Mechanism Design with Aftermarkets: On the Impossibility of Pure Information Intermediation

Preliminary and Incomplete

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Abstract

A mediator, with no prior information and no control over the market protocol, attempts to redesign the information structure in the market by running an information intermediation mechanism with transfers that first elicits information from an agent, and then discloses information to another market participant (third party). The note establishes a general impossibility result: If the third party has full bargaining power in the interaction with the agent, all incentive-compatible information intermediation mechanisms are uninformative about the agent's type.

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1 Introduction

This note establishes an impossibility result in a setting where a mediator, with no ex-ante information, attempts to redesign the information structure in a market. The result provides additional context for mechanism design with aftermarkets, a problem considered in Dworczak (2016a,b), and clarifies the relationship between a number of recent papers on information and mechanism design.

The model is roughly as follows. A mediator proposes an *information intermediation mechanism* to an agent who has a privately observed type. That is, under commitment, the mediator sends signals and exchanges transfers with the agent as a function of the agent's report to the mechanism. There is no budget balance requirement, and the mediator can subsidize the agent. The signal released by the mechanism is publicly observed. Subsequently, in the *aftermarket*, the agent interacts with a third party who makes a decision about the final allocation. Under the assumption that the third party has full bargaining power and can use randomization and transfers, I prove that the mediator cannot release any information that would change the outcome of the aftermarket interaction (Theorem 1).

Bergemann, Brooks and Morris (2015) study a related model which can be interpreted in the following way: The mediator has full access to the information of the agent, and thus the information intermediation mechanism does not need to be incentive-compatible (and there is no need to use transfers). In the aftermarket, the agent buys an object from a monopolist (the third party). They show that by disclosing information about the agent's value to the monopolist, the mediator can induce a large set of different outcomes (see Section 2 for details). In contrast, by adding the incentive-compatibility requirement, this note shows that the mediator with no prior knowledge of the agent's type cannot influence the outcome *at all*.

Because of the assumption about the full bargaining power of the third party, the model is a special case of sequential agency (see, for example, Calzolari and Pavan, 2009). In a sequential agency model, the agent contracts sequentially with multiple principals. In my case, the agent contracts with the mediator, and then with the third party. What distinguishes my model is the fact that the first-stage principal (the mediator) does not control any "physical" allocation, highlighting the role of information disclosure. Theorem 1 is related to a result about optimality of privacy in Calzolari and Pavan (2006b) who show conditions under which the first-stage principal should not disclose any information to maximize her profits. Under the strong assumption that the first-stage principal only controls signals and transfers, Theorem 1 offers a stronger conclusion – privacy is not only optimal but in fact the only feasible choice for the mediator in the pure information intermediation model.[1]

When the mediator acts as a mechanism designer who does control a physical allocation of a good, the model becomes a special case of mechanism design with aftermarkets considered by Dworczak (2016a,b) and is also closely related to the model of Calzolari and Pavan (2006a). In those models, the mediator (designer) can use the allocation rule to elicit and disclose information about the agent. The important contribution of this note is to show that the ability to disclose any information about the agent's type hinges on using a non-constant allocation rule. To elicit and disclose any information to an aftermarket where the third party has full bargaining power, the mechanism designer has to offer the good with at least two distinct probabilities.[2]

While this note does not consider any specific application, its conclusion provides a useful point of reference for market design problems in which the market regulator attempts to redesign the information structure. One important example of such a setting is the financial over-the-counter market. The Dodd-Frank Act and the introduction of the Trade Reporting and Compliance Engine (TRACE) were regulatory attempts to increase transparency in those otherwise opaque markets.[3] Many financial benchmarks, such as LIBOR, are also a source of pre-trade transparency.[4] Implementability of such information schemes hinges to a large degree on the presence of multiple agents who observe the same information.[5] For example, LIBOR aggregates reports about borrowing costs from multiple banks, and discloses a trimmed average. In my model, only one agent has access to private information. As evidenced by the so-called LIBOR scandal in which banks purposefully misreported their private information, incentive-

[1] Mathematically, the assumptions of Theorem 1 are not comparable to the assumptions of the main result of Calzolari and Pavan (2006b) because I consider a much more general type space for the agent and a more general decision space for the third party. Economically, my assumptions are more restrictive. However, Calzolari and Pavan (2006b) assume that the first-stage principal's utility is quasi-linear in transfers, and show optimality in this setting. My impossibility result covers cases in which the mediator does not derive utility from transfers, for example, when the mediator is a benevolent regulator.

[2] In the model of Calzolari and Pavan (2006a), the third party does not have full bargaining power but this does not change the conclusion because information about the agent is irrelevant when the agent makes the offer.

[3] See Asquith, Covert and Pathak (2013) and Bessembinder and Maxwell (2008) for institutional details and empirical analysis.

[4] LIBOR is the London Interbank Offered Rate. See Hou and Skeie (2013) for details.

[5] See for example Duffie, Dworczak and Zhu (2016).

compatibility is crucial if an information scheme is to fulfill its role in real-life markets.[6] The note shows in a formal model that imposing incentive-compatibility can drastically constrain the informativeness of information schemes.

2 An example

A monopolist with no production costs sells an object to a consumer who is privately informed about her value for the good. The value is captured by a one-dimensional privately observed type θ, distributed according to uniform distribution on $[0, 1]$.

Assuming that the monopolist has full bargaining and commitment power, the optimal selling mechanism is to make a take-it-or-leave-it offer to the consumer: Buy the good at a price of $1/2$ or do not trade at all. As a result, the monopoly profit is $1/4$, and the consumer enjoys an information rent of $1/8$. This is illustrated by point A in Figure 2.1.

Suppose, however, that there is a mediator who can observe the type of the consumer and send arbitrary signals to the monopolist, under full commitment. For example, the mediator can commit to disclosing the type of the consumer to the monopolist. In this case, the monopolist perfectly price-discriminates, and we reach point B in Figure 2.1, with a socially optimal outcome but all rents accruing to the seller. More surprisingly, it is also possible to implement the socially optimal outcome while maximizing the surplus of the consumer, corresponding to point C in Figure 2.1. More generally, Bergemann, Brooks and Morris (2015) show that any combination of the producer and consumer surplus in the gray triangle in Figure 2.1 can be achieved by the mediator.

Suppose now that the mediator does not have access to the information of the consumer directly (the mediator knows the distribution of θ but not its realization). Instead, again with full commitment power, the mediator can run an arbitrary *information intermediation mechanism* with transfers. That is, before the consumer interacts with the monopolist, the consumer participates in a mechanism created by the mediator where the allocation space consists of all possible signals. For example, the mediator can send a binary signal, high or low, charge the consumer for choosing the high signal, and subsidize her for choosing the low signal. Because different types of the consumer have different preferences over price distributions in the second stage (stemming from different valuations for the good), the hope is that by setting up transfers correctly, the

[6] See Hou and Skeie (2013) and Official Sector Steering Group (2014) for details.

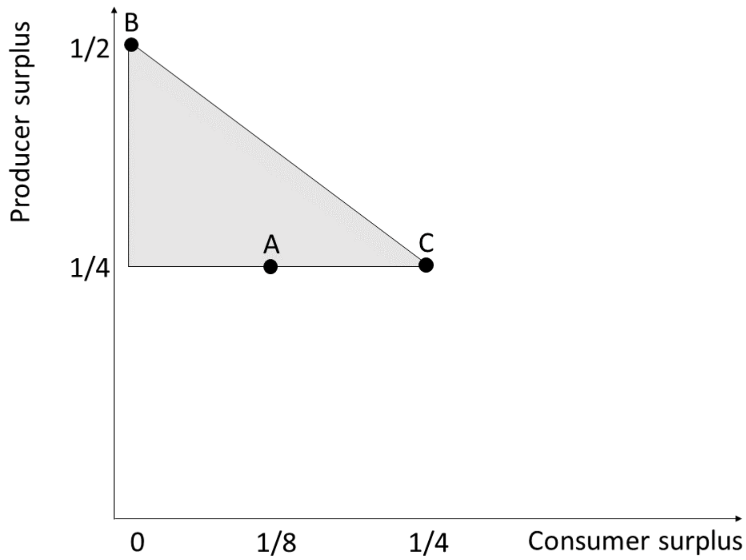

Fig. 2.1: The surplus triangle (Bergemann, Brooks and Morris, 2015)

mediator can disclose information about the consumer to the monopolist. That turns out not to be the case:

Fact 1. *The only outcome that can be induced by the mediator who does not have access to consumer's information is point A in Figure 2.1.*

The mediator cannot influence the information structure in any way, and the only feasible outcome is the one that would arise with no information disclosure (point A). Fact 1 is a special case of the main result of the paper (see Section 4).

Intuitively, the impossibility in the above example is a consequence of misaligned preferences of the consumer and the monopolist. It is the *high* types of the consumer who are willing to pay most for signals that lead to low prices quoted by the monopolist because they are the ones who buy with the highest probability. (Low types of the consumer have a lower willingness to pay because they rarely trade, for example, type 0 never buys from the monopolist at any positive price.) At the same time, a low price is quoted by the monopolist only if the monopolist believes the type of the consumer to be *low*. If the mediator tries to send a signal that leads to a low price quoted by the monopolist, then high types want to select that signal in the mechanism more than low types, and thus a low price cannot be optimal for the monopolist given this signal.

Similar intuition is discussed by Calzolari and Pavan (2006a) and Calzolari and Pavan (2006b) in the context of their results. A formalization of the notion of misaligned preferences (in the above sense) is considered in Dworczak (2016b), where the condition is phrased in terms of sub/supermodularity of the consumer's and the monopolist's utility functions in a model where the monopolist quotes one of two possible prices (high or low).

3 Model

In this section, I introduce the formal model. The model features three players: an agent, a third party, and a mediator. In the first stage, the mediator, who has full commitment power, offers an information intermediation mechanism to the agent. The mechanism sends signals as a function of the report of the agent. In the second stage, the agent interacts with the third party in the aftermarket.

The agent has a privately observed type $\theta \in \Theta$, where Θ is an arbitrary type space, and θ is distributed according to some distribution F. In the first stage, the mediator offers a direct mechanism (π, t) to the agent.[7] In the mechanism (π, t), $\pi : \Theta \to \Delta(\mathcal{S})$ denotes a signal function, where $\mathcal{S}$ is some arbitrary signal space (chosen by the mediator), and $t : \Theta \to \mathbb{R}$ is the transfer function. If the agent reports $\hat{\theta}$, she pays $t(\hat{\theta})$ (the transfer can be negative), and a signal s is drawn from the distribution $\pi(\cdot | \hat{\theta})$ and announced publicly.

In the second stage (the aftermarket), the third party observes the signal s released by the first-stage mechanism (but nothing else), and acts as a mechanism designer. There is an abstract decision space Y with $y \in Y$ denoting a generic element. The third party proposes an *aftermarket mechanism* to the agent, under commitment, with possible use of randomization and transfers. Formally, the third party chooses a direct mechanism $(\mathbf{y}, \tau)$, where $\mathbf{y} : \Theta \to \Delta(Y)$, and $\tau : \Theta \to \mathbb{R}$. The agent reports $\hat{\theta}$ to the aftermarket mechanism, pays $\tau(\hat{\theta})$ to the third party, and then a decision y is drawn from the distribution $\mathbf{y}(\hat{\theta})$ and implemented. I will denote by $d\mathbf{y}(\cdot | \theta)$ the probability distribution over Y given the decision rule $\mathbf{y}$ and report θ.[8]

The ex-post utility of the agent is given by $u(y, \theta) - t_a$, where y denotes the decision taken by the third party and $t_a \in \mathbb{R}$ is the overall net transfer from the agent to other players. The agent can also decide not to participate in either stage, in which case

[7] Because the mediator has commitment power, by the Revelation Principle, it is without loss of generality to look at direct mechanisms.

[8] Because the first-stage information intermediation mechanism has no direct externalities for the aftermarket (except, of course, for the signal that is sent), it is without loss of generality to look at direct mechanisms offered by the third party. See for example Calzolari and Pavan (2006b).

the payoff to all players is normalized to zero. The third party maximizes $v(y,\,\theta,\,t_{tp})$, where t_{tp} denotes the net transfer from the agent to the third party. I assume that $v(y,\,\theta,\,t_{tp})$ is non-decreasing in t_{tp} but impose no other restrictions. The agent and the third party are expected utility maximizers. The preferences of the mediator are not specified because they will not play any role in the analysis.

Throughout, in cases when Θ, $\mathcal{S}$ or Y are infinite, I assume that they are endowed with respective $\sigma-$ fields, and the payoff functions, as well as the mechanisms proposed by the mediator and the third party, are measurable.

I describe the optimization problems of the third party and the mediator, respectively, in more detail, starting from the second stage.

The third party's problem Let F^s_π, for $s \in \mathcal{S}$, denote the posterior distribution of θ conditional on signal s, given disclosure rule π in the first-stage mechanism. The solution to the third party's problem is obtained by maximizing, for any $s \in \mathcal{S}$,

$$\mathbb{E}_{\theta\sim F^s_\pi}\left[\int_Y v(y,\,\theta,\,\tau(\theta))d\mathbf{y}(y|\,\theta)\right]$$

over all mechanisms $(\mathbf{y},\,\tau)$ that satisfy the individual-rationality and incentive-compatibility constraints: For all $\theta \in \Theta$,

$$\int_Y u(y,\,\theta)d\mathbf{y}(y|\,\theta) - \tau(\theta) \geq 0, \tag{A-IR}$$

$$\theta \in \underset{\hat{\theta}}{\operatorname{argmax}} \int_Y u(y,\,\theta)d\mathbf{y}(y|\,\hat{\theta}) - \tau(\hat{\theta}). \tag{A-IC}$$

I let $\mathcal{Y}^s_\pi$ denote the set of allocation rules $\mathbf{y} : \Theta \to \Delta(Y)$ that (together with some transfers) solve the above problem, for any given π and s. Finally, I let $\mathcal{Y}_\emptyset$ denote the corresponding set in the case when no information is revealed by the first-stage mechanism.

The mediator's problem By releasing information in the first-stage mechanism, the mediator induces a distribution of posterior beliefs held by the third party, thereby influencing the choice of an aftermarket mechanism. I assume that in cases when $\mathcal{Y}^s_\pi$ is multi-valued (that is, when the third party is indifferent among several mechanisms), the mediator can select the aftermarket mechanism.[9] Let $U^s_\pi(\theta)$ denote the correspond-

[9] The selection is made independently of the (reported) type of the agent.

ing information rent of the agent in the aftermarket mechanism, i.e. the expected utility of type θ who participates in the aftermarket mechanism selected from $\mathcal{Y}^s_\pi$. The information intermediation mechanism (π, t) is *truthful* if it is individually-rational and incentive-compatible: For all $\theta \in \Theta$,

$$\int_S U^s_\pi(\theta) d\pi(s|\,\theta) - t(\theta) \geq 0, \tag{I-IR}$$

$$\theta \in \operatorname*{argmax}_{\hat{\theta}} \int_S U^s_\pi(\theta) d\pi(s|\,\hat{\theta}) - t(\hat{\theta}). \tag{I-IC}$$

For any π supported by a truthful information intermediation mechanism, there is a corresponding set of decision rules $\mathbf{x} : \Theta \to \Delta(Y)$ that can be implemented by the mediator. Denote by $\bar{\mathcal{X}}$ the set of all decisions rules that can be induced by some truthful (π, t), that is

$$\bar{\mathcal{X}} = \{\mathbf{x} : \Theta \to \Delta(Y) : \exists \text{ truthful } (\pi, t),\, \mathbf{x}(\cdot \,|\, \theta) = \int_S \mathbf{y}^s(\cdot\, |\theta) d\pi(s|\,\theta),\, \mathbf{y}^s \in \mathcal{Y}^s_\pi,\, \forall \theta \in \Theta\}.$$[10]

The set $\bar{\mathcal{X}}$ contains all final outcomes that can be induced by the mediator through information disclosure in the first stage.

4 Main result

The following result establishes a general impossibility result for pure information intermediation.

Theorem 1. $\bar{\mathcal{X}} = \mathcal{Y}_\emptyset$. *That is, the mediator cannot influence the outcome of the aftermarket by releasing information.*

Theorem 1 states that the mediator cannot have any non-trivial impact on the information structure of the aftermarket. If the mechanism (π, t) attempts to send informative signals that change the decisions of the third party, then the agent will misreport in the mechanism, rendering the signals uninformative.

Formally, signals can be informative about the type of the agent but only if this information is irrelevant for the third party. For example, if the third party's utility is constant in θ, the mediator can disclose the type of the agent perfectly. This is possible,

[10] For two functions $\mathbf{x}, \mathbf{x}' : \Theta \to \Delta(Y)$, $\mathbf{x}(\cdot \,|\, \theta) = \mathbf{x}'(\cdot \,|\, \theta)$ denotes equality of the respective measures on Y conditional on θ, i.e. that $\mathbf{x}(A \,|\, \theta) = \mathbf{x}'(A \,|\, \theta)$ for all measurable subsets A of Y.

however, only because the choice of the mechanism by the third party is independent of her beliefs.

Fact 1 is a special case of Theorem 1. Regardless of the beliefs induced by the first-stage mechanism, quoting a price is within the set of optimal mechanisms for the monopolist (third party).[11] Therefore, the setting of Section 2 satisfies the assumptions of Theorem 1.

To prove Theorem 1 (see Section 5), I consider a relaxed problem in which the mediator can choose a "grand mechanism" by jointly choosing the mechanisms for both stages, subject to three necessary conditions coming from the original problem. First, the grand mechanism has to be incentive-compatible if the agent reports the same type in both stages. Second, the agent cannot find it profitable to misreport in the first-stage mechanism, and then refuse to participate in the second-stage mechanism. Third, the expected profit of the third party under the grand mechanism cannot be lower than her no-information profit in the original problem (i.e. in the case when no information is revealed by the first-stage mechanism). I then show that these three constraints imply that any decision rule that can be implemented by the mediator is also a solution to the third party's design problem with no additional information in the original model.

The mediator's inability to influence the outcome of the aftermarket hinges on three assumptions about the third party, namely that the third party can use *(i)* randomization, *(ii)* transfers, and *(iii)* has commitment power. If any of the above elements is missing, the mediator might be able to influence the outcome of the aftermarket (precisely because she can use randomization, transfers, and has commitment power). See Dworczak (2016b) for a related discussion and examples.

The conclusion of Theorem 1 fails also when the mediator additionally controls "physical" allocation of some good. For example, suppose that the mediator sells an object to the agent, and in the aftermarket the third party makes an offer to repurchase the object from the agent. This is a model considered by Calzolari and Pavan (2006a), and a special case of the model considered by Dworczak (2016a,b). When the mediator controls the allocation of the object, the allocation rule can be used as a leverage to elicit information. This is only possible when the good is offered to different types with different probabilities. If the allocation rule is constant in the type, Theorem 1 applies and no information can be revealed.

[11] This follows formally from the results of Skreta (2006) who considers a model of sequential mechanism design in a setting where the monopolist has imperfect commitment power.

5 Proof

In the last section of the paper, I prove Theorem 1.

First, it is obvious that $\mathcal{Y}_\emptyset \subseteq \bar{\mathcal{X}}$, because, by definition, decision rules in $\mathcal{Y}_\emptyset$ can be achieved by the mediator by not disclosing any information.

Conversely, suppose that a decision rule $\mathbf{x} : \Theta \to \Delta(Y)$ belongs to $\bar{\mathcal{X}}$, i.e. it is implementable by some truthful first-stage mechanism (π, t) offered by the mediator. Then, it has to be the case that $\mathbf{x}$ is implementable in the relaxed problem in which the mediator controls the choice of the aftermarket mechanism subject to conditions that are necessary in the original (unrelaxed) problem:

1. The "grand mechanism" is incentive-compatible for the agent. That is, taking into account the transfers from both stages, and the final decision rule, the agent must find it profitable to report her true type:

$$\int_Y u(x,\, \theta) d\mathbf{x}(x|\, \theta) - \tau(\theta) - t(\theta) \geq \int_Y u(x,\, \theta) d\mathbf{x}(x|\, \hat{\theta}) - \tau(\hat{\theta}) - t(\hat{\theta}),\ \forall \theta,\, \hat{\theta} \in \Theta. \tag{5.1}$$

2. In the original problem, the agent always has the option to refuse to participate in the aftermarket. Thus, it has to be the case that her ex-ante expected payoff in the grand mechanism is at least equal to the payoff she would receive by optimally reporting in the first-stage mechanism, and not participating in the second-stage mechanism:

$$\int_Y u(x,\, \theta) d\mathbf{x}(x|\, \theta) - \tau(\theta) - t(\theta) \geq \max_{\hat{\theta}} \{-t(\hat{\theta})\},\ \forall \theta \in \Theta. \tag{5.2}$$

3. Finally, no matter what information the first-stage mechanism reveals in the original problem, the third party cannot have an expected utility that is lower than her utility if no information is revealed. Denoting the no-information utility by $v^\star$, it has to be that

$$\mathbb{E}_\theta \left[\int_Y v(x,\, \theta,\, \tau(\theta)) d\mathbf{x}(x|\, \theta) \right] \geq v^\star. \tag{5.3}$$

Summarizing, if $\mathbf{x}$ is implementable, then equations (5.1) - (5.3) hold for some transfer functions t and τ.

Define $\bar{t}(\theta) \equiv t(\theta) + \max_{\hat{\theta}}\{-t(\hat{\theta})\} + \tau(\theta)$. Then, the problem can be written as

$$\int_Y u(x,\theta)d\mathbf{x}(x|\,\theta) - \bar{t}(\theta) \geq \int_Y u(x,\theta)d\mathbf{x}(x|\,\hat{\theta}) - \bar{t}(\hat{\theta}),\ \forall\theta,\ \hat{\theta} \in \Theta, \tag{5.4}$$

$$\int_Y u(x,\theta)d\mathbf{x}(x|\,\theta) - \bar{t}(\theta) \geq 0,\ \forall\theta \in \Theta, \tag{5.5}$$

$$\mathbb{E}_\theta\left[\int_Y v(x,\theta,\tau(\theta))d\mathbf{x}(x|\,\theta)\right] \geq v^\star. \tag{5.6}$$

Equations (5.4) and (5.5) imply that the aftermarket mechanism $(\mathbf{x},\bar{t})$ satisfies individual-rationality (A-IR) and incentive-compatibility (A-IC), and is hence feasible for the third party in the original problem. By definition of $v^\star$, it thus has to be the case that

$$\mathbb{E}_\theta\left[\int_Y v(x,\theta,\bar{t}(\theta))d\mathbf{x}(x|\,\theta)\right] \leq v^\star.$$

Because v is non-decreasing in the transfer received by the third party, and $\bar{t}(\theta) \geq \tau(\theta)$, for all θ, using inequality (5.6), we conclude that

$$v^\star \leq \mathbb{E}_\theta\left[\int_Y v(x,\theta,\tau(\theta))d\mathbf{x}(x|\,\theta)\right] \leq \mathbb{E}_\theta\left[\int_Y v(x,\theta,\bar{t}(\theta))d\mathbf{x}(x|\,\theta)\right] \leq v^\star.$$

Thus, all above inequalities are in fact equalities, and in particular,

$$\mathbb{E}_\theta\left[\int_Y v(x,\theta,\bar{t}(\theta))d\mathbf{x}(x|\,\theta)\right] = v^\star.$$

But this means that $\mathbf{x}$ (along with $\bar{t}$) is optimal for the aftermarket design problem of the third party when no information is disclosed. Thus, $\mathbf{x}$ belongs to $\mathcal{Y}_\emptyset$. □

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