Pareto Improvements in the Contest for College Admissions

Wojciech Olszewski and Ron Siegel*

June 2019

Abstract

Many countries base college admissions on a centrally-administered test. Students take the test and are assigned to colleges based on the rank order of their performance. Students invest a great deal of effort to improve their performance on the test, and there is growing concern about the large costs associated with such test-preparation activities. We consider modifying the tests by introducing performance-disclosure policies that pool intervals of performance rankings, and investigate how such policies can improve students' welfare in a Pareto sense. Pooling affects the equilibrium allocation of students to colleges, which hurts some students and benefits others, but also affects the effort students exert. We characterize the Pareto frontier of Pareto improving policies, and also identify improvements that are robust to the distribution of college seats. We illustrate some of our results by applying them to estimate derived from SAT data.

1 Introduction

College and university admissions in many countries are determined by students' performance on a centrally-administered test. This is the case, for example, for most colleges and universities in Brazil, China, Russia, South Korea, and Turkey. The students with the highest performance are admitted to the best colleges, those ranked slightly below them are admitted to the next best colleges, and so on. In many other countries factors such as high-school grades are also considered, but centralized test results are still of paramount importance in the college admissions process.

*Department of Economics, Northwestern University, Evanston, IL 60208 (e-mail: wo@northwestern.edu) and The Pennsylvania State University, University Park, IL 16802 (e-mail: rus41@psu.edu). We are indebted to Bela Szabadi for outstanding research assistantship. We thank Dawei Fang, Alex Frankel, Navin Kartik, Emir Kamenica, Shengwu Li, Philipp Strack, Jun Xiao, and seminar participants at Bocconi, Cambridge, Carlo Alberto, Cincinnati, European University Institute, Hong Kong, Johns Hopkins, Kyoto, Leicester, Northwestern, Oxford, Penn State, SKKU, Virginia, Wisconsin, Yonsei, the AEA Winter Meetings in Chicago, the Southampton Winter Workshop in Economic Theory, the Pennsylvania Economic Theory Conference, and the SAET Annual Conference in Taipei for very helpful comments and suggestions. Financial support from the NSF (grant SES-1325968) is gratefully acknowledged.

Consequently, students invest a great deal of time and effort preparing for these tests. In many Asian countries, including China, Japan, South Korea, and Taiwan, students attend specialized "cram schools,"[1] which focus on improving students' performance on the tests. This often consists of rote learning, solving a large number of practice problems, and practicing test-taking strategies tailored to the specific test. In other countries, students hire tutors, buy books, and take specialized courses, all geared entirely toward improving their test scores. These activities likely improve students' performance on the test, but are far less likely to generate substantial long-term improvements in students' productive human capital.[2] These activities do, however, carry significant costs in terms of time, money, and effort. In South Korea, for example, it is not uncommon for high school students to spend several hours a day in cram schools, and the high stakes competition for college admissions is seen as one of the main causes for the high rates of unhappiness and suicide among teenagers.[3] Similar concerns have also been raised in the United States.[4]

Addressing this important issue is more difficult than it might initially appear. Passing laws to prohibit or limit test-preparation activities may be both difficult and ineffective.[5] Changing the admissions process may also be impractical. First, it is not clear what a better system would look like. For example, accurate tests lead to better students being admitted to better colleges, and other systems may lead to different outcomes, which may or may not be preferred. Second, implementing a new system may be expensive and technically difficult. Third, a new system that makes some students worse off would likely face significant resistance, even if it made other students better off.

This paper investigates simple modifications to admissions processes based on centralized tests that can make all students better off. We model college admissions as a contest with many players (students) and many prizes (college seats). Students exert costly and wasteful effort and are admitted to colleges based on the rank order of their performance. We consider performance-disclosure policies, which coarsen students' rank order by pooling together intervals of performance and assigning the same score to all performances

[1] The term comes from the word "cramming," which also attests to the style and content of the instruction.

[2] For example, McIntyre and Munson (2008) investigated cramming in the context of a college class and showed that long-term retention of the material was far lower for "high crammers" than for "low crammers" (27 percent versus 82 percent).

[3] See, for example, Matthew Carneys' discussion "South Korean education success has its costs in unhappiness and suicide rates" from June 15, 2015 on the Australian Broadcasting Corporation. The high suicide rate among teenagers is frequently attributed to their and others' expectations for them to do well in the competition for college admissions (Ahn and Baek (2013)).

[4] For example, Hsin and Xie (2014) report that the high academic effort Asian-Americans exert leads to lower subjective well being and to psychological and social difficulties. Schwartz (2016) discusses (without modeling formally) the psychological and other costs to students, and suggests that a lottery among those passing a threshold might increase welfare relative to the current admissions process. Bodoh-Creed and Hickman (2015) provide additional references, and also estimate the cost of competition in monetary terms.

[5] For example, In a 2014 New York Times article, (https://www.nytimes.com/2014/08/02/opinion/sunday/south-koreas-education-system-hurts-students.html), Se-Woong Koo reports that many South Korean presidents tried to limit cram schools' activities, including passing a 10 p.m. mandatory closure time. But even this restriction was circumvented "by operating out of residential buildings or blacking out windows so that light cannot be seen from outside."

in an interval. If many students obtain the same score, they are randomly admitted to the corresponding fraction of colleges. For example, a "top pooling" policy that pools some fraction (an interval) of the top performing students leads to these students being randomly assigned to the same fraction of the top college seats.[6] An attractive property of performance-disclosure policies is that they do not require changing the tests or introducing new components to the admissions process. They also respect the property that a higher score leads to a better college assignment than a lower score.

A real-world example of a coarse performance-disclosure policy is the one recently adopted by the South Korean Ministry of Education for the College Scholastic Ability Test (CSAT), which determines college admissions in South Korea. Until 2018, each part of the test was graded on a 0-100 or 0-50 scale. Starting in 2018, scores of 90-100 in the English component of the CSAT are reported as one grade, scores of 80-89 as another grade, etc. "Students in the same graded classification will all be considered on an equal playing field in the college admissions process, regardless of their numerical scores.[7]" The goal is to reduce costly competition between students,[8] while recognizing that the assortativity of the admissions process will be reduced as well.[9] One possible concern with such a policy, however, is that even if its overall effect on student welfare is positive, it may be that some students are harmed while others benefit.

We are interested in performance-disclosure policies that benefit all students, and refer to such policies as Pareto improving. In particular, we do not need to consider welfare trade-offs across students. A key finding of our analysis is that Pareto improving policies often exist. This may seem surprising, since a fixed set of college seats implies that a student can be admitted to a better college only if some other student is admitted to a worse college. The crucial element that makes Pareto improvements possible is that test preparation is costly. The costs students incur, as well as the resulting college assignment, are determined in equilibrium, and the equilibrium is affected by the performance-disclosure policy. Relative to the baseline contest with no coarsening, introducing a performance-disclosure policy leads to some students being admitted to better colleges; this makes them better off even if they incur higher costs, as long as the cost increase is not too

[6] A similar policy was proposed by Barry Schwartz in a 2007 LA times article (http://articles.latimes.com/2007/mar/18/opinion/op-schwartz18/2) in order "to dramatically reduce the pressure and competition that our most talented students now experience." "the names of all the "good enough" students could be placed in a metaphorical hat, with the 'winners' drawn at random for admission. Though a high school student will still have to work hard to be "good enough" for Yale, she won't have to distort her life in the way she would if she had to be the 'best.' The only reason left for participating in all those enrichment programs would be interest, not competitive advantage." We provide a framework to formalize, evaluate, and compare top pooling and other performance-disclosure policies.

[7] Korea JoongAng Daily, October 10, 2015. "CSAT English section to take absolute grade scale."

[8] This "... grading system aims to reduce excessive competition among test-takers." "'We are trying to alleviate unnecessary and exorbitant competition between students who are competing with one another to gain one or two points more,' said Kim Doo-yong, a ministry official." Ibid.

[9] "In the last mock exam..., 23 percent of total examinees scored in the first grade... but only 4.64 percent of total examinees received perfect scores, which by current standards means they would have been the only ones to classify in the first grade." Ibid.

large. Other students are admitted to worse colleges; if they also incur lower costs they are made better off as long as the reduction in the costs is large enough.

We characterize the Pareto improving policies and rank them in a Pareto sense. We also characterize the Pareto frontier of such policies. We do this initially for top pooling policies and then for single-interval policies that do not necessarily pool the top performing students. The characterization shows that pooling a performance interval is Pareto improving if and only if the student with the highest performance in the interval benefits from the policy. This in turn happens if the population distribution of student ability conditional on the same interval (in percentile terms) first-order stochastically dominates (FOSD) the uniform distribution. We then generalize this condition for policies with multiple pooling intervals.

We then consider robust Pareto improving policies, which are Pareto improving for any distribution of college seats. We characterize the robust Pareto improving policies and show that the Pareto optimal policy among them is unique. This policy consists of pooling on each of the maximal intervals on which the conditional distribution of student ability FOSD the uniform distribution. This characterization may be particularly useful for empirical work, because using the characterization only requires obtaining an estimate of the students' ability distribution. Given this distribution, it is straightforward to derive the Pareto optimal policy, which is robust to the distribution of college seats. As an illustration of what such a policy could look like, we use SAT data as a proxy for the distribution of student ability and apply our characterization of the robust Pareto optimal policy. This leads to a "bottom pooling" policy, which pools together approximately 80 percent of the lowest-performing students. Such a policy is consistent with the admission policies of many lower-ranked colleges in the United States, but we stress that our use of SAT data is only an illustration and that deriving an estimate of the student ability distribution in any concrete setting requires a separate and serious empirical investigation.

The rest of the paper is organized as follows. Section 1.1 reviews the related literature. Section 2 introduces the model. Section 3 presents the equilibrium and the notion of Pareto improvement. Section 4 investigates top pooling. Section 5 investigates policies with multiple pooling intervals. Section 6 derives the conditions for robust Pareto improvements and illustrates the results using SAT data. Section 7 concludes. The appendix contains proofs, examines peer effects and positive externalities, and extends our results for top pooling to more general student utility functions.

1.1 Relation to the literature

College admissions feature prominently in the matching literature, beginning with Gale and Shapley's (1962) seminal contribution. The focus of much of this work is on stability and efficiency in the presence of heterogeneous student preferences, while abstracting from the effort students exert. Since we are interested in Pareto improvements, endogenous effort choice is an important feature of our framework.

Condorelli's (2012) paper is probably the most closely related to our paper. He characterizes the ex-ante efficient allocations of heterogeneous objects to heterogeneous agents with private valuations. We are interested in ex-post Pareto improvements. The difference between ex-ante and ex-post Pareto seems essential in the context of our application, because the former allocations may involve a better treatment of some types of agents at the expense of other types, and be controversial in the context of college admissions. And the set of ex-ante optimal allocations can be completely different from those that are ex-post Pareto improving, as we discuss in Section 2.1 below. In addition, although he provides an elegant general solution to the problem he studies, his solution delivers little insight regarding pooling intervals of performance in the context of college admissions, which is the focus of our work. His main take-away insights apply when all players' type distributions have monotone hazard rates. Our results do not require such a condition.

Several papers compare allocating objects via contests[10] and lotteries from the perspective of contestants' welfare (see Taylor et al. (2003), Koh et al. (2006), Hoppe et al. (2009), and Chakravarty and Kaplan (2013)). The most closely related work is by Hoppe et al. (2009). They consider a two-sided matching model with ex-ante symmetric agents on each side, in which assortative matching takes place based on costly signals. They provide conditions (expressed in terms of monotone failure rates) under which random matching leads to ex-ante higher welfare than assortative matching, and show that random matching is Pareto improving for agents on one side if the distribution of types of that side first-order stochastically dominates the uniform distribution.

Hafalir et al. (2018) investigate a model of college admissions with entrance exams and two colleges with different qualities. They compare centralized admissions, in which students can apply to both colleges, and decentralized admissions, in which each student can apply to only one college. They show that lower ability students prefer the decentralized setting and higher ability students prefer the centralized setting. Fang and Noe (2018) consider a selection contest with identical prizes, and show that pooling a larger number of the top performers than the number of prizes can sometimes lead to lower risk taking without reducing winner quality. Fang, Noe, and Strack (2018a) consider a large contest framework similar to ours to investigate the effect of different university grading curves when post-graduation salaries depend on inferences employees make from grades about student ability and human capital accumulation.

Ostrovsky and Schwarz (2010) investigate information disclosure policies by schools when students are passive and exert no effort, and focus on the amount of information schools reveal in equilibrium. In our analysis, performance disclosure policies affect students' efforts, and this determines which policies are Pareto preferred. A more recent contribution by Boleslavsky and Cotton (2015) considers schools' incentives to invest in quality when they can choose imperfectly informative grading policies. As a result of this strategic choice, schools have a greater incentive to invest in quality, which can increase welfare. Gottlieb and Smetters (2014) investigate why MBA students vote for grade non-disclosure policies when employers make inferences about students' abilities based on the disclosed information. Frankel and Kartik (2017) study a signaling model with players that are heterogeneous in two dimensions, and show that in equilibrium the market is unable to learn either dimension.

[10] Contests in these papers typically have the form of waiting in line.

Dubey and Geanakoplos (2010) consider a game of status between students. A student's status is equal to the difference between the number of students with a lower grade and the number of students with a higher grade. In particular, the aggregate allocation value of status is always 0. A student's performance is a noisy measure of his costly effort, and, similarly to our model, a grading policy pools intervals of performance. The focus is on characterizing grading policies that maximize effort. Such policies involve some pooling, and with heterogeneous students necessary conditions for such policies are derived. Coarse grades also arise in the setting of Harbaugh and Rasmusen (2018), in which a sender can choose whether to certify his privately-known quality. Certification schemes with coarse grades can result in more information by inducing the sender to certify a larger set of qualities.

Our paper also contributes to the theory of all-pay contests. Most papers in this literature focus on settings with two players, ex-ante symmetric players, or identical prizes. Olszewski and Siegel (2016a) introduced the approximation approach to large contests, which makes it possible to study contests with many ex-ante asymmetric players and heterogeneous prizes, as we do here. Olszewski and Siegel (2016c) use this approach to study performance-maximizing contests.[11] Bodoh-Creed and Hickman (2018a) use a similar (and independently developed) approach to study quotas and affirmative action in college admissions. While there are several technical differences between their model and ours,[12] the main differences are in the design instruments they consider (quotas and affirmative action) and their focus on aggregate welfare as opposed to our focus on Pareto improvements. They also allow for productive effort, which is potentially important in college admissions settings that take into account factors like high school performance. One of their findings is that using a lottery to assign students to colleges would generate higher aggregate student welfare than a contest for college admissions. Our investigation of optimal category rankings shows that a pure lottery can be improved upon for all students by partitioning the set of students into several categories based on their performance and using a separate assignment lottery for each category.

2 The baseline contest

A large number of players (students) compete for prizes (college seats) by taking a test. Each prize is characterized by its known value $y \in [0, 1]$, and each player is characterized by her ability (type) $x \in [0, 1]$, which affects her cost of performance on the test and/or her prize valuation. Each player's type is drawn from a player-specific distribution, independently across players. This accommodates ex-ante asymmetry across players. After privately observing her type, each player exerts costly effort to achieve her desired performance $t \geq 0$ on the test. The test may have several parts or be comprised of several examinations,

[11] Fang, Noe, and Strack (2018b) study the effect of different prize structures on aggregate effort in symmetric all-pay auctions with complete information.

[12] They consider a more general utility function but assume that the limit distribution of college seats is atomless, and consider two groups of students, the minority and the majority, such that students within each group are ex-ante symmetric.

provided that they are weighted in a way that produces a single number (the performance) according to which players are ranked. The player with the highest performance obtains the highest prize, the player with the second-highest performance obtains the second-highest prize, and so on. Some prizes may be identical, to allow for multiple seats in a given college (or tier of colleges). Ties are resolved by a fair lottery. The utility of a type x player who chooses performance t and obtains prize y is

$$f(x)\, y - \frac{c(t)}{g(x)}, \tag{1}$$

where c is strictly increasing and twice continuously differentiable, and $\lim_{t\to\infty} c(t) = \infty$.[13] Function c captures the cost of performance, function $f \geq 0$ captures the effect of the player's type on her prize valuation, and function $g \geq 0$ captures the effect of the player's type on her cost of performance. We let $f(x)\, g(x) = x$, so a higher type corresponds to higher ability. Two special cases (which are assumed in most of the contest literature) are

$$xy - c(t), \tag{2}$$

in which the player's type only affects her prize valuation, and

$$y - \frac{c(t)}{x}, \tag{3}$$

in which the player's type only affects her performance cost. Utilities (1) for different functions f and g are strategically equivalent, because for each type x multiplying (1) by $g(x)$ gives (2). Throughout our analysis we will assume utility (2). This is for convenience only - as we now discuss, all our results hold without change for any utility (1) (and for the special case (3)).

2.1 Discussion of the model

The above setup accommodates heterogeneity in college quality and student ability, and models costly test preparation as a strategic choice. Like any other model, it abstracts from certain realistic and potentially important aspects. First, the model stipulates a common ordinal ranking of college quality across students.[14] Second, the model abstracts from factors that are not controlled by the players and may affect their performance ("noise"). These two assumptions are made for tractability (but they may also be fairly realistic in some settings). Third, the model assumes that test preparation is costly, as in Spence (1973). This cost is captured by function c, and should be interpreted as net of any direct benefit from the preparation activities. This is most appropriate for activities specifically geared toward improving students' performance on the test, as discussed in the introduction. The model is not as suitable for countries in which other activities, such as taking AP classes, play an important role in college admissions and may have significant

[13] The linearity of y is a normalization; we can replace y in players' utility with $h(y)$, where h is strictly increasing and twice continuously differentiable and $h(y) = 0$, without affecting any of the results.

[14] Homogeneous ordinal preferences are also assumed in some matching papers on school choice (for example Lien, Zheng, and Zhong (2017)).

direct benefits at moderate levels of investment. But even there the costs exceed the benefits past a certain point. Bodoh-Creed and Hickman (2018b) provide support for this in the context of college admissions in the United States.[15] In such cases, function c can be thought of as a simplification that assumes that all preparation activities are costly.

Fourth, similarly to much of the matching literature, peer effects are absent: a student's valuation for being admitted to a college does not depend on the other students admitted to the same college. In fact, peer effects can be accommodated without changing the substance of any of our results. This is done in Online Appendix D. The idea is that in a large contest each student is fairly certain about the equilibrium distribution of student types admitted to the various colleges. We can therefore replace the value y of being admitted to a specific college with another value that includes the peer effects generated by the set of students admitted to that college. The rest of the analysis is unchanged. Finally, in Online Appendix E we show that our results for top pooling generalize to separable utility functions of the form $h(x,y) - c(t)$ and $h(y) - c(x,t)$ that satisfy some conditions.[16]

It is also important to point out that while utilities (1) for different functions f and g are strategically equivalent, different functions f have different implications for aggregate welfare. Setting aside players' performance, if $f(x)$ increases in x (e.g., utility (2)), then the allocation of prizes that maximizes aggregate welfare allocates the highest prize to the player with the highest type, the second highest prize to the player with the second highest type, etc. But the opposite is true if $f(x)$ decreases in x. And if $f(x)$ is independent of x (e.g., utility (3)), then all prize allocations generate the same aggregate welfare. These different functional forms make no difference for our analysis, however, because we focus on performance-disclosure policies that make all students better off, and in fact all types of students better off (we provide a precise definition in Section XXX below). Since each individual player's ranking of prize-performance pairs is the same for all utility functions (1), our results for hold for all utility functions (1). In particular, we do not need to take a stand on aggregate welfare or on whether awarding higher prizes to players with higher types is desirable from a welfare perspective. In contrast, the ex-ante efficient allocations (Condorelli (2012)), which maximize the sum of expected utilities, differ across the functional forms. For example, under utility (3) the ex-ante efficient allocations are those that induce a performance of 0, that is, all the possible lotteries over prizes (including deterministic allocations), which do not depend on players' performance.

[15] They study a rich data set and a contest model in which effort can be productive, and show that for most students most of the effort is wasteful (and the wasted effort is three times higher than the productive effort for the middle 50 percent of the learning cost distribution).

[16] Similar results can be obtained for category rankings, and even for some more general utility functions of the form $h(x,y) - c(x,t)$. But the form of such general results is more involved.

3 Equilibrium

A direct equilibrium analysis of the baseline contest described in Section 2 is intractable because the equilibria generally involve mixed strategies and are not symmetric. In particular, higher types do not always choose higher performance, and therefore do not always obtain better prizes. Since the contests we consider have many players and prizes, we can make use of the tractable approach to studying the equilibria of large contests, which was developed in Olszewski and Siegel (2016a). They show that all the equilibria of such large contests are closely approximated by the unique single-agent mechanism in a specific environment that implements the assortative allocation of prizes to agent types and gives the lowest type a utility of 0. More precisely, denote by F the average distribution of players' types and suppose that it has a continuous, strictly positive density f, and denote by G the empirical distribution of prizes, which need not be be continuous or have full support. For example, G may consist of atoms that represent colleges (or tiers of colleges). The size of each atom represents the number of seats in the college. The *assortative allocation* assigns to each type x prize

$$y^A(x) = G^{-1}(F(x)),$$

where

$$G^{-1}(z) = \inf\{y : G(y) \geq z\} \text{ for } 0 \leq z \leq 1.$$

That is, the quantile in the prize distribution of the prize assigned to type x is the same as the quantile of type x in the type distribution. It is well known (see, for example, Myerson (1981)) that the unique incentive-compatible mechanism that implements the assortative allocation and gives type $x = 0$ utility 0 specifies for every type x performance

$$t^A(x) = c^{-1}\left(xy^A(x) - \int_0^x y^A(\tilde{x})\, d\tilde{x}\right). \tag{4}$$

This implies type x obtains utility

$$U(x) = xy^A(x) - c(t^A(x)) = \int_0^x y^A(\tilde{x})\, d\tilde{x}. \tag{5}$$

Roughly speaking, the approximation shows that in any equilibrium of a large contest a player with type x with high probability chooses a performance close to $t^A(x)$ and obtains a prize close to $y^A(x)$, which gives her a utility close to $U(x)$. See Olszewski and Siegel (2016a) for a precise statement and additional details.

The intuition for why this single-agent mechanism approximates the equilibria of large (finite) contests is that, given players' equilibrium strategies, with a large number of players the law of large numbers implies that each bid leads to an almost deterministic rank-order quantile (in the distribution of bids) and thus to an almost deterministic prize. In the limit we obtain an "inverse tariff" that maps bids to prizes. Utility (1) implies that higher types choose higher bids from any tariff, so the mechanism induced by the inverse tariff implements the assortative allocation. Any player can bid 0 and obtain the lowest prize, so the utility of type 0 is 0.

In the rest of paper we focus on the approximating single-agent mechanism to investigate how different performance-disclosure policies affects students' welfare in a Pareto sense, which we define below. As discussed in the introduction, the potential for Pareto improvements exists because performance is costly: if pooling reduces students' performance, all students could be made better off even though the allocation of college seats changes.

3.1 The notion of Pareto improvements

We study the approximating mechanisms under various performance-disclosure policies, and use the term "Pareto-improving" in reference to the utility of the types in these approximating mechanisms. A performance-disclosure policy is Pareto improving if all types are better off, and there is a positive measure of types that are strictly better off. Such an improvement implies that in a sufficiently large contest some players are strictly better off and no player is worse off by more than an arbitrarily small amount; moreover, the sum of these small amounts across all players who are worse off is arbitrarily small compared to the gains of the players who are strictly better off. The colleges in our model simply serve students and are not treated as agents in the analysis of Pareto-improving policies. One could alternatively assume that colleges represent the rest of the society, for which they generate some positive externalities. This approach is briefly described in Section C of the appendix,

4 Top pooling

We begin by considering "top q pooling," in which a fraction q of the highest performing students are pooled. These students still obtain the best college seats, but the allocation of these seats to the students is random. Thus, to study the effect of top pooling we can simply consider a contest in which the top fraction q of prizes are replaced with mass q of identical prizes whose value is equal to the average value of the top prizes. To do this, let $x^* = F^{-1}(1-q)$ be the type whose quantile in the average type distribution is $1-q$, and let G^q be the prize distribution that results from replacing the top mass q of prizes in distribution G with a mass q of prize

$$y(q) = \frac{\int_{1-q}^{1} G^{-1}(z)\, dz}{q} = \frac{\int_{x^*}^{1} y^A(x) dF(x)}{1-F(x^*)}.$$

That is, $(G^q)^{-1}(F(x)) = G^{-1}(F(x))$ for $x \le x^*$, and $(G^q)^{-1}(F(x)) = y(q)$ for $x > x^*$. The corresponding assortative allocation $y^{A,q}$ satisfies

$$y^{A,q}(x) = (G^q)^{-1}(F(x)). \tag{6}$$

The unique single-agent mechanism that implements this allocation and gives type $x = 0$ a utility of 0 specifies performance

$$t^{A,q}(x) = c^{-1}\left(xy^{A,q}(x) - \int_0^x y^{A,q}(\tilde{x})\, d\tilde{x}\right). \tag{7}$$

Consider how this mechanism compares with the one in Section 3, which implements the assortative allocation y^A, and in which the performance t^A is given by (4). By definition of G^q and $y^{A,q}$, we have that $y^{A,q}(x) = y^A(x)$ and $t^{A,q}(x) = t^A(x)$ for $x \leq x^*$, and $y^{A,q}(x) = y(q)$ and $t^{A,q}(x) = M$ for $x > x^*$, where

$$M = c^{-1}\left(x^* y(q) - \int_0^{x^*} y^A(\tilde{x})\, d\tilde{x}\right) = c^{-1}\left(x^* \frac{\int_{x^*}^1 y^A(\tilde{x}) dF(\tilde{x})}{1 - F(x^*)} - \int_0^{x^*} y^A(\tilde{x})\, d\tilde{x}\right). \tag{8}$$

Type x^* is a threshold type, above which pooling occurs in large contests: all higher types choose the same performance M and obtain the same lottery over prizes. Since there is a one-to-one correspondence between q and x^*, in what follows we also refer to top q pooling as "top pooling with threshold x^*." To gain some intuition for performance M, notice that (5) and (8) imply that

$$x^* y^A(x^*) - c(t^A(x^*)) = x^* \frac{\int_{x^*}^1 y^A(x) dF(x)}{1 - F(x^*)} - c(M), \tag{9}$$

that is, type x^* is indifferent between choosing performance $t^A(x^*)$ and obtaining prize $y^A(x^*)$ and choosing performance M and obtaining a prize randomly from the mass $1 - F(x^*)$ of the highest prizes. (Note that $t^A(x^*) < M$.)

4.1 Welfare comparisons

To understand the welfare effect of top pooling, we compare each type's utilities in the approximating mechanisms with and without top pooling. We assume that not all prizes are identical in the top mass $q = 1 - F(x^*)$ of prizes.[17]

Proposition 1 *Consider top pooling with threshold x^*.*

(a) The utility of types $x < x^*$ is not affected.

(b) The utility of type $x > x^*$ increases if and only if

$$\frac{\int_{x^*}^1 y^A(\tilde{x}) dF(\tilde{x})}{1 - F(x^*)} \geq \frac{\int_{x^*}^x y^A(\tilde{x})\, d\tilde{x}}{x - x^*}. \tag{10}$$

(c) The gain in utility for types $x > x^*$ first increases and then decreases in type. Thus, there is a type x^{**} in $(x^*, 1]$ such that the utility of types x in (x^*, x^{**}) strictly increases, and the utility of types $x > x^{**}$ strictly decreases.

(d) Top pooling is Pareto improving if and only if it increases the utility of type 1, that is,

$$\frac{\int_{x^*}^1 y^A(\tilde{x}) dF(\tilde{x})}{1 - F(x^*)} \geq \frac{\int_{x^*}^1 y^A(\tilde{x})\, d\tilde{x}}{1 - x^*}. \tag{11}$$

[17] If they are identical, then top pooling has no effect.

Proposition 1 shows that the effect of top pooling on players' welfare depends on their types. Players with types lower than x^* are unaffected, since their performance does not change and is lower than the performance of the players with types higher than x^*. Consequently, the prize they obtain also does not change. Players with intermediate types (in (x^*, x^{**})) benefit, but the reason for this varies across the players. Players with intermediate types close to x^* choose a higher performance with top pooling (since $t^A(x^*) < M$), and obtain a prize lottery that is better than the prize they obtain without top pooling (because prize $y^A(x^*)$, which is in quantile $1-q$ of the prize distribution, is the lower bound of the support of the prize lottery). If type x^{**} is close to 1, then intermediate types close to x^{**} benefit even though they obtain a prize lottery that is worse than the prize they obtain without top pooling (because prize $y^A(1)$ is the best prize); they benefit because they exert less effort with top pooling than without it (since $M < t^A(1)$). This tradeoff goes in the other direction for players with high types (above x^{**}), so they are made worse off.

The tradeoff is captured by (10), which, by multiplying both sides by $x - x^*$, compares the gain in utility of type $x > x^*$ relative to type x^* with and without top pooling. Note that the left-hand side of (10) is the average of $y^A(\tilde{x})$ across types $\tilde{x}$ that choose performance M. This term is independent of x. The right-hand side of (10) is the average of $y^A(\tilde{x})$ across all types lower than x that choose performance M. This term increases in x. In addition, the average in the first term is taken with respect to the actual (truncated) distribution of types, while the average in the second term is taken with respect to the (truncated) uniform distribution.

Proposition 1 shows that types slightly higher than x^* benefit from top pooling,[18] but high types may or may not benefit. This depends on whether type 1 benefits, in which case all types higher than x^* do. The two possibilities are depicted in Figure 1, which illustrates the utility gain resulting from top pooling as a function of type. The left-hand side corresponds to top pooling with $x^{**} < 1$, so it is not Pareto improving, and the right-hand side corresponds to top pooling that is Pareto-improving, so $x^{**} = 1$.

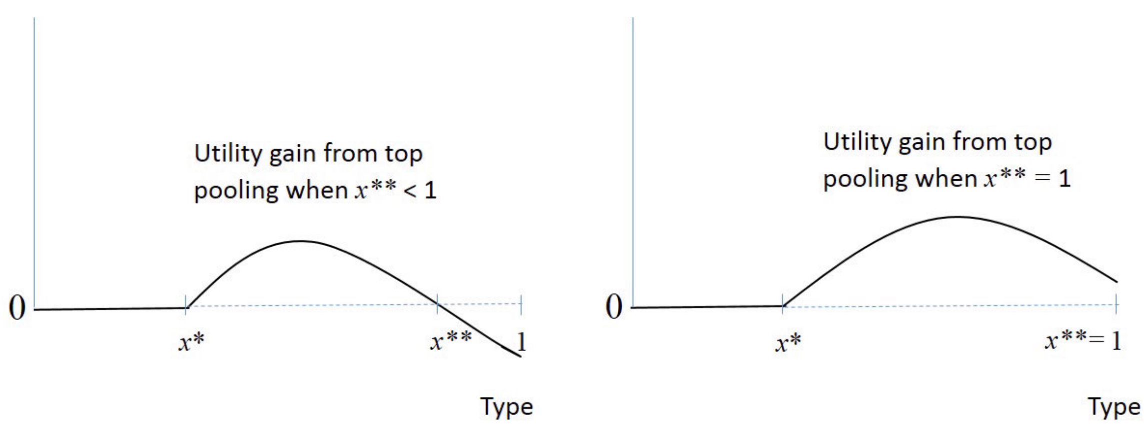

Figure 1: Utility gain from top pooling

[18] This is because the marginal equilibrium utility, or the marginal information rent, is equal to a type's prize, which is higher with top pooling for types slightly higher than x^*.

The following example illustrates the results from Proposition 1.

Example 1 *Suppose that F and G are uniform. Consider utility (2) with $c(t) = t$.*[19] *The assortative allocation is $y^A(x) = x$, and the approximating mechanism specifies performance $t^A(x) = x^2/2$. The payoff of type x is $x^2/2$.*

Under top pooling with threshold x^*, every type $x < x^*$ chooses performance $\frac{1}{2}x^2$ and obtains prize x, and every type $x > x^*$ chooses performance M and obtains a prize drawn uniformly from interval $[x^*, 1]$. Performance M is given by

$$\frac{1}{2}(x^*)^2 = x^* \frac{1+x^*}{2} - M,$$

so $M = x^*/2$. Thus, the payoffs are $\frac{1}{2}x^2$ for $x < x^*$ and

$$x\frac{1+x^*}{2} - \frac{x^*}{2} \geq \frac{1}{2}x^2$$

for $x > x^*$. All top pooling thresholds are Pareto-improving.

Example 1 shows that there may exist multiple Pareto-improving pooling thresholds. It is therefore reasonable to ask whether these thresholds can be Pareto ranked. In Example 1, the derivative of the payoff of every type $x > x^*$ with respect to the threshold type x^* is $(x-1)/2$, so all types prefer $x^* = 0$, i.e., the Pareto preferred top pooling is a lottery over all prizes. In general, however, Pareto-improving pooling thresholds are not Pareto ranked, as the following example shows.

Example 2 *Let $F = G$ have density $f = g = 7/4$ on intervals $[0, 1/4]$ and $[3/4, 1]$, and density $f = g = 1/4$ on interval $(1/4, 3/4)$. Consider utility (2) with $c(t) = t$. The assortative allocation is $y^A(x) = x$, and the approximating mechanism specifies performance $t^A(x) = x^2/2$. The payoff of type x is $x^2/2$.*

Top pooling with threshold $x^* = 0$, which is a lottery, is Pareto improving, since the expected prize of $1/2$ at performance 0 gives each type x utility $x/2$, which exceeds $x^2/2$. Now consider top pooling with threshold $x^* = 1/2$. For this threshold we have $M = 19/64$, since

$$\frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{2}\left(\frac{1}{8}\frac{5}{8} + \frac{7}{8}\frac{7}{8}\right) - \frac{19}{64}.$$

Type $x = 1$ benefits from this top pooling, since $((1/8)(5/8) + (7/8)(7/8)) - (19/64) > 1/2$. It is therefore Pareto improving, by part (d) of Proposition 1. Type $x = 1$ (as well as slightly lower types) also prefer this top pooling to a pure lottery. However, types in the interval $(0, 1/2)$ have the opposite preference, because a lottery gives each of them an expected utility of $x/2$, and top pooling with threshold $x^* = 1/2$ gives each of them an expected utility of $x^2/2$.

The following result clarifies when Pareto-improving pooling thresholds are Pareto-ranked.

[19] Recall that all our results hold for utility (1).

Proposition 2 *Suppose that $x_1^* < x_2^*$ are Pareto-improving top pooling thresholds.*

(a) If type $x = 1$ weakly prefers x_1^* to x_2^*, then types x in $(x_1^*, 1)$ strictly prefer x_1^* to x_2^*, so x_1^* is Pareto preferred to x_2^*.

(b) If type $x = 1$ strictly prefers x_2^* to x_1^*, then x_1^* and x_2^* are not Pareto ranked. There is an x^{**} in $(x_2^*, 1)$ such that types x in (x_1^*, x^{**}) strictly prefer x_1^*, and types $x > x^{**}$ strictly prefer x_2^*.

Figure 2 illustrates the two parts of Proposition 2. The left-hand side corresponds to part (a), and the right-hand side corresponds to part (b).

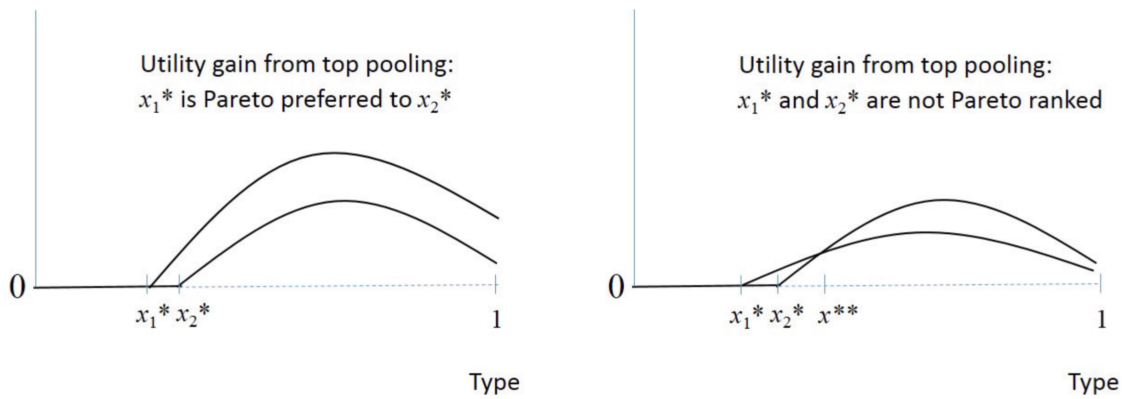

Figure 2: Comparing the utility gain from pooling thresholds $x_1^* < x_2^*$

Proposition 2 leads to a simple description of the Pareto frontier of top pooling thresholds. To see this, consider the function φ that assigns to any threshold x^* the utility of type $x = 1$ in the approximating mechanism with this threshold. Denote by ϕ the lowest monotone function that is pointwise weakly higher than φ. Then, the Pareto frontier consists of all the thresholds x^* at which $\varphi(x^*) = \phi(x^*)$ (see Figure 3).

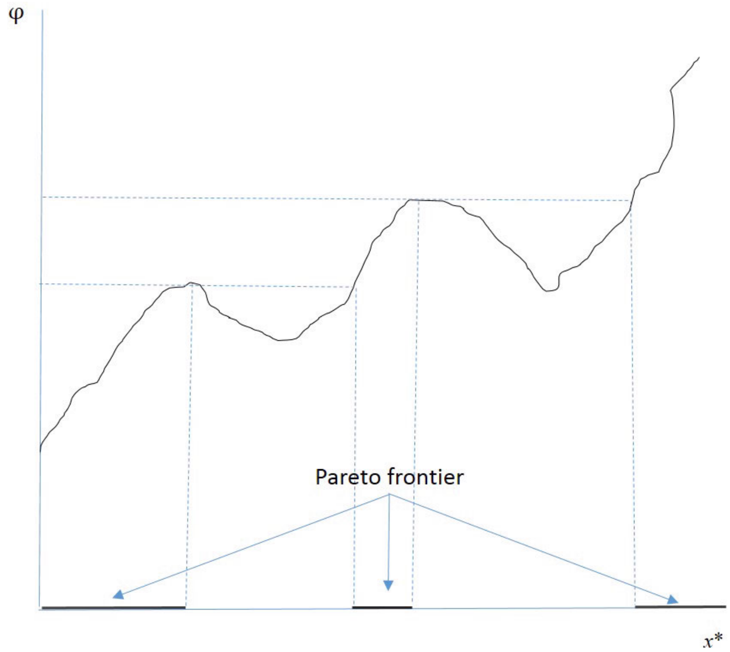

Figure 3: Pareto frontier of top pooling thresholds

When we consider only a finite set X^* of top pooling thresholds, Proposition 2 implies that the Pareto frontier of X^* consists of the threshold x^* that is most preferred by type 1 among all the thresholds in X^*, the threshold that is most preferred by type 1 among the thresholds in X^* that are lower than x^*, and so on.

5 Category Rankings

We now consider more general performance disclosure policies, which may include one or more pooled intervals of performance ranking. We investigate how different policies affect students' welfare, and identify the Pareto improving ones. We will use the term "category rankings" to describe such policies. A category ranking is a monotone partition of the players according to the ranking of their performance. One example is partitioning them above and below the median performance. Another example is partitioning them according to whether their performance is below the 10-th percentile, between the 10-th percentile and the 20-th percentile, etc. A category ranking induces a partition of the set of prizes, and the prizes within each element of the partition are randomly assigned to the players in the corresponding element of the category ranking.

Formally, a category ranking is a monotone partition $\mathcal{J}$ of the set $[0,1]$ of quantiles into singletons and left-open intervals. The intervals are $J_k = \left(q_k^l, q_k^h\right]$ for $1 \leq k \leq K \leq n$, where $0 \leq q_1^l < q_1^h \leq \cdots \leq q_K^l < q_K^h \leq 1$. The interpretation is the fraction $q_k^h - q_k^l$ of players whose performance quantile rankings lie in J_k are grouped together (any rule can be used to break ties in the ranking of two or more players who choose the same performance). Prizes are assigned in decreasing value to the partition elements, and distributed according to a fair lottery among the players in each partition element. To describe the approximating mechanism, denote by $G^{\mathcal{J}}$ the distribution of prizes when the prizes in each interval J_k are replaced with an equal mass of prize

$$y(J_k) = \frac{\int_{q_k^l}^{q_k^h} G^{-1}(z)dz}{q_k^h - q_k^l} = \frac{\int_a^b h(y^A(x))dF(x)}{F(b) - F(a)} \tag{12}$$

for $a = F^{-1}\left(q_k^l\right)$ and $b = F^{-1}\left(q_k^h\right)$. The corresponding assortative allocation $y^{A,\mathcal{J}}(x)$ satisfies

$$y^{A,\mathcal{J}}(x) = \left(G^{\mathcal{J}}\right)^{-1}(F(x)). \tag{13}$$

The unique incentive-compatible mechanism that implements this allocation and gives type $x = 0$ a utility of 0 specifies performance

$$t^{A,\mathcal{J}}(x) = c^{-1}\left(xy^{A,\mathcal{J}}(x) - \int_0^x y^{A,\mathcal{J}}(\tilde{x})\, d\tilde{x}\right). \tag{14}$$

Note that a category ranking induces a partition $\mathcal{I}$ of the set of types $X = [0,1]$ into singletons and K intervals $I_k = \left(F^{-1}\left(q_k^l\right), F^{-1}\left(q_k^h\right)\right]$, such that all types in interval I_k choose the same performance and obtain the same prize $y(J_k)$ in the approximating mechanism, and singleton types obtain the prize they did in the original approximating mechanism. Thus, the assortative allocation and approximating mechanism can be equivalently defined from the partition $\mathcal{I}$ of types (instead of the partition $\mathcal{J}$) by letting $G^{\mathcal{I}}$ coincide with $G^{\mathcal{J}}$ and defining $y^{A,\mathcal{I}}(x)$ and $t^{A,\mathcal{I}}(x)$ as in (13) and (14) with $\mathcal{I}$ instead of $\mathcal{J}$.

Thus, from the perspective of the approximating mechanism, a category ranking $\mathcal{J}$ corresponds to a partition $\mathcal{I}$ of the set of types into singletons and a finite number of intervals. In what follows, it will be convenient to consider such partitions of the set of types and the corresponding approximating mechanisms. We will abuse terminology slightly by also referring to such partitions $\mathcal{I}$ of the type interval $[0,1]$ as category rankings.

5.1 The added value of category rankings

Top pooling is a particular kind of category ranking: top pooling with threshold x^* is the category ranking $\mathcal{I} = \{(x^*, 1]\} \cup \{\{x\} : x \le x^*\}$. The richer set of outcomes that can be generated by category rankings may include outcomes that are Pareto preferred to all outcomes that can be generated by top poolings. This is what the following example demonstrates.

Example 3 *Let $F = G$ have density $f = g = 5/4$ on interval $[0, 3/4]$, and density $f = g = 1/4$ on interval $[3/4, 1]$. Consider utility (2) with $c(t) = t$. The assortative allocation is $y^A(x) = x$, and the approximating mechanism specifies performance $t^A(x) = x^2/2$. The payoff of type x is $x^2/2$.*

Top pooling with threshold $x^* = 3/4$ is Pareto improving. Indeed, the corresponding performance M is given by

$$\frac{1}{2}\left(\frac{3}{4}\right)^2 = x^*\left(\frac{1+\frac{3}{4}}{2}\right) - M,$$

which gives $M = 3/8$. Types in $(3/4, 1]$ choose performance M, and each of them obtains a prize drawn uniformly from interval $(3/4, 1]$. The utility of type $x = 1$ is equal to $1/2$ both with and without top pooling. So, by part (d) of Proposition 1, top pooling with threshold $x^* = 3/4$ is Pareto improving. One can readily check that top pooling with any threshold $x^* > 3/4$ is also Pareto improving and gives type $x = 1$ utility $1/2$. We will show that top pooling with any threshold $x^* < 3/4$ is not Pareto improving. So, by part (a) of Proposition 2, the threshold $x^* = 3/4$ is the Pareto preferred one.

To see that no threshold $x^* < 3/4$ is Pareto improving, recall that the performance $M < 3/8$ satisfies

$$\frac{1}{2}(x^*)^2 = x^* E[y \mid t = M] - M,$$

where $E[y \mid t = M]$ is the expected prize contingent on choosing performance M. The utility of type $x = 1$ is thus

$$E[y \mid t = M] + \underbrace{\frac{1}{2}(x^*)^2 - x^* E[y \mid t = M]}_{-M} < \frac{1}{2},$$

because $E[y \mid t = M] < (1 + x^*)/2$ for $x^* < 3/4$.

Top pooling with threshold $x^* = 3/4$ is the category ranking that pools together the top $1/16$ of the types and leaves the other types as singletons. However, this category ranking is Pareto inferior to the category ranking that pools together the top $1/16$ of the types, and pools together the bottom $15/16$ of the types. Indeed, under this category ranking, the bottom $15/16$ of the types exert no effort and obtain an expected prize of $3/8$, while the top $1/16$ of the types choose performance $3/8$ and obtain an expected prize of $7/8$. Under the former category ranking, the top $1/16$ of the types also choose performance $3/8$ and obtain an expected prize of $7/8$, but the bottom $15/16$ of the types x obtain a lower utility of $x^2/2$.

5.2 Welfare comparisons

Consider first single-interval category rankings, that is, category rankings of the form $\mathcal{I} = \{(x^*, x^{**}]\} \cup \{\{x\} : x \leq x^* \text{ or } x > x^{**}\}$ for some types $0 \leq x^* < x^{**} \leq 1$; top pooling is a special case in which $x^{**} = 1$. As in the case of top pooling, we assume that not all of the prizes in quantiles $[F(x^*), F(x^{**})]$ are identical.[20] The following results generalizes Proposition 1.

Proposition 3 *(a) The utility of type $x \in (x^*, x^{**}]$ increases as a result of the single-interval category ranking $\mathcal{I}$ if and only if*

$$\frac{\int_{x^*}^{x^{**}} y^A(\tilde{x}) dF(\tilde{x})}{F(x^{**}) - F(x^*)} \geq \frac{\int_{x^*}^{x} y^A(\tilde{x})\, d\tilde{x}}{x - x^*}.$$

(b) The category ranking $\mathcal{I}$ is Pareto improving if and only if

$$\frac{\int_{x^*}^{x^{**}} y^A(\tilde{x}) dF(\tilde{x})}{F(x^{**}) - F(x^*)} \geq \frac{\int_{x^*}^{x^{**}} y^A(\tilde{x})\, d\tilde{x}}{x^{**} - x^*}. \tag{15}$$

[20] If they are identical, then the category ranking has no effect.

The intuition for Proposition 3 is similar to the one underlying Proposition 1, applied to types in the interval $(x^*, x^{**}]$. In particular, if a type $x \in (x^*, x^{**}]$ benefits from the category ranking, then all types in the interval $(x^*, x]$ benefit as well. Types $x \leq x^*$ are clearly not affected by the category ranking, and the derivative of the utility of types $x > x^{**}$ is equal to $y^A(x)$ both in the original contest and under the category ranking. Thus, if type x^{**} is better off under the category ranking, then so are all types higher than x^{**}, which gives part (b).

For category rankings that include more than one interval, a generalization of the conditions in Proposition 3 provides sufficient conditions for a category ranking to increase the utility of a type and to be Pareto improving, but these conditions are no longer necessary. This is because pooling on an interval may increase the utility of types above the interval to such a degree that even if these types are pooled in a way that lowers their utility, the overall effect may be to increase their utility relative to the baseline contest.

To obtain the sufficient conditions, consider a category ranking $\mathcal{I}$ that includes precisely the $K \geq 2$ intervals $I_1, \dots, I_K$, where $I_k = (x_k^*, x_k^{**}]$ and $x_k^{**} \leq x_{k+1}^*$ for $k < K$. The effect of the category ranking can be described as follows. For each $k < K$ let G^k be the distribution of prizes when the prizes corresponding to intervals $I_1, \dots, I_k$ are replaced by their averages. Then, the contest with the category ranking that pools only intervals $I_1, \dots, I_{k+1}$ is the same as the contest with the single-interval category ranking that pools only interval I_{k+1} but starts with prize distribution G^k. Proposition 3 describes the effect of this single-interval category ranking on a baseline contest with prize distribution G^k. By induction on k we immediately obtain the following corollary of Proposition 3.

Proposition 4 *(a) The utility of type $x \in (x_k^*, x_k^{**}]$ increases as a result of the category ranking $\mathcal{I}$ if*

$$\frac{\int_{x_k^*}^{x_k^{**}} y^A(\tilde{x})dF(\tilde{x})}{F(x_k^{**}) - F(x_k^*)} \geq \frac{\int_{x_k^*}^{x} y^A(\tilde{x})\,d\tilde{x}}{x - x_k^*} \text{ and } \frac{\int_{x_j^*}^{x_j^{**}} y^A(\tilde{x})dF(\tilde{x})}{F(x_j^{**}) - F(x_j^*)} \geq \frac{\int_{x_j^*}^{x_j^{**}} y^A(\tilde{x})\,d\tilde{x}}{x_j^{**} - x_j^*} \text{ for all } j < k.$$

(b) The category ranking $\mathcal{I} = \{I_1, ..., I_k\}$ is Pareto improving if

$$\frac{\int_{x_j^*}^{x_j^{**}} y^A(\tilde{x})dF(\tilde{x})}{F(x_j^{**}) - F(x_j^*)} \geq \frac{\int_{x_j^*}^{x_j^{**}} y^A(\tilde{x})\,d\tilde{x}}{x_j^{**} - x_j^*} \text{ for all } j \leq k.$$

Characterizing the Pareto frontier of category rankings is more complicated than for top poolings. In the appendix we provide a method for checking whether a category ranking belongs to the Pareto frontier of category rankings, and we illustrate its usefulness with an example.

6 Robust Pareto improvements

The results in the previous sections suggest that Pareto improvements exist in some college admissions settings, but to apply these results, we need to construct estimates of the type distribution F and the prize distribution G (which affects the results via the assortative allocation y^A).

We now present simpler results that rely only on properties of the type distribution F and correspond to Propositions 1, 2, 3, and 6. These results are potentially more useful for empirical work, because they are more robust and easier to apply.[21] We present first the results which involve only distribution F, because they are simpler. We will use the term "robust Pareto improvement" as shorthand for "weakly better for every type for any functions c and G, and a Pareto improvement for some functions."

The first result characterizes robust Pareto improving top pooling thresholds. It follows immediately from part (d) of Proposition 1 and the definition of first-order stochastic dominance (FOSD).

Corollary 1 *Type x^* is a robust Pareto improving top pooling threshold if and only if the distribution F truncated below type x^* FOSD the uniform distribution truncated below this x^*.*

Consider two robust Pareto improving top pooling thresholds $x_1^* < x_2^*$. By definition of top pooling, the effect of top pooling with threshold x_2^* is identical to the effect of using a baseline contest and modifying the prize distribution by changing the prizes allocated to types $(x_2^*, 1]$ from what is specified by G to a mass $1 - F(x_2^*)$ of the average (according to distribution G) of these prizes. Then, by Corollary 1, top pooling with threshold x_1^* leads to a further robust Pareto improvement. This proves the following result.

Corollary 2 *If $x_1^* < x_2^*$ are robust Pareto improving top pooling thresholds, then top pooling with threshold x_1^* is robust Pareto preferred to top pooling with threshold x_2^*. Thus, the Pareto frontier of robust Pareto improving top pooling thresholds is a singleton, which is the lowest robust Pareto improving top pooling threshold.*

Corollary 2 explains why in Example 1 lower top pooling thresholds are Pareto preferred to higher ones, and why a lottery is Pareto preferred to any positive top pooling threshold.[22]

The next result characterizes robust Pareto improving single-interval category rankings. It follows immediately from part (b) of Proposition 1 and the definition of FOSD.

Corollary 3 *Category ranking $\mathcal{I} = \{(x^*, x^{**}]\} \cup \{\{x\} : x \leq x^* \text{ or } x > x^{**}\}$ is robust Pareto improving if and only if distribution F truncated below x^* and above x^{**} FOSD the uniform distribution truncated below x^* and above x^{**}.*

Consider two robust Pareto improving single-interval category rankings $\mathcal{I}_1$ and $\mathcal{I}_2$, with corresponding pooled type intervals $I_1 = (x_1^*, x_1^{**}]$ and $I_2 = (x_2^*, x_2^{**}]$. Suppose $I_1 \subseteq I_2$. The effect of pooling on interval I_1 is identical to the effect of using a baseline contest and modifying the prize distribution by changing the prizes allocated to types $(x_1^*, x_1^{**}]$ from what is specified by G to a mass $F(x_1^{**}) - F(x_1^*)$ of the average of these prizes. By Corollary 3, applying the category ranking $\mathcal{I}_2$ leads to a further robust Pareto improvement. Thus, $\mathcal{I}_2$ is robust Pareto preferred to $\mathcal{I}_1$.

[21] An estimate of the distribution F is essential for all of our results.

[22] Example 2 fails this condition, because F does not FOSD the uniform distribution on $[0, 1]$.

Similarly, if I_1 and I_2 are disjoint, then the two-interval category ranking with pooled type intervals I_1 and I_2 is robust Pareto preferred to $\mathcal{I}_1$ and $\mathcal{I}_2$. Finally, suppose that I_1 and I_2 intersect. In this case, the following lemma and our result for the case $I_1 \subseteq I_2$ imply that the single-interval category ranking with pooled type interval $I_1 \cup I_2$ is robust Pareto preferred to $\mathcal{I}_1$ and $\mathcal{I}_2$.

Lemma 1 *Consider two intervals $I_1 = (x_1, x_3]$ and $I_2 = (x_2, x_4]$ for $0 \leq x_1 < x_2 < x_3 < x_4 \leq 1$. If for each interval I_1 and I_2, F restricted to the interval FOSD the uniform distribution restricted to the interval, then F restricted to the union $I_1 \cup I_2$ of the intervals FOSD the uniform distribution restricted to $I_1 \cup I_2$.*

These observations immediately lead to the following result.

Proposition 5 *The Pareto frontier of robust Pareto improving category rankings is a singleton $\mathcal{I}^{PF}$, which consists of the maximal intervals on which F restricted to each interval FOSD the uniform distribution restricted to the interval, along with singletons for all other types.*

The following example illustrates Proposition 5.

Example 4 *Let F have density $f = 4/3$ on interval $[0, 1/4]$, $f = 2/3$ on interval $(1/4, 1/2]$, $f = 1/2$ on interval $(1/2, 3/4]$, and $f = 3/2$ on interval $(3/4, 1]$. (Notice that for robust Pareto improvements we do not specify functions c and G.) Then, F restricted to interval $[1/2, 1]$ FOSD the uniform distribution restricted to the same interval, because the former has an increasing density and the latter has a constant density.*

On interval $[1/4, 1]$, the uniform distribution has density $4/3$, and F restricted to this interval has density 1 on interval $(1/4, 1/2]$, density $3/4$ on interval $(1/2, 3/4]$, and density $9/4$ on interval $(3/4, 1]$. So F also FOSD the uniform distribution, when both are restricted to interval $(1/4, 1]$.

Distribution F does not, however, FOSD the uniform distribution when both are restricted to a longer interval that contains $[1/4, 1]$, because their densities on interval $[0, 1/4]$ are $4/3$ and 1, respectively. In addition, F "weakly" FOSD the uniform distribution when both are restricted to interval $[0, 1/4]$. Thus, by Proposition 5, the unique robust Pareto improving category ranking on the Pareto frontier consists of two intervals: $[0, 1/4]$ and $(1/4, 1]$. It is easy to construct examples in which a single Pareto-frontier category ranking consists of any finite number of intervals.

6.1 An illustration with data

The key to using Proposition 5 in a concrete college admissions setting is to obtain an estimate of the ability distribution F in the student population. This is a serious empirical exercise that is beyond the scope of this paper. Instead, we use publicly available data to illustrate the kind of results one could obtain once an estimate of the distribution F has been generated. Specifically, we use SAT composite score data for 2011-2015, made publicly available by College Board.[23] The data provide a distribution of SAT scores, which

[23] www.collegeboard.org

closely approximates a truncated normal distribution (see Figure 4). For the purpose of the illustration only, we assume that college admissions are determined solely based on students' SAT scores, and that the score distribution reflects the ability distribution in the student population. The latter assumption is consistent with the "curving" that SAT score undergo, and which can be thought of as reflecting the test designers' model of the population ability distribution. According to this view, the role of the test is not to provide an estimate of the population ability distribution, but rather to identify the location of each student within the distribution. The location of each student's score in the score distribution corresponds to the location of his ability in the population ability distribution.

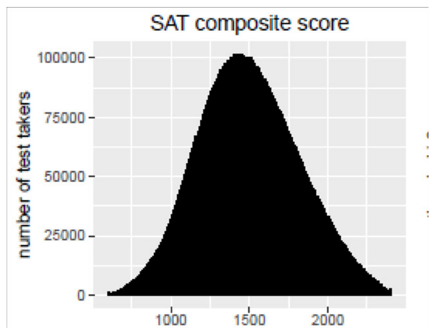

Figure 4: SAT score distribution

We apply Proposition 5 to this distribution of SAT scores and identify the unique robust Pareto improving category ranking. This ranking is illustrated in Figure 5 and consists of three intervals of pooling: 600 to 1830, 2310 to 2320, and 2390 to 2400. The latter two intervals are insignificant,[24] but the first corresponds to "bottom pooling," in which more than eighty percent of the lower-performing students are pooled. Such bottom pooling is indeed prevalent: many lower-ranked colleges accept a large fraction of the applicants, so the applicants' incentives to compete are weak. Of course, our finding that bottom pooling based on SAT data is robust Pareto improving is just an illustration of how to use our theoretical results and cannot be translated to policy recommendations. Our theoretical results can likely be used to derive policy recommendations in settings where a centralized test is the primary determinant of college admissions, but doing this would require a serious empirical investigation.

[24] The scores are reported in increments of 10, so each interval consists of two scores.

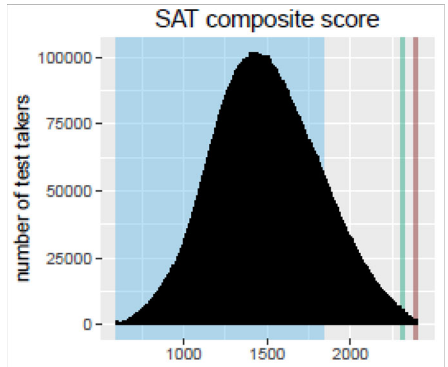

Figure 5: The unique robust Pareto improving category ranking

7 Conclusion

This paper investigated how to improve college admissions settings based on centralized tests. Students engage in test-preparation activities to improve their ranking, but these activities are costly. Our main message is that coarse performance disclosure policies can benefit all students, regardless of their ability. These policies take a simple form and are easy to implement.

Our characterization of robust Pareto improvements, and Proposition 5 in particular, identify Pareto improvements that apply to any distribution of valuations of college seats. This may be particularly useful for empirical work, since characterizing these Pareto improvements only requires estimating the ability distribution in the student population. We provide an illustration based on SAT data, but a serious empirical estimation in a concrete setting is left for future work. The Pareto improving performance disclosure policies resulting from such an exercise can be used as a basis for policy recommendations, which have the potential to significantly improve all students' welfare in college admissions settings that are based on centralized tests.

A Pareto frontier of category rankings

Using Proposition 3, we provide a method for checking whether a category ranking belongs to the Pareto frontier of category rankings. For this, we will need another concept. Let $\mathcal{I}$ be a category ranking, and let $x^* < x^{**}$ be an arbitrary pair of types that belong to two different elements $I \neq I'$ (intervals or singletons) of I, so $x^* \in I \in \mathcal{I}$ and $x^{**} \in I' \in \mathcal{I}$. We define a new category ranking $\mathcal{I}(x^*, x^{**})$ that groups all types between x^* and x^{**} into one category as follows: (i) if $I = (a, b]$ and $I' = (a', b']$, replace I, I', and all elements of I between I and I' with $(a, x^*]$, $(x^*, x^{**}]$, and $(x^{**}, b']$; (ii) if $I = \{x^*\}$ and $I' = (a', b']$, replace I' and all elements of I between I and I' with $(x^*, x^{**}]$, and $(x^{**}, b']$; (iii) if $I = (a, b]$ and $I' = \{x^{**}\}$, replace I, I', and all elements of I between I and I' with $(a, x^*]$ and $(x^*, x^{**}]$; (iv) if $I = \{x^*\}$ and $I' = \{x^{**}\}$, replace I' and all elements of I between I and I' with $(x^*, x^{**}]$.

Proposition 6 *A category ranking $\mathcal{I}$ belongs to the Pareto frontier of category rankings if and only if there is no pair of types $x^* < x^{**}$ such that*

$$x^* = a \text{ for some } I = (a, b] \in \mathcal{I} \text{ or } x^* = d \text{ for some } I = \{d\} \in \mathcal{I} \text{ and } x^{**} \in I' \neq I \in \mathcal{I},$$
$$\text{and type } x^{**} \text{ weakly prefers ranking } \mathcal{I}(x^*, x^{**}) \text{ to ranking } \mathcal{I}.$$

Proposition 6 helps to characterize the Pareto frontier by substantially reducing the set of category rankings to which any given ranking must be compared, as the following example demonstrates.

Example 5 *Revisit Example 3. It is easy to verify that any interval that satisfies condition (15) must be contained in $(0, 3/4]$ or $(3/4, 1]$. Thus, any candidate for a Pareto-improving category ranking consists of an interval partition of $(0, 3/4]$ and an interval partition of $(3/4, 1]$. But by the general payoff formula (17) in the proof of Proposition 6, if a partition of $(0, 3/4]$ (or a partition of $(3/4, 1]$) includes more than one element, then $(x^*, x^{**}] = (0, 3/4]$ ($(x^*, x^{**}] = (3/4, 1]$, respectively) violates the condition from Proposition 6. Indeed, the payoffs for x^{**} are equal under $\mathcal{I}$ and under $\mathcal{I}(x^*, x^{**})$. Thus, the Pareto frontier has only one element, the category ranking $\{(0, 3/4], (3/4, 1]\}$.*

B Proofs

Proof of Proposition 1. Part (a) follows because with top pooling types $x < x^*$ choose effort $t^A(x)$ and obtain prize $y^A(x)$. For part (b), note that the utility of type x^* is the same in the approximating mechanisms of the original contest and in the one with top pooling. Consider first the utility of a type $x > x^*$ in the approximating mechanism of the original contest. By (5), this utility exceeds that of type x^* by

$$\int_{x^*}^{x} h\left(y^A(\tilde{x})\right) d\tilde{x}.$$

In the approximating mechanism with top pooling, the utility of type x exceeds that of type x^* by

$$(x - x^*) \frac{\int_{x^*}^{1} h(y^A(\tilde{x})) dF(\tilde{x})}{1 - F(x^*)},$$

since both types' performance is M, and both types' prize is chosen randomly from the mass $1 - F(x^*)$ of the highest prizes. Thus, top pooling increases the utility of type x if and only if (10) holds.

For part (c), note that the derivative with respect to x of the utility gain of type x is

$$\frac{\int_{x^*}^{1} h(y^A(\tilde{x}))dF(\tilde{x})}{1 - F(x^*)} - h\left(y^A(x)\right). \tag{16}$$

The fraction in (16) is a weighted average of $h\left(y^A(\tilde{x})\right)$ over types in $[x^*, 1]$, so (16) is positive for types x close to x^*, monotonically decreases as x increases, and becomes negative for types x close to 1. Thus, the utility gain resulting from top pooling for types $x > x^*$ first increases and then decreases in the type. In particular, the utility of all types $x < 1$ strictly increases if the utility of type 1 weakly increases, which gives part (d).

Proof of Proposition 2. Recall that given a threshold x^*, the utility with top pooling of type $x > x^*$ is

$$U^{x^*}(x) = (x - x^*)\frac{\int_{x^*}^{1} h(y^A(\tilde{x}))dF(\tilde{x})}{1 - F(x^*)} + \int_0^{x^*} h\left(y^A(\tilde{x})\right)d\tilde{x}.$$

This means that $U^{x^*}(x)$ can be represented as $x\phi(x^*) + \psi(x^*)$, where

$$\phi(x^*) = \frac{\int_{x^*}^{1} h(y^A(\tilde{x}))dF(\tilde{x})}{1 - F(x^*)}$$

and

$$\psi(x^*) = \int_0^{x^*} h\left(y^A(\tilde{x})\right)d\tilde{x} - x^*\frac{\int_{x^*}^{1} h(y^A(\tilde{x}))dF(\tilde{x})}{1 - F(x^*)}.$$

Notice that $\phi(x^*)$ increases in x^*, because as x^* increases $\phi(x^*)$ becomes an average over higher values of $h(y^A(\tilde{x}))$.

Thus, $(x-1)\phi(x_1^*) > (x-1)\phi(x_2^*)$ for all $x < 1$, and if $\phi(x_1^*) + \psi(x_1^*) \geq \phi(x_2^*) + \psi(x_2^*)$, then

$$x\phi(x_1^*) + \psi(x_1^*) = (x-1)\phi(x_1^*) + \phi(x_1^*) + \psi(x_1^*) >$$

$$> (x-1)\phi(x_2^*) + \phi(x_2^*) + \psi(x_2^*) = x\phi(x_2^*) + \psi(x_2^*),$$

which yields part (a) for types $x > x_2^*$. Lower types higher than x_1^* also strictly prefer pooling with threshold x_1^* to x_2^*, since they strictly prefer pooling with threshold x_1^* to no pooling (part (c) of Proposition 1 with $x^{**} = 1$), and their utility with threshold x_2^* is equal to their utility with no pooling (part (a) of Proposition 1). Types lower than the threshold x_1^* obtain the same prize and their performance is the same with both thresholds.

Suppose now that type 1 strictly prefers x_2^* to x_1^*. Observe first that there exists an $x^{**} > x_2^*$ such that types x in (x_2^*, x^{**}) strictly prefer x_1^* and types $x > x^{**}$ strictly prefer x_2^*. Indeed, since $\phi(x_1^*) < \phi(x_2^*)$, this follows from the fact that

$$[x\phi(x_2^*) + \psi(x_2^*)] - [x\phi(x_1^*) + \psi(x_1^*)] = x[\phi(x_2^*) - \phi(x_1^*)] + [\psi(x_2^*) - \psi(x_1^*)]$$

strictly increases in x, and type x_2^* strictly prefers x_1^* to x_2^*. This observation about type x_2^*'s preferences follows from the fact that x_1^* is Pareto improving, part (c) of Proposition 1 applied to x_1^*, and the fact that type x_2^* is indifferent between pooling with threshold x_2^* and no pooling. Since all types close to 1 strictly prefer x_2^* to x_1^*, we have that $x^{**} < 1$, and because type x_2^* strictly prefers x_1^* to x_2^*, we have that $x^{**} > x_2^*$. Types in (x_1^*, x_2^*) strictly prefer x_1^* to x_2^* for the same reason that type x_2^* does, and types lower than x_1^* are indifferent between x_1^*, x_2^*, and no pooling.

Proof of Proposition 3. Let $r^* = F(x^*)$ and $r^{**} = F(x^{**})$. Then, any type $x < x^*$ provides effort $t(x) = t^A(x)$ and obtains prize $y(x) = y^A(x)$. Types in $(x^*, x^{**}]$ provide a certain effort t and obtain a fair lottery over prizes $y \in (G^{-1}(r^*), G^{-1}(r^{**})]$. Since type x^* is indifferent between the two options, we have that

$$x^* h(y^A(x^*)) - c(t^A(x^*)) = \int_0^{x^*} h\left(y^A(\tilde{x})\right) d\tilde{x} = x^* \frac{\int_{x^*}^{x^{**}} h(y^A(\tilde{x}))dF(\tilde{x})}{F(x^{**}) - F(x^*)} - c(t).$$

By (5) we have that

$$U^{\mathcal{I}}(x) - U(x) = (x - x^*)\frac{\int_{x^*}^{x^{**}} h(y^A(\tilde{x}))dF(\tilde{x})}{F(x^{**}) - F(x^*)} - \int_{x^*}^{x} h\left(y^A(\tilde{x})\right) d\tilde{x}$$

$$= (x - x^*)\left[\frac{\int_{x^*}^{x^{**}} h(y^A(\tilde{x}))dF(\tilde{x})}{F(x^{**}) - F(x^*)} - \frac{\int_{x^*}^{x} h\left(y^A(\tilde{x})\right) d\tilde{x}}{x - x^*}\right].$$

Thus, (15) is a necessary and sufficient condition for types x in $(x^*, x^{**}]$ to be better off. To show that (15) is a necessary and sufficient condition for Pareto improvement, observe that types $x \le x^*$ are indifferent. Any type $x > x^{**}$ obtains prize $y^A(x)$ and has payoff

$$U^{\mathcal{I}}(x) = \int_0^{x^*} h\left(y^A(\tilde{x})\right) d\tilde{x} + (x^{**} - x^*)\frac{\int_{x^*}^{x^{**}} h(y^A(\tilde{x}))dF(\tilde{x})}{F(x^{**}) - F(x^*)} + \int_{x^{**}}^{x} h\left(y^A(\tilde{x})\right) d\tilde{x},$$

which is no lower than $U(x)$ if and only if condition (15) is satisfied.

The last equality is obtained directly from (5) by noticing that the contest under our category ranking $\mathcal{I}$ is equivalent to a baseline contest in which prizes $y^A(\tilde{x})$, for $\tilde{x}$ in $(x^*, x^{**}]$, are replaced with the certainty equivalents of the lottery faced by types $\tilde{x}$ in $(x^*, x^{**}]$ under our category ranking $\mathcal{I}$.

Proof of Proposition 6. It will be helpful to provide first a general formula for the utility of type $x \in [0,1]$ under category ranking $\mathcal{I}$. This utility exceeds $U(x)$ given by (5) by the expression

$$\sum_{(a,b]\in\mathcal{I}, a<b<x} \left[(b-a)\frac{\int_a^b h(y^A(\tilde{x}))dF(\tilde{x})}{F(b) - F(a)} - \int_a^b h\left(y^A(\tilde{x})\right) d\tilde{x}\right] + \tag{17}$$

$$(x-a)\frac{\int_a^b h(y^A(\tilde{x}))dF(\tilde{x})}{F(b) - F(a)} - \int_a^x h\left(y^A(\tilde{x})\right) d\tilde{x} \text{ for } x \in (a,b] \in \mathcal{I}.$$

This formula follows directly from the fact that types $\tilde{x} \in (a,b] \in \mathcal{I}$ obtain a fair lottery over prizes $y^A(\tilde{x}')$ for $\tilde{x}' \in (a,b]$.

We will first show that when a pair $x^* < x^{**}$ satisfies the condition in Proposition 6, the category ranking $\mathcal{J} = \mathcal{I}(x^*, x^{**})$ Pareto improves over $\mathcal{I}$. Types $x \in [0, x^*]$ are obviously indifferent between the two category rankings. By assumption, the utility of type x^{**} is no lower under $\mathcal{J}$ than under $\mathcal{I}$. We will now show that the utility of types $x \in (x^*, x^{**})$ is strictly higher under $\mathcal{J}$ than under $\mathcal{I}$. Indeed, the derivative on $(x^*, x^{**}]$ of type x's utility under $\mathcal{J}$, $U^{\mathcal{J}}(x)$, is constant and equal to

$$\frac{\int_{x^*}^{x^{**}} h(y^A(\tilde{x}))dF(\tilde{x})}{F(x^{**}) - F(x^*)}.$$

In turn, the derivative on $(x^*, x^{**}]$ of type x's utility under $\mathcal{I}$, $U^{\mathcal{I}}(x)$, is equal to $h\left(y^A(x)\right)$ if x does not belong to any non-degenerate interval $(a, b] \in \mathcal{I}$, and is equal to

$$\frac{\int_a^b h(y^A(\tilde{x}))dF(\tilde{x})}{F(b) - F(a)}$$

if $x \in (a, b] \in \mathcal{I}$. This means that the derivative increases in x, and increases strictly except on intervals $(a, b] \in \mathcal{I}$. So, $U^{\mathcal{I}}(x)$ is a convex non-linear function. Since $U^{\mathcal{J}}(x)$ is linear on $(x^*, x^{**}]$, $U^{\mathcal{I}}(x^*) = U^{\mathcal{J}}(x^*)$, and $U^{\mathcal{I}}(x^{**}) \leq U^{\mathcal{J}}(x^{**})$, we obtain that $U^{\mathcal{I}}(x) \leq U^{\mathcal{J}}(x)$ for all $x \in (x^*, x^{**}]$, and the inequality is strict for all types $x \in (x^*, x^{**}]$. Similarly, the derivative of $U^{\mathcal{J}}(x)$ on $(x^{**}, b']$ exceeds that of $U^{\mathcal{I}}(x)$ if $a' < x^{**} < b'$ for some $(a', b'] \in \mathcal{I}$, and the two derivatives are equal for $x > b'$, which completes the proof that $\mathcal{J}$ Pareto improves over $\mathcal{I}$.

Suppose now that another category ranking $\mathcal{I}'$ Pareto improves over $\mathcal{I}$. Recall that $\mathcal{I}$ consists of singletons and a finite number of intervals $[x_1, x_1'], (x_2, x_2'], ..., (x_k, x_k']$, with $x_i' < x_{i+1}$. Denote by x' the highest type such that $\mathcal{I}$ and $\mathcal{I}'$ coincide up to x', and suppose that x' is the lower endpoint of an interval $(x_l, x_l']$ in $\mathcal{I}$. (A similar argument to the one that follows applies if x' is a singleton.)

Now, x' must be the endpoint of a non-trivial interval in $\mathcal{I}'$, which we denote by $(x^*, x^{**}]$, where $x' = x^* < x^{**}$. Otherwise, for types x slightly higher than x_l the utility of these types under $\mathcal{I}$ would exceed their utility under $\mathcal{I}'$ by (17). It also cannot be that $x^{**} < x_l'$, since it would then follow from (17) that x^{**} strictly prefers $\mathcal{I}$ to $\mathcal{I}'$.

Thus $x_l' < x^{**}$, and since $\mathcal{I}'$ Pareto improves over $\mathcal{I}$, type x^{**} weakly prefers $\mathcal{I}'$ to $\mathcal{I}$. And since (by (17)) the payoff of type x^{**} under any ranking depends only on the intervals up to the one that contains x^{**}, type x^{**} is indifferent between ranking $\mathcal{I}'$ and ranking $\mathcal{J} = \mathcal{I}(x^*, x^{**})$, and therefore prefers ranking $\mathcal{J}$ to ranking $\mathcal{I}$.

Proof of Lemma 1. We have to show for every $x \in I_1 \cup I_2$ that

$$\frac{F(x) - F(x_1)}{F(x_4) - F(x_1)} \leq \frac{x - x_1}{x_4 - x_1}. \tag{18}$$

Consider first $x \in I_1$. By the definition of FOSD on I_1, we have

$$\frac{F(x) - F(x_1)}{F(x_3) - F(x_1)} \leq \frac{x - x_1}{x_3 - x_1},$$

so for (18) it suffices to show that

$$\frac{F(x_3) - F(x_1)}{F(x_4) - F(x_1)} \leq \frac{x_3 - x_1}{x_4 - x_1}.$$

Suppose instead that

$$\frac{F(x_3)-F(x_1)}{F(x_4)-F(x_1)} > \frac{x_3-x_1}{x_4-x_1}. \tag{19}$$

This implies that

$$\frac{F(x_4)-F(x_3)}{F(x_4)-F(x_1)} < \frac{x_4-x_3}{x_4-x_1} \tag{20}$$

In addition,

$$\frac{F(x_3)-F(x_2)}{F(x_3)-F(x_1)} = 1 - \frac{F(x_2)-F(x_1)}{F(x_3)-F(x_1)} \geq 1 - \frac{x_2-x_1}{x_3-x_1} = \frac{x_3-x_2}{x_3-x_1}, \tag{21}$$

which multiplied by (19) gives

$$\frac{F(x_3)-F(x_2)}{F(x_4)-F(x_1)} > \frac{x_3-x_2}{x_4-x_1}. \tag{22}$$

Dividing (20) by (22) we obtain

$$\frac{F(x_4)-F(x_3)}{F(x_3)-F(x_2)} < \frac{x_4-x_3}{x_3-x_2} \Rightarrow \frac{F(x_3)-F(x_2)}{F(x_4)-F(x_3)} + 1 > \frac{x_3-x_2}{x_4-x_3} + 1 \Rightarrow$$

$$\frac{F(x_4)-F(x_2)}{F(x_4)-F(x_3)} > \frac{x_4-x_2}{x_4-x_3} \Rightarrow \frac{F(x_4)-F(x_3)}{F(x_4)-F(x_2)} < \frac{x_4-x_3}{x_4-x_2}.$$

This last inequality is a contradiction, since FOSD on I_2 implies the opposite weak inequality, similarly to (21). Therefore, (18) holds for $x \in I_1$.

Now consider $x \in [x_3, x_4]$. Instead of (18) we will show the equivalent inequality

$$\frac{F(x_4)-F(x)}{F(x_4)-F(x_1)} \geq \frac{x_4-x}{x_4-x_1}.$$

From FOSD on I_2 we have

$$\frac{F(x_4)-F(x)}{F(x_4)-F(x_2)} \geq \frac{x_4-x}{x_4-x_2}.$$

Thus, it suffices to show that

$$\frac{F(x_4)-F(x_2)}{F(x_4)-F(x_1)} \geq \frac{x_4-x_2}{x_4-x_1}.$$

This inequality holds, because otherwise we would have

$$\frac{F(x_2)-F(x_1)}{F(x_4)-F(x_1)} > \frac{x_2-x_1}{x_4-x_1},$$

which would violate (18) for $x = x_2 \in I_1$.

C Externalities

In order to focus on the competitive aspect of college admissions, we modeled performance as a costly choice variable that is used to rank students. Students' effort may have additional, external effects, both because it affects the learning environment, which affects all students, and because the level of education may affect society more generally.[25] Our analysis showed that top pooling decreases the performance of players

[25] This is also true for college students, who often learn from one another, and also affects the classes they take, because classes are often adjusted to the students taking them.

with high types and increases the performance of some players with lower types. The aggregate effect on performance is therefore ambiguous.

When performance costs are convex, one sufficient condition for top pooling to increase aggregate performance in a large contest is that the marginal performance cost at the performance M is sufficiently high. The intuition is as follows. In the original contest, players with different types can separate by choosing different levels of performance. Around M (in the original contest), it suffices to choose slightly higher performance levels in order to separate from players with lower types, because the marginal cost of performance is high. Top pooling prevents this separation, but the slight reduction in performance from high types who now choose M is outweighed by lower types who face a lower marginal cost of increasing their performance to reach M. This and other sufficient conditions[26] can be combined with the conditions under which top pooling is Pareto improving to obtain conditions under which top pooling is socially beneficial even in the presence of performance externalities.

Another form of externality, which we dub "match externality," concerns the social cost (beyond students' individual utility) of the departure of the allocation from assortative matching. Our results above show that assortative matching is obtained in the baseline contest. The associated performance is, however, costly. It may be socially beneficial to impose a category ranking that reduces the match quality along with the associated performance. To highlight this possibility, consider utility (3) with linear functions c and h and suppose that types and prizes are distributed uniformly. Suppose also that society suffers a loss $-c\,|x-y|^q$ if type x obtains prize y. Then, for $q>2$, it can be shown that any category ranking that maximizes welfare includes a finite number of intervals, none of them degenerate, and the length of these intervals decreases when one moves from the left to the right. The intuition is that intervals reduce competition and therefore reduce performance, which increases welfare, but decrease the match quality, which decreases welfare. The second effect is more important when players' types are higher, so the intervals shrink as types increase.[27]

D Peer effects

We now model peer effects in a way that does not change any of our results, and requires only a transformation of the prize distribution. Consider a limit prize distribution that consists of atoms, where each atom represents a mass of seats in a particular college. Students who attain a particular college experience peer effects from other students attending the same college.

To model this, consider an equilibrium of a finite contest and denote by $I(y)$ the set of players admitted to university y (for a particular realization of types and bids). The utility of a player of type x admitted to

[26] Additional sufficient conditions are given by Olszewski and Siegel (2016b). They also show that with linear or concave costs effort caps (which are equivalent to top pooling in large contests) do not increase aggregate effort.

[27] For $q=2$ the optimal category ranking consists of an interval with lower endpoint 0 and singletons above the upper bound of the interval.

university y by bidding t is

$$xy+x\frac{\sum_{i\in I(y)}p\left(x_{i}\right)}{\left|I\left(y\right)\right|}-c\left(t\right)=x\underbrace{\left(y+\frac{\sum_{i\in I(y)}p\left(x_{i}\right)}{\left|I\left(y\right)\right|}\right)}_{\tilde{y}}-c\left(t\right),$$

where $p\left(x\right)$ captures the peer effect exerted by a player of type x. We refer to $\tilde{y}$ as the effective prize for player i, which is the sum of the value of the college and the average peers effects of the other students attending the college. Note that the effective prize depends on the equilibrium because it is determined by the equilibrium allocation of prizes.

The limit approximating mechanism still implements the assortative allocation. To see this, consider two universities $y<y'$ and the corresponding limit effective prizes $\tilde{y}$ and $\tilde{y}'$. If the allocation of students to universities is such that $\tilde{y}\geq\tilde{y}'$, then all types would make the lower bid to get into $\tilde{y}$. But then ties will be broken randomly, which would generate the same peer effects for both prizes, so $y<y'$ would give us $\tilde{y}<\tilde{y}'$, a contradiction. If $\tilde{y}<\tilde{y}'$, then standard mechanism design results tell us that higher types get higher prizes, so it will be the highest types that are assigned to y', and the lower types will be assigned to y. Thus, in the limit we get the assortative allocation.

This means that in the limit approximating mechanism, for each prize y in the support of the limit prize distribution G we have that the effective prize is

$$\tilde{y}=y+\frac{\int_{x_{L}^{y}}^{x_{H}^{y}}p\left(\tilde{x}\right)dF\left(\tilde{x}\right)}{F\left(x_{H}^{y}\right)-F\left(x_{L}^{y}\right)}=\frac{\int_{x_{L}^{y}}^{x_{H}^{y}}\left(y+p\left(\tilde{x}\right)\right)dF\left(\tilde{x}\right)}{F\left(x_{H}^{y}\right)-F\left(x_{L}^{y}\right)},\tag{23}$$

where (x_{L}^{y},x_{H}^{y}) is the interval of types that are allocated prize y in the assortative allocation (so $x_{L}^{y}=F^{-1}\left(\lim_{y'\uparrow y}G\left(y'\right)\right)$ and $x_{H}^{y}=F^{-1}\left(G\left(y\right)\right)$). Now, replace the limit prize distribution G with distribution $\tilde{G}$ in which every prize y is replaced with the effective prize $\tilde{y}$. The assortative allocation y^{A} is replaced with $\tilde{y}^{A}$, so $\tilde{y}^{A}\left(x\right)$ is the effective prize for type x under the assortative allocation. Then, all our results on the characterization of Pareto improvements continue to hold.

To this, it is enough to consider two consecutive prizes and determine the effect of pooling the types that are allocated these prizes. Denote by $y<y'$ two consecutive prizes in the support of the limit prize distribution G, so $y=y^{A}\left(x\right)$ for x in $(x_{L}^{y},x_{H}^{y}]$ and $y'=y^{A}\left(x\right)$ for x in $(x_{L}^{y'},x_{H}^{y'}]$ (with $x_{H}^{y}=x_{L}^{y'}$). By pooling types on interval $[x_{L}^{y},x_{H}^{y'}]$, the two prizes y and y' are combined to create a certainty equivalence prize y''. The corresponding effective prize is

$$\tilde{y}''=\frac{\int_{x_{L}^{y}}^{x_{H}^{y}}ydF\left(\tilde{x}\right)+\int_{x_{L}^{y'}}^{x_{H}^{y'}}y'dF\left(\tilde{x}\right)+\int_{x_{L}^{y}}^{x_{H}^{y'}}p\left(\tilde{x}\right)dF\left(\tilde{x}\right)}{F(x_{H}^{y'})-F\left(x_{L}^{y}\right)}=\frac{\left(F\left(x_{H}^{y}\right)-F\left(x_{L}^{y}\right)\right)\tilde{y}+\left(F\left(x_{H}^{y}\right)-F\left(x_{L}^{y}\right)\right)\tilde{y}'}{F(x_{H}^{y'})-F\left(x_{L}^{y}\right)}=$$

$$\frac{\int_{x_{L}^{y}}^{x_{H}^{y}}\tilde{y}^{A}\left(\tilde{x}\right)dF\left(\tilde{x}\right)+\int_{x_{L}^{y'}}^{x_{H}^{y'}}\tilde{y}^{A}\left(\tilde{x}\right)dF\left(\tilde{x}\right)}{F(x_{H}^{y'})-F\left(x_{L}^{y}\right)}=\frac{\int_{x_{L}^{y}}^{x_{H}^{y'}}\tilde{y}^{A}\left(\tilde{x}\right)dF\left(\tilde{x}\right)}{F(x_{H}^{y'})-F\left(x_{L}^{y}\right)},$$

where the first equality follows from (23). As in the proof of Proposition 3, pooling is a Pareto improvement if and only if

$$\tilde{y}''-\frac{\int_{x_{L}^{y}}^{x_{H}^{y'}}\tilde{y}^{A}\left(x\right)dx}{x_{H}^{y'}-x_{L}^{y}}\geq0\iff\frac{\int_{x_{L}^{y}}^{x_{H}^{y'}}\tilde{y}^{A}\left(x\right)dF\left(x\right)}{F(x_{H}^{y'})-F\left(x_{L}^{y}\right)}\geq\frac{\int_{x_{L}^{y}}^{x_{H}^{y'}}\tilde{y}^{A}\left(x\right)dx}{x_{H}^{y'}-x_{L}^{y}}.$$

E More general utility functions

We will now show how our results for top pooling can be extended to more general student utility functions.

Consider the following, separable utility functions: $h(x,y) - c(t)$ and $h(y) - c(x,t)$, where $h(x,0) = c(x,0) = 0$ for all x, c is strictly increasing in t when $x > 0$ and decreasing in x when $t > 0$, and h is strictly increasing in y when $x > 0$ and strictly increasing in x when $y > 0$. These utilities generalize utilities (2) and (3), respectively. We will now describe how the results of Section 4 can be extended to these utility functions.[28] Part (a) of Proposition 1 follows from the results of Olszewski and Siegel (2016a), as these results hold for the more general utility functions. (Of course, the formula defining M will be different for the more general utility functions.)

Consider first the utility $h(x,y) - c(t)$. The efforts $t^A(x)$ satisfy the following equations

$$t^A(x) = c^{-1}\left(h(x, y^A(x)) - \int_0^x h_1\left(z, y^A(z)\right) dz\right), \tag{24}$$

and

$$h(y^A(x)) - c(x, t^A(x)) = -\int_x^1 c_1(z, t^A(z)) dz.$$

respectively. The former equation determines $t^A(x)$. By differentiating the latter equation, assuming that F and G are differentiable, we obtain the following differential equation

$$h'(y^A(x))(y^A)'(x) = c_2(x, t^A(x))(t^A)'(x).$$

This equation, together with the initial condition $t^A(0) = 0$, uniquely determines $t^A(x)$, assuming that the involved functions satisfy the Lipschitz condition.

For the utility $h(x,y) - c(t)$ we have that

$$U(x) = h(x, y^A(x)) - c(t^A(x)) = \int_0^x h_1\left(z, y^A(z)\right) dz$$

and

$$U^{x^*}(x) = \frac{\int_{x^*}^1 h(x, y^A(z))dF(z)}{1-F(x^*)} - c(M) = \frac{\int_{x^*}^1 h(x, y^A(z))dF(z)}{1-F(x^*)} - \frac{\int_{x^*}^1 h(x^*, y^A(z))dF(z)}{1-F(x^*)} + \int_0^{x^*} h_1\left(z, y^A(z)\right) dz.$$

This yields

$$U^{x^*}(x) - U(x) = \frac{\int_{x^*}^1 h(x, y^A(z))dF(z)}{1-F(x^*)} - \frac{\int_{x^*}^1 h(x^*, y^A(z))dF(z)}{1-F(x^*)} - \int_{x^*}^x h_1\left(z, y^A(z)\right) dz.$$

Comparing this expression to 0, we obtain an analogue of condition (10) from part (b) of Proposition 1.

The derivative of $U^{x^*}(x) - U(x)$ is equal to

$$\frac{\int_{x^*}^1 h_1(x, y^A(z))dF(z)}{1-F(x^*)} - h_1\left(x, y^A(x)\right).$$

[28] The results of Section 4 can be extended to a more general separable utility function $h(x,y) - c(x,t)$. However, the results for this more general function would be a combination the results for the two more specific functions, and the analysis would be less transparent.

If we assume that $h_1(x,y)$ increases in y, then this derivative is positive at $x = x^*$ and negative at $x = 1$. Assuming that the derivative changes its sign only once, we obtain part (c) of Proposition 1; in addition, this yields the following analogue of the condition from part (d):

$$\frac{\int_{x^*}^1 h(1, y^A(z))dF(z)}{1 - F(x^*)} - \frac{\int_{x^*}^1 h(x^*, y^A(z))dF(z)}{1 - F(x^*)} - \int_{x^*}^1 h_1\left(z, y^A(z)\right) dz \geq 0.$$

For the utility $h(y) - c(x,t)$ we have that

$$U(x) = h(y^A(x)) - c(x, t^A(x)) = -\int_0^x c_1(z, t^A(z))dz$$

and

$$U^{x^*}(x) = \frac{\int_{x^*}^1 h(y^A(z))dF(z)}{1 - F(x^*)} - c(x, M) = c(x^*, M) - c(x, M) - \int_0^{x^*} c_1(z, t^A(z))dz,$$

which yields

$$U^{x^*}(x) - U(x) = c(x^*, M) - c(x, M) + \int_{x^*}^x c_1(z, t^A(z))dz.$$

Comparing this expression to 0, we obtain an analogue of condition (10) from part (b) of Proposition 1.

The derivative of this expression is $c_1(x, t^A(x)) - c_1(x, M)$. Assume that $c_1(x,t)$ decreases with t. Then, $c_1(x, t^A(x)) - c_1(x, M) \geq 0$ when $t^A(x) \leq M$, and $c_1(x, t^A(x)) - c_1(x, M) \leq 0$ when $t^A(x) \geq M$. Since $U^M(x) - U(x) = 0$ when $x = x^*$, part (c) of Proposition 1 must hold; in addition, this yields the following analogue of the condition from part (d):

$$c(x^*, M) - c(1, M) + \int_{x^*}^1 c_1(z, t^A(z))dz \geq 0.$$

Proposition 2 also generalizes to utilities $h(x,y) - c(t)$ and $h(y) - c(x,t)$, and its proof requires only minor changes. Consider first the utility $h(x,y) - c(t)$. we have

$$U^{x^*}(x) = \frac{\int_{x^*}^1 h(x, y^A(z))dF(z)}{1 - F(x^*)} - \frac{\int_{x^*}^1 h(x^*, y^A(z))dF(z)}{1 - F(x^*)} + \int_0^{x^*} h_1\left(z, y^A(z)\right) dz,$$

so, $U^{x^*}(x)$ can be represented as $\phi(x, x^*) + \psi(x^*)$, where

$$\phi(x, x^*) = \frac{\int_{x^*}^1 h(x, y^A(z))dF(z)}{1 - F(x^*)}$$

and

$$\psi(x^*) = \int_0^{x^*} h_1\left(z, y^A(z)\right) dz - \frac{\int_{x^*}^1 h(x^*, y^A(z))dF(z)}{1 - F(x^*)}.$$

Assume that for all $x'' > x'$, the difference $h(x'', y) - h(x', y)$ strictly increases in y. This implies that $\phi(x, x_1^*) - \phi(1, x_1^*) > \phi(x, x_2^*) - \phi(1, x_2^*)$. Together with $\phi(1, x_1^*) + \psi(x_1^*) \geq \phi(1, x_2^*) + \psi(x_2^*)$, this yields part (a).

To show part (b), suppose that type 1 strictly prefers x_2^* to x_1^*. By the assumption that $h(x'', y) - h(x', y)$ strictly increases in y, it follows that $U^{x_1^*}(x) - U^{x_2^*}(x)$ strictly decreases in x. Thus, there exists an x^{**} such that types $x < x^{**}$ strictly prefer x_1^* and types $x > x^{**}$ strictly prefer x_2^*. And since all players with types close to 1 strictly prefer x_2^* to x_1^*, we have that $x^{**} < 1$. Finally, it can be readily checked that the derivative of ψ with respect to x^* is negative. Hence, $\psi(x_1^*) > \psi(x_2^*)$, and so $x^{**} > 0$.

For the utility $h(y) - c(x,t)$, we have that

$$U^{x_1^*}(x) = c(x^*, M) - c(x, M) - \int_0^{x^*} c_1(z, t^A(z))dz.$$

So, $U^{x_1^*}(x)$ can be represented as $\phi(x, x_1^*) + \psi(x_1^*)$, where

$$\phi(x, x_1^*) = -c(x, M_1)$$

and

$$\psi(x_1^*) = c(x^*, M) - \int_0^{x^*} c_1(z, t^A(z))dz.$$

Assume that for all $x'' > x'$, the difference $c(x', M) - c(x'', M)$ strictly increases in M. This implies that $\phi(x, x_1^*) - \phi(1, x_1^*) > \phi(x, x_2^*) - \phi(1, x_2^*)$. Together with $\phi(1, x_1^*) + \psi(x_1^*) \geq \phi(1, x_2^*) + \psi(x_2^*)$, this yields part (a).

To show part (b), suppose that a player of type 1 strictly prefers x_2^* to x_1^*. By the assumption that $c(x', M) - c(x'', M)$ strictly increases in M, it follows that $U^{x_1^*}(x) - U^{x_2^*}(x)$ strictly decreases in x. Thus, there exists an x^{**} such that types $x < x^{**}$ strictly prefer x_1^* and types $x > x^{**}$ strictly prefer x_2^*. And since all players with types close to 1 strictly prefer x_2^* to x_1^*, we have that $x^{**} < 1$. Finally, it can be readily checked that the derivative of ψ with respect to x^* is negative. Hence, $\psi(x_1^*) > \psi(x_2^*)$, so $x^{**} > 0$.

References

[1] Ahn S-Y and Baek H-J (2013): "Academic achievement-oriented society and its relationship to the psychological well-being of Korean adolescents," In: Yi CC (eds) The psychological well-being of East Asian youth, quality of life in Asia, 265–279.

[2] Bodoh-Creed, A. and B. Hickman (2018a): "College Assignment as a Large Contest," Journal of Economic Theory, 175: 88-126.

[3] Bodoh-Creed, A. and B. Hickman (2018b): "Pre-College Human Capital Investment and Affirmative Action: A Structural Policy Analysis of US College Admissions," Mimeo.

[4] Boleslavsky, R. and C. Cotton (2015): "Grading Standards and Education Quality," American Economic Journal: Microeconomics, 7, 248–279.

[5] Chakravarty, S. and T. R. Kaplan (2013): "Optimal allocation without transfer payments," Games and Economic Behavior, 77(1), 1-20.

[6] Che, Y.-K. and I. Gale (1998): "Caps on Political Lobbying," American Economic Review, 88, 643-651.

[7] Condorelli, D. (2012): "What money can't buy: Efficient mechanism design with costly signals," Games and Economic Behavior, 75(2), 613-624.

[8] Dubey, P. and J. Geanakoplos (2010): "Grading exams: 100, 99, 98, . . . or A, B, C?" Games and Economic Behavior, 69, 72-94.

[9] Fang, D. and T. Noe (2018): "Lowering the bar and limiting the field: The effect of strategic risk-taking on selection contests," Mimeo.

[10] Fang, D., T. Noe, and P. Strack (2018a): "A is the new B: Contests for grades under rational expectations," Mimeo.

[11] Fang, D., T. Noe, and P. Strack (2018b): "Turning Up the Heat: The Demoralizing Effect of Competition in Contests," Journal of Political Economy, forthcoming.

[12] Frankel, A. and N. Kartik (2017): "Muddled Information," Journal of Political Economy, forthcoming.

[13] Gale, D. and L. Shapley (1962): "College Admissions and the Stability of Marriage," American Mathematical Monthly, 69(1), 9-15.

[14] Gottlieb, D. and K. Smetters (2014): "Grade Non-Disclosure," Mimeo.

[15] Hafalir, I., Hakimov, R., Kübler, D., and M. Kurino (2018): "College Admissions with Entrance Exams: Centralized versus Decentralized," Journal of Economic Theory, 176: 886-934.

[16] Harbaugh, R. and E. Rasmusen (2018): "Coarse grades: Informing the public by withholding information," American Economic Journal: Microeconomics, 10(1): 210-35.

[17] Hoppe, H., Moldovanu, B., and A. Sela (2009): "The Theory of Assortative Matching Based on Costly Signals," Review of Economic Studies, 76, 253-281.

[18] Hsin, A. and Y. Xie (2014): "Explaining Asian Americans' academic advantage over whites," Proceedings of the National Academy of Sciences, 111(23), 8416–8421.

[19] Koh, W. T., Z. Yang and L. Zhu (2006): "Lottery rather than waiting-line auction," Social Choice and Welfare, 27, 289-310.

[20] Lien, J., J. Zheng, and X. Zhong (2017): "Ex-ante fairness in the Boston and serial dictatorship mechanisms under pre-exam and post-exam preference submission," Games and Economic Behavior, 100, 98-120.

[21] Olszewski, W. and R. Siegel (2016a): "Large Contests," Econometrica 84, 835-854.

[22] Olszewski, W. and R. Siegel (2016b): "Bid Caps in Large Contests," Games and Economic Behavior, forthcoming.

[23] Olszewski, W. and R. Siegel (2016c): "Performance-Maximizing Large Contests," Mimeo.

[24] Ostrovsky, M. and M. Schwarz (2010): "Information Disclosure and Unraveling in Matching Markets," American Economic Journal: Microeconomics, 2(2), 34-63.

[25] McIntyre, S. and J. Munson (2008): "Exploring Cramming: Student Behaviors, Beliefs, and Learning Retention in the Principles of Marketing Course," Journal of Marketing Education, 30(3), 226-243.

[26] Spence, M. (1973). "Job Market Signaling." Quarterly Journal of Economics, 87, 355-374.

[27] Taylor, G. A., K. K. Tsui, and L. Zhu (2003): "Lottery or waiting-line auction?" Journal of Public Economics, 87, 1313-1334.