppose that \( z_k^t = x_k^t \) for \( k = 0, ..., m(t) \); in particular, \( l(t) = m(t) \). Then, \( z_{l+1}^t \leq x_{l+1}^t \) for \( k = 1, ..., m(t+1) \); in particular, \( l(t+1) \geq m(t+1) \).

**Proof.** For \( k = 1 \), if \( m(t) < n \), then \( z_{l+1}^t = x_{l+1}^t/(1 + c) \), which is smaller than \( x_{l+1}^t = x_{l+1}^t/2 \). If \( m(t) = n \) and \( x_1^t > x_{l+1}^t \) for some \( l \), then

\[
z_{l+1}^t = \frac{x_1^t - (x_l^t - x_{l-1}^t)}{1 + c},
\]

for an \( l \) with the smallest \( x_l^t - x_{l-1}^t \), which again is smaller than \( x_{l+1}^t \) obtained by the same equation with \( c = 1 \). Finally, suppose \( m(t) = n \) and \( x_1^t \leq x_l^t - x_{l-1}^t \) for all \( l \). Take the smallest \( l \) such that either (a) \( x_1^t = x_l^t - x_{l-1}^t \) and the design consumed by \( x_l^t - x_{l-1}^t \) in period \( t \) has a higher priority than the design of the elite in period \( t \), or (b) \( x_1^t < x_l^t - x_{l-1}^t \). If this \( l \) is such that \( x_1^t = x_l^t - x_{l-1}^t \), then \( z_{l+1}^t = x_{l+1}^t \). If this \( l \) is such that \( x_1^t < x_l^t - x_{l-1}^t \), then

\[
z_{l+1}^t = \frac{cx_1^t + cx_{l-1}^t + x_l^t}{1 + c},
\]

which is smaller than \( x_{l+1}^t \) obtained by the same equation with \( c = 1 \).

If \( k > 1 \), note first that \( z_{k+1}^t, x_{k+1}^t \in (x_{k'-1}^t, x_{k'}^t) \). Indeed, if \( z_{k+1}^t, x_{k+1}^t \in (x_{k'-1}^t, x_{k'}^t) \), then \( z_1^t, z_n^t \in (x_{k'-1}^t, x_{k'}^t) \) for the lowest \( k' > k'' \) such that either (a) \( x_{k'}^t = x_{k'}^t - x_{k'-1}^t \) and the design consumed in period \( t \) by \( x_{k'-1}^t, x_{k'}^t \) has a higher priority than the one consumed by \( x_{k'-1}^t, x_{k'}^t \) or (b) \( x_{k'}^t - x_{k'-1}^t > x_{k'}^t - x_{k'-1}^t \). In particular, if this \( k' \) is such that (a) is
satisfied, then $z_{k+1}^t = x_{k+1}^t = x_k^t$, while if the $k'$ is such that (b) is satisfied, then

$$z_{k+1}^t = \frac{x_k^t + cx_{k'-1}^t + c(x_{k''}^t - x_{k''-1}^t)}{1 + c},$$

which is smaller than $x_{k+1}^t$ obtained by the same equation with $c = 1$. If there does not exist $k'$ such that (a) or (b) is satisfied, then $z_{k+1}^t = x_{k+1}^t = 1$.  

At least for $n = 3$, we also find finite-period cycles in the two alternative settings.

**Proposition 5.** (i) There exists a finite-period cycle $C$ spanning 30 periods for the preferences with proportional weighting. The cycle $C$ is obtained by replicating the columns below with cutpoints three times, and shifting the design from the last column of the previous replica to the first column, the first column of the previous replica to the second column, etc.

<table>
<thead>
<tr>
<th>$y_0^t$</th>
<th>design</th>
<th>$y_1^t$</th>
<th>design</th>
<th>$y_2^t$</th>
<th>design</th>
<th>$y_3^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 1$</td>
<td>0</td>
<td>$A$</td>
<td>0.0766523</td>
<td>$B$</td>
<td>0.687199</td>
<td>$C$</td>
</tr>
<tr>
<td>$t = 2$</td>
<td>0</td>
<td>$A$</td>
<td>0.431257</td>
<td>$B$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$t = 3$</td>
<td>0</td>
<td>$C$</td>
<td>0.215629</td>
<td>$A$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$t = 4$</td>
<td>0</td>
<td>$B$</td>
<td>0.107814</td>
<td>$C$</td>
<td>0.79738</td>
<td>$A$</td>
</tr>
<tr>
<td>$t = 5$</td>
<td>0</td>
<td>$B$</td>
<td>0.526485</td>
<td>$C$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$t = 6$</td>
<td>0</td>
<td>$A$</td>
<td>0.263243</td>
<td>$B$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$t = 7$</td>
<td>0</td>
<td>$C$</td>
<td>0.131621</td>
<td>$A$</td>
<td>0.909588</td>
<td>$B$</td>
</tr>
<tr>
<td>$t = 8$</td>
<td>0</td>
<td>$C$</td>
<td>0.613219</td>
<td>$A$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$t = 9$</td>
<td>0</td>
<td>$B$</td>
<td>0.306609</td>
<td>$C$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$t = 10$</td>
<td>0</td>
<td>$A$</td>
<td>0.153305</td>
<td>$B$</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

(ii) For $c = 1.5$, there exists a finite-period cycle $D$ spanning 15 periods for the preferences with asymmetric weighting. The cycle $D$ is obtained by replicating the columns below with cutpoints three times, and shifting the design from the last column of the previous replica to the first column, the first column of the previous replica to the second column, etc.
Simulations suggest that under the proportional weighting the dynamics of demand converge to cycle C for all initial cutpoints. Moreover, cycle D seems to be the unique finite-period cycle under the asymmetric weighting with $c = 1.5$. The simulation results, however, are inconclusive regarding the convergence of the dynamics to cycle D. Mathematica codes for the simulations are provided in the online appendix.

The cycle D for the case that $c = 1.5$ has been provided as an example for the existence of finite-period cycles under asymmetric weighting. We also find finite-period cycles for other values of $c$. However, the structure of these cycles depends on $c$ and a characterization of cycles for general $c$ is complex.

Finally, one could also wish to consider other weighting functions, which depend on the distance between $i$ and $j$ in a nonlinear manner, or vary with $i$. However, such models lose a lot of its tractability. For example, if we replace $i - j$ by a function $g(i - j)$ in (5), then even if the demand has an interval structure in period $t$, it can no longer have this kind of structure in period $t + 1$.

To see intuitively why, suppose that agent $i$ from some interval $(x_{k-1}^l, x_k^l)$ is indifferent between the design of the agents from this interval and the design of the agents from $(x_{l-1}^l, x_l^l)$, $l < k$. If function $g$ is very steep above $i - x_{l-1}^l$ and very flat below $i - x_{l-1}^l$, then it is possible that in interval $(x_{k-1}^l, x_k^l)$ there will be types lower than $i$ and types higher than $i$ that prefer in period $t + 1$ the design of the agents from $(x_{l-1}^l, x_l^l)$ to their own current design.
6 Conclusion

This paper proposes a model, which shows – with a minimal structure on heterogeneous (anti-)conformity preferences – how demand for goods waxes and wanes in cycles. The overall demand dynamics generated by the model are in line with traditional sociological fashion theories. The demands for various designs of the same good over time follow repetitive bell-shaped cycles. In each of these cycles, the demand for a design trickles through the society. A distinctive feature of the model is that the dynamics of demand converge to an infinitely repeated unique finite-period cycle. A promising direction for future research is including firms into the model as strategic actors which can set prices and decide about introducing new and withdrawing old designs.
References


URL: [http://www.kellogg.northwestern.edu/research/math/papers/940.pdf](http://www.kellogg.northwestern.edu/research/math/papers/940.pdf)


