Housing Booms and the U.S. Productivity Puzzle

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Abstract

The United States has been experiencing a slowdown in productivity growth for more than a decade. I exploit geographic variation across U.S. Metropolitan Statistical Areas (MSAs) to investigate the link between the 2006-2012 decline in house prices (the housing bust) and the productivity slowdown. Instrumental variable estimates support a causal relationship between the housing bust and the productivity slowdown. The results imply that one standard deviation decline in house prices translates into an increment of the productivity gap—i.e. how much an MSA would have to grow to catch up with the trend—by 6.9p.p., where the average gap is 14.51%. Using a newly-constructed capital expenditures measure at the MSA level, I find that the long investment slump that came out of the Great Recession explains a large fraction of this effect. Next, I document that the housing bust led to the investment slump and, ultimately, the productivity slowdown, mostly through the collapse in consumption expenditures that followed the bust. Lastly, I construct a quantitative general equilibrium model that rationalizes these empirical findings, and find that this mechanism is behind roughly 50 percent of the productivity slowdown.

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1 Introduction

The United States has been experiencing a slowdown in labor productivity growth. Figure 1 displays the post-1995 evolution of labor productivity together with several trends based on the average growth rates of periods characterized by relatively different productivity dynamics. The series followed the trend until the second half of the 2000s, when it slowed down and then increased significantly during the Great Recession (2008-2009). Strikingly, the series did not continue growing with the trend after this sharp increase, but it stagnated at a very slow pace of growth for many years thereafter. This consistently slow growth opened up a gap that got increasingly wider over the years and that is estimated to account for up to a 16% loss in GDP, which is equivalent to $23,400 per household in the U.S. (Syverson, 2017). Even though the growth rate of labor productivity has been recovering significantly since 2016, these facts present a puzzle: what are the main drivers of this slowdown?

The previous literature has mainly focused on relating this slowdown in productivity to long-run, acyclical forces, such as an increasing difficulty to measure productivity, a slowdown in the entry rate of new business, or that new innovations—for some reason—might be less productive than before. In contrast, this paper addresses the question of whether part of this slowdown in productivity could be an endogenous response from the Great Recession.

In particular, this paper investigates the link between the 2006-2012 decline in house prices (the housing bust) and the labor productivity slowdown. To this end, I follow an approach that exploits geographic variation across U.S. Metropolitan Statistical Areas (MSAs). This approach has several advantages. First, geographic cross-sectional analyses offer much greater variation than time-series. Second, this variation gets rid of relevant recession-independent (“secular”) forces and nationwide shocks that are common across areas and that, over a horizon of the length of this analysis, could impact the dynamics of productivity and confound the results. Third, this type of analysis is more directly tied to the primitives of the economic environment as policy responses are largely “differenced out”, offering much sharper theoretical predictions (Nakamura and Steinsson, 2014). This approach is founded on the finding that the slowdown in productivity is greatly heterogeneous across space, as it

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1 This kind of sharp increase in productivity is a consistent pattern across recessions. A hypothesis is that it might be due to changes in the composition of the labor force as a consequence of the large amount of job losses—disproportionately concentrated on low skill workers (see Charles et al., 2018b, for evidence in the context of the Great Recession)—that occurs during recessions.

2 This was—to an important extent—arguably unanticipated, as the Congressional Budget Office (CBO)’s successive projections (some of them collected in figure B.1) show.

3 See, for example, Mokyr (2014a), Guvenen et al. (2017), Fernald (2015), Decker et al. (2017), and Gordon (2017).
was the magnitude of the housing cycle of the 2000s (Ferreira and Gyourko, 2011) and the depth of the Great Recession (Mian and Sufi, 2014). All these aspects make the U.S. MSAs a uniquely equipped laboratory for this investigation.

I measure productivity at the MSA level as real GDP per worker. I measure the MSA productivity slowdown as the difference between the log value of the pre-2001 trend extrapolated to 2015 and the log value of the series in the same year. I call this object the productivity gap, and it denotes how much the series would have to grow—at a given year—to catch up with its underlying trend. The average productivity gap, across MSAs, is 14.51%.

I structure the paper in three main parts. The first part analyzes the direct relationship between the housing bust and the productivity slowdown. I first show that the productivity gap is, on average, substantially greater in MSAs that experienced larger house price declines. However, there might be confounding variables that could affect the housing bust and the productivity slowdown at the same time. To address endogeneity concerns, I follow an IV
approach based on the extensive literature that exploits geographic variation in housing supply elasticity to instrument for the housing boom-bust cycle\textsuperscript{4}. To address residual concerns about self-selection of the pre-sample labor force and industrial composition, I include a set of controls that summarizes MSA size and demographics, the pre-sample composition of the labor force, and the pre-sample industrial composition; and I also include the pre-sample labor productivity levels.

The results imply that one standard deviation decline in house prices translates into an increment of the productivity gap by 6.9 percentage points. A simple back-of-the-envelope calculation suggests that the housing bust explains around 50\% of the productivity gap. In addition, a year-by-year analysis shows that this effect got increasingly larger over the years, starting in 2009, reaching its peak around 2014-2015, and decreasing progressively after that.

The second part of the paper explores the mechanisms that connect the housing bust to the productivity slowdown. I proceed in two steps. First, I examine the extent to which the housing bust affected the productivity gap \emph{via} the long slump in investment that followed the Great Recession. To this end—and given the lack of a comprehensive measure of capital expenditures beyond the national level—I directly construct a measure of capital expenditures at the MSA level through confidential Census Bureau microdata from the Annual Capital Expenditures Survey. I provide evidence that supports that this slump in investment is an important pathway through which the housing bust affected the productivity slowdown.

Second, I examine the two main channels that could explain how the housing bust affected the slump in investment and, ultimately, the productivity gap. On the one hand, the housing bust led to a slump in consumption expenditures (Mian et al., 2013) that may have consistently discouraged corporate investment. On the other hand, firms in MSAs that experienced larger declines in house prices might have suffered limitations in their access to external funding that could have affected their investment decisions—which I call the ‘credit supply hypothesis’. I provide empirical evidence that supports a central role for the former and a limited one for the later.

In the third part of the paper, I build a quantitative general equilibrium model that accounts for these empirical findings. I use the model to rationalize the empirical results and, mainly, shed light on the quantitative importance of the documented mechanism.

The model starts from a New Keynesian framework with nominal rigidities and includes three key ingredients. First, the model incorporates a vintage capital structure, where each period there is a new generation of machines that are more productive than the ones built in the previous period.
in the previous period; this ingredient directly links investment to productivity through the pace of technology diffusion. Second, the model includes a putty-clay production function with irreversible investment, where the firm can freely choose the characteristics of capital goods at the time of construction, but these stay fixed thereafter; this ingredient, among other things, makes the degree of persistence of the productivity dynamics more in line with the data. Both ingredients, together, affect the way productivity responds to changes in investment, generating more empirically plausible dynamics. Lastly, the model incorporates housing, borrowers and lenders, and a borrowing constraint based on housing as collateral; this ingredient is a parsimonious way, standard in the literature, to generate consumption responses to large movements in house prices.

In the presence of nominal rigidities, a shock to consumer demand leads to a decline of the returns to investment, which importantly slows down the pace at which new investment-specific technology is incorporated to the firms and reduces the stock of capital per worker. Both forces eventually create a slump in labor productivity in a setup where the technology frontier keeps growing as usual, connecting with the evidence that shows no significant contraction in measures of innovation such as R&D expenditures and patents.3

The model does well at matching a number of key moments in the data that are not targeted in the calibration, and suggests that the housing bust accounts for roughly half of the measured labor productivity gap in the United States. These findings contrast with the common argument that attributes the productivity slowdown solely to secular forces.

Related Literature

This paper contributes towards three main strands of literature. First, this paper adds to the literature that studies the causes of the U.S. productivity slowdown. One explanation for the slowdown is that it is, to a great extent, illusory and due to a measurement problem.4 The idea is that, for several reasons, the recent productivity gains are not reflected in the productivity statistics. Syverson (2017), however, provides extensive empirical evidence that suggests that this hypothesis is greatly at odds with the data, which is also complemented by the findings of Nakamura and Soloveichik (2016); Byrne et al. (2016); and Cardarelli and Lusinyan (2015). Another hypothesis, based on Byrne et al. (2013), Fernald (2015) and Cette et al. (2015), ties the slowdown to the reversal of the productivity accelerations

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3See appendix G and, in particular, figure G.1. Moreover, extensive literature has argued that it is not a lack of productivity-enhancing innovations what constraints productivity growth (see, for example, Brynjolfsson and McAfee, 2014; and Mokyr, 2014b).

4See, for example, Brynjolfsson and McAfee (2014); Mokyr (2014a); Guvenen et al. (2017); Byrne et al. (2018); Feldstein (2015); and Smith (2015).
in the manufacturing and the utilization of information and communication technologies (ICTs) during the late 1990s and early 2000s. Gordon (2017) points out several potential explanations and ties the current productivity slowdown to the one of the late 1970s, also viewing the period of the late 1990s and early 2000s as a one-off aberration. While these hypotheses may be part of the forces behind the productivity slowdown, reversal and one-off aberration stories are unlikely the main driver as they are directly challenged by figure 1, which clearly shows a slowdown in productivity even when compared to the average growth rate during the 1948-1994 and 1980-1994 periods. Lastly, Cowen (2011) raises several reasons why innovation may have slowed down, and Decker et al. (2017) and Alon et al. (2018) suggest a role for the slowdown in the U.S. business dynamism. This paper focuses on the link between the 2006-2012 housing bust and the productivity slowdown.

On the theoretical front, Anzoategui et al. (2019) offers a model of endogenous growth based on R&D and technology adoption to analyze the role of these variables as sources of business cycle persistence, which suggests an important role for adoption versus a small role for exogenous TFP movements to account for the recent productivity decline. I see this paper as complementary to theirs, as I develop an empirical analysis that exploits geographic variation to document a causal relationship between the housing bust and the productivity slowdown and to disentangle the mechanisms that link both.

Second, this paper contributes to the emerging literature that studies the consequences of the 2006-2012 housing bust. Mian et al. (2013) and Guren et al. (2018) follow different empirical strategies to study the impact of the housing bust on consumption expenditures based on the housing wealth effect, and Berger et al. (2017) analyzes these findings from the theoretical point of view. On different contexts, and to name just a few examples, Mian and Sufi (2014) studies the effects on employment; Charles et al. (2018a) finds that the housing cycle had important effects on college attendance; and Stroebel and Vavra (2019) finds a causal response of local retail prices to house price changes during the boom and bust. This paper is the first to study the effects of the housing bust on the productivity slowdown and, to the best of my knowledge, it is even the first to analyze productivity across space.

Lastly, this paper is also related to the vast literature that studies the aftermath of the Great Recession. Lo and Rogoff (2015) reviews the plausible reasons behind the relatively slow growth after the Great Recession, and points to demand factors—in particular, the still critical problem of debt even years after the large deleveraging that immediately followed the Great Recession—as the main obstacles to economic growth. The central role of weak demand in the aftermath of the Great Recession has also been emphasized by Summers (2016 and 2017), through the premise that under a declining supply we should see inflation
to accelerate, whereas the low growth tendency of inflation and quantity points to the demand side. This paper complements these ideas, showing that the collapse and slow recovery of consumption expenditures that followed the Great Recession explain most of the documented effect of the housing bust on the productivity slowdown. Moreover, the paper provides direct empirical support for a mechanism that exemplifies the so-called ‘inverse Say’s law’, or the idea that weak demand—when sustained over a period of time—can affect the economy’s potential output.

Hall (2015) carries out a time series growth accounting decomposition that breaks down output growth into total factor productivity growth, capital growth and labor growth, and estimates that the 13.3 percentage points shortfall in output with respect to trend in 2013 can be attributed as 3.5 to total factor productivity, 3.9 to the capital input, and 5.9 to elements related to the labor input. I see this paper as complementary, as both share the fundamental role for the capital stock in the aftermath of the Great Recession but, in contrast, this paper focuses on studying the productivity slowdown instead of the growth of GDP and, in particular, it analyzes the link between the housing bust and the productivity slowdown and documents the mechanisms behind.

A fundamental paper in this literature is Yagan (2019), which exploits variation across local labor markets together with longitudinal linked employment-employee administrative data to isolate the long-term employment impacts of the Great Recession. The key finding is that exposure to a severe local Great Recession made the local working-age population substantially less likely to be employed in 2015, years after the end of the Great Recession and despite the recovery of the local unemployment rate. This paper is obviously different as it studies the productivity slowdown instead of employment, but both emphasize the long-lasting effects of the Great Recession on the economy.

**Layout.** The rest of the paper proceeds as follows. Section 2 describes the data. Section 3 introduces the measurement details and the geographic heterogeneity of the productivity slowdown. Sections 4 and 5 discuss the empirical methodology and the empirical results. Section 6 presents the model and discusses the model results. Section 7 concludes.

## 2 Data

This section introduces a brief description of the main data sources (see appendix A for more details). The unit of observation is the MSA, which is defined by the U.S. Office of Management and Budget as a contiguous area of at least 50,000 people, delineated on the
basis of a central urban cluster. According to the 2010 census, 83.6% of Americans live in MSAs. The rationale to choose MSAs is two-fold: they constitute local labor markets (Moretti, 2010)—unlike counties—, and they are the smallest geographic delineation for which one can measure labor productivity at this time.

**Labor productivity.** Labor productivity is measured as real GDP per worker, and comes from the Bureau of Economics Analysis (BEA). For some robustness checks I use hours worked from the Current Employment Statistics (CES) of the Bureau of Labor Statistics (BLS) and from the American Community Survey (ACS). Given that the GDP data at the MSA level is only available from 2001, I also use personal income (also from the BEA) to construct the trend during the years where GDP is not available, which allows me to go back to 1969\(^7\). Appendix A shows that the growth of personal income is highly correlated with the growth of GDP, at the MSA level, during the years that both series overlap, and at the state level for the rest of years.

**House prices.** The house price indexes come from the Federal Housing Finance Agency (FHFA). I compute the log difference between the years 2006 and 2012. I standardize to facilitate the interpretation of the results.

**Consumption.** There is no direct measure of consumption expenditures at the MSA level (the finest level of geography aggregation is the state). Some of the regional measures for local consumption expenditures previously used in the literature are car purchases and credit card data (for both of them see, for example, Mian et al., 2013), but they are both proprietary. Publicly available measures include retail sales (Fishback et al., 2005), which are only available at a geographically disaggregated level during the Economic Census years (every five years); and retail sales tax data, which are noisy and only available for a subset of regions (Garrett et al., 2005). My main measure of consumption expenditures at the MSA level extrapolates national retail sales to MSAs with the use of MSA retail GDP from the BEA. Appendix A provides details about this procedure and shows that this measure, when aggregated to the state level to compare it with direct consumption expenditures measures, responds nearly one-for-one with changes in consumption expenditures for each state. In a robustness check, I use retail employment per capita from the quarterly census of employment and wages following Guren et al. (2018)\(^8\).

\(^7\)Section 4.3 shows that the results are robust to a variety of alternative approaches and sources used to construct the trend.

\(^8\)The rationale for proxying local consumption expenditures with retail sales, gdp or employment, and why they track consumption so closely across studies (apart from appendix A of this paper see, for example,
**Capital expenditures.** There is, to the best of my knowledge, no publicly available (I believe not even proprietary), comprehensive measure of capital expenditures at any geographic aggregation finer than the nation as a whole. The data series that probably gets closest to it at the MSA level is the public statistics on capital expenditures from the Census of Manufacturers, which is only available for the Economic Census years and is restricted to manufacturing. I directly construct capital expenditures measures at the MSA level with the use of restricted-access Census Bureau microdata from the Annual Capital Expenditures Survey. Appendix A explains the details of this procedure. I use the capital expenditures measure from the Census of Manufacturers in a robustness check.

**Other variables.** Other variables include population and the share of workers with a college degree from the decenial census; the employment share of different sectors from the BLS; the GDP share of different sectors, income per capita and wages from the BEA; the housing supply elasticity measures of Saiz (2010), summary of deposits and commercial and industrial loans data from the Federal Deposit Insurance Corporation (FDIC), etc. See text and appendix A for further details.

### 3 Measuring the Productivity Slowdown across MSAs

This section discusses the procedure to measure the productivity slowdown at the MSA level and introduces the geographic variation that I exploit in the rest of the paper.

I measure labor productivity at the MSA level as real GDP (in constant 2009 dollars) per worker\(^9\).

As figure B.2 shows, the growth rate of labor productivity is greatly heterogeneous across MSAs. Hence, measuring the productivity slowdown at the local level by considering simple growth rates could lead to largely misleading cross-sectional results. Instead, a precise measure should consist of some sort of deviation with respect to a long-run trend.

\(^9\)There are, to my knowledge, only two sources of hours worked data at the MSA level: the American Community Survey (ACS) and the Current Employment Statistics (CES). The problem with the ACS’s measure of hours is that it is only available for a subset of MSAs, as the smallest geographic area is the Public Use Microdata Area (PUMA), which is a geographic unit that contains at least 100000 people and only overlaps with MSAs with 100000 people or more (hence, the PUMA areas do not encompass those MSAs with 50000 to 100000 people). The problem with the CES’s measure of hours is that it is only available after 2006. These problems with the existing measures of hours worked are the motivation to take output per worker as the baseline definition. However, as shown in section 4.3, the results are robust to measure labor productivity using any of these two measures of hours worked.

*Kaplan et al., 2016; and Guren et al., 2018*, is that retail is an intermediate input for household consumption, since one has to purchase something to be able to consume it.
The problem with the construction of the trend is that the GDP series is available only from 2001 onward. To overcome this challenge, I use MSA personal income to construct the trend during the years where the GDP series is not available, which allows me to go back to 1969. Appendix A shows that the growth rates of both series—within MSA and across years—are very similar for the years where they overlap\(^\text{10}\). The results are similar when only using the short period 2001-2006 to construct the trend and under a variety of alternative approaches detailed in section 4.3.

![Figure 2](image.png)

**Figure 2**: Illustration of the construction of the productivity gap for Miami (large productivity slowdown) and Pittsburgh (small productivity slowdown). The solid line is the real labor productivity series. The dashed line is the 1969-2001 trend.

Figure 2 illustrates the construction of the measure of the productivity slowdown at the MSA level. First, I fit a flexible polynomial through the 1969-2001 values of the series to approximate the long-run trend\(^\text{11}\). Then, I extrapolate the 1969-2001 trend to year 2015. Finally, the measure of the labor productivity slowdown is the difference between the log of the extrapolated trend component in 2015 and the log of the actual series in the same year\(^\text{12}\). I call this difference the *productivity gap*, and it stands for how much labor productivity would have to grow, at a given year, to catch up with the trend. Figure 2 displays two examples of the construction of the productivity gap: Miami, on the left, which registers one of the largest productivity gaps, 24.35%; and Pittsburgh, on the right, which displays quite a mild

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\(^{10}\)They also track each other very well at the national level for relatively high frequencies.

\(^{11}\)For the baseline results, I fit a fifth degree polynomial. The results are virtually the same for any reasonable degree larger than one. I choose the period 1969-2001 because the series begin in 1969, and to be conservative and stay away from the years of the housing boom (2002-2006), which are relatively unstable in some dimensions and could—in principle—distort the long-run trend measure. In any case, section 4.3 shows that the results are robust to the specific period chosen as well as a variety of alternative approaches to measure the trend.

\(^{12}\)I have chosen 2015 as the benchmark year in the analysis because it is the most recent year where all the objects of the analysis are available (in particular, capital expenditures are still not available for 2016 onward at this time). In any case, I also report the main results across years below.
slowdown and no significant productivity gap. The average productivity gap, across MSAs, is 14.51%.

Figure 3 shows the geographic distribution of the productivity gap across MSAs. The spectrum goes from areas in dark red and brown, where the productivity gap is very large; through yellow and light blue, where the productivity gap is much smaller; and all the way to dark blue colors, where the productivity gap is even negative—meaning that the labor productivity series is actually above the trend. Figure B.3 displays a nonparametric probability density estimate of the productivity gap across MSAs.

4 The Housing Bust and the Productivity Slowdown

This section analyzes the direct relationship between the housing bust and the productivity slowdown. It is structured as follows. Subsection 4.1 details the empirical strategy. Subsection 4.2 describes the main results and, finally, 4.3 introduces the robustness checks.
4.1 Empirical Strategy

I begin by examining the cross-sectional correlation between the housing bust and the productivity slowdown. Figure 4 is a scatterplot that combines, at the MSA level, the productivity gap with the (standardized) 2006-2012 log change in house prices (housing bust)\(^\text{13}\). The size of the dots corresponds to the MSA population in 2000. We can see that the productivity gap is significantly greater in those MSAs that experienced larger house price declines. This relationship is confirmed by the OLS regression of the following empirical specification:

\[
\Delta \pi_i = \beta_0 + \beta_1 \Delta H_i + X_i \beta_2 + \epsilon_i. \tag{1}
\]

The unit of observation is the MSA, \(\Delta \pi_i\) is the productivity gap in 2015, \(\Delta H_i\) is the housing bust as measured above, and \(X_i\) is a vector of controls\(^\text{14}\). The results are reported in Table C.1. Column (1) suggests that a one standard deviation decline in house prices over the period 2006-2012 is associated with an increase of the productivity gap by 5.7 percentage points in 2015 (the average productivity gap is 14.51%).

**What are the productivity consequences of each standard deviation decline in house prices?** Identifying this effect is complicated because there might be an unobserved third factor that could be behind the housing bust and, at the same time, be able to generate persistent effects on productivity\(^\text{15}\). In such a case, the OLS estimates would suffer from omitted variable bias.

To address this challenge, I follow the extensive literature that exploits geographic variation in housing supply elasticities to instrument for the housing boom-and-bust cycle (see, among many others, Mian and Sufi, 2011, 2013 and 2014; Giroud and Mueller, 2017; Stroebel and Vavra, 2019; Guren et al., 2018). The idea is that heterogeneity in housing supply elasticities, across MSAs, leads to heterogeneous responses of local house prices to a similar housing demand shock.

The particular measure of housing supply elasticity that I use comes from Saiz (2010). This consists in an index that measures the ease with which the supply of new housing can be expanded in MSAs, based on geographic characteristics. This index assigns high

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\(^{13}\)I standardize the 2006-2012 log change in house prices to facilitate the interpretation of the results, since the average—across MSAs—house price decline is 18.45%.

\(^{14}\)This vector includes population in year 2000; the share of workers with a college degree, income per capita and average wage in 2001; the employment shares of services, manufacturing, construction, and IT technology industries in 2001; and the labor productivity levels in 2001. I justify these choices below.

\(^{15}\)It is relevant to emphasize the term *persistent*, because if the effects are purely transitory they would not show up in the productivity gap measure—as this is computed in 2015 with respect to the pre-2001 trend, whereas the housing bust expands the 2006-2012 period.
elasticity scores mainly to areas with a flat topology and suitable for relatively easy urban development, and low elasticity scores essentially to areas where steep slopes and large bodies of water make habitable land availability more restricted\textsuperscript{16}.

The first and second stages of the IV analysis are the regressions of equations

\[ \Delta H_i = \gamma_0 + \gamma_1 H S_i + X_i \gamma_2 + \omega_i \]  \hspace{1cm} (2)

and

\[ \Delta \pi_i = \beta_0 + \beta_1 \Delta \overline{H}_i + X_i \beta_2 + \epsilon_i. \]  \hspace{1cm} (3)

The unit of observation is the MSA. All terms have been defined above except $H S_i$, which denotes the housing supply elasticity; $\omega_i$ and $\epsilon_i$, which are the error terms. The coefficient

\textsuperscript{16}Formally, the estimates of housing supply elasticity are the predicted values from regression six in Table III 6 in Saiz (2010).
of interest is $\beta_1$, which identifies the causal effect of the housing bust on the productivity gap\(^{17}\). The vector of controls, $X_i$, includes a variety of pre-existing characteristics of the MSA, having to do with size and demographics (population in 2000 and income per capita in 2001), the composition of the labor force (the share of workers with a college degree and average wage in 2001), and the industrial composition (the shares of services, manufacturing, construction, and IT technology industries in 2001\(^{18}\)). Importantly, $X_i$ also contains the labor productivity levels at year 2001.

Table C.2 reports the results of the first stage regression, supported also by figure B.4. Both show that the instrument is highly predictive—across MSAs—of the housing bust.

The exclusion restriction requires that, conditional on the controls, the housing supply elasticities affect the 2015 productivity gap only through its impact on the housing bust\(^{19}\).

To support the valicity of Saiz (2010)'s housing supply elasticity as an instrument for the housing bust, table C.3 shows that the instrument is uncorrelated with the pre-sample labor productivity levels and with the change in labor productivity (2002-2006 with respect to 1998-2002). Furthermore, Mian and Sufi (2011) shows that the instrument is not correlated with the change in wage growth (2002-2006 with respect to 1998-2002), the employment share in the construction sector in year 2006, and the employment growth in the construction sector 2002-2006; whereas Stroebel and Vavra (2019) finds no relationship between the instrument and income growth during the boom-and-bust cycle.

A residual concern is that the geographic characteristics that make housing supply inelastic could also be behind, for different reasons, of some sort of self-selection that makes the pre-sample labor force (Davidoff et al., 2016) and industrial composition consistently different between elastic and inelastic MSAs. This could bias the results if, for instance, industry specific shocks had been able to affect, at the same time, the housing bust and the productivity gap, and the shocked industries were concentrated in more or less elastic MSAs\(^{20}\). To address this and other potential concerns, the IV regression includes a rich

\(^{17}\)In fact, the coefficient $\beta_1$ is expected to be negative, so its absolute value identifies the increase in the productivity gap as a consequence of a standard deviation decline in 2006-2012 house prices.

\(^{18}\)The baseline specification is based on employment shares, but using real GDP shares yields virtually the same results.

\(^{19}\)It is important to notice that the instrument, housing supply elasticity, is time-invariant; while the instrumented variable, house price change, is time-variant. The instrument summarizes certain pre-existing characteristics that are intrinsic to a given MSA (such as land availability, proximity to steep slopes or water, etc), and generally time-invariant, instead of denoting a shock. The instrument, hence, does not predict house price changes by itself, but through the interaction with time-variant shocks (or, in other words, at times where these taking place). An example of one of such time-variant shocks is a national housing demand shock.

\(^{20}\)Appendix F will specifically address this potential concern, as well as others.
set of controls that summarizes the pre-sample composition of the labor force (the share of workers with a college degree and the average wage in 2001) and industrial composition (the shares of services, manufacturing, construction, and IT technology industries in 2001). This set of controls also includes the labor productivity levels in year 2001, which control for other pre-existing productivity differences across MSAs.

### 4.2 Main Results

Table 1 reports the estimates from the second-stage regression. Column (1) presents the baseline results, with an estimate of -6.9. This implies that a standard deviation decline in house prices translates, on average, into an increment of the productivity gap by 6.9 percentage points, where the average productivity gap is 14.51%.

<table>
<thead>
<tr>
<th>Productivity gap</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>House price 2006-2012</td>
<td>-0.069 ***</td>
<td>-0.066***</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Include baseline controls</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>First Stage F-statistic</td>
<td>137.60</td>
<td>128.20</td>
</tr>
<tr>
<td>First Stage (F-test) p-value</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Mean dependent variable</td>
<td>0.1451</td>
<td>0.1451</td>
</tr>
<tr>
<td>N (Number of MSAs)</td>
<td>243</td>
<td>243</td>
</tr>
</tbody>
</table>

NOTE.—IV results, equation 3. House price change is standarized (standard deviation is 19.90, average is -18.45). Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.

Column (2) presents the results of a specification that includes no controls. The results are robust to the addition of the controls, which helps to alleviate potential concerns on the exclusion restriction.

Figure 5 collects the IV estimates from the baseline specification (equation 3) for each year, 2009-2017. In particular, each estimate and confidence interval comes from a separate regression, where the right hand side of equation 3 stays unchanged while the productivity gap is recalculated for each specific year\(^{21}\). The figure displays both the estimates and

\(^{21}\)The procedure is exactly the one described in section 3, but applied to the other years. In particular, the
the 95% confident intervals. The IV estimate is very close to zero and insignificant before 2009, helping to alleviate endogeneity concerns\textsuperscript{22}. The effect of the housing bust on the productivity gap has been widening over the years, starting to be significant in 2009, and reaching its peak around 2014-2015—much after the end of the Great Recession—at the previously pointed out point estimate of -6.9. After this, the point estimates suggests that it has started to decrease\textsuperscript{23}.

Figure 5: IV estimates from the main specification (equation 3), year by year (see text). House price change is standardized. Baseline controls are enumerated in the text. Robust standard errors 95\% confident intervals.

Finally, the effect of the housing bust on the productivity gap is also important in terms of its economic size. This is easy to see from a simple back-of-the-envelope calculation. The 2006-2012 log change in U.S. house prices is -21.33\% (FHFA annual house price in-productivity gap for year, say, 2012, is calculated as the difference between the log value of the extrapolated, same trend component, in 2012, and the log value of the actual series in 2012.\textsuperscript{22}It is important to notice the difference in timing with respect to the beginning of the housing bust in 2006. Under reverse causality, the effect would have been expected to become significant by 2006 or earlier. At the same time, if both the productivity slowdown and the housing bust had been generated by a third factor, the effect would have been more likely to become significant closer in time to the beginning of the housing bust.\textsuperscript{23}Of course this last interpretation is based on the point estimates only, as the confidence intervals are large enough to not to be able to reject that the 2016-2017 estimates are equal to the 2014-2015 ones.
index). Transforming this number to cross-MSA standard deviations of the housing bust (the standard deviation is 19.1%), and applying it to the benchmark estimate from the main specification, implies that the housing bust increased the 2015 productivity gap by 7.71 percentage points. In comparison, the national 2015 productivity gap is 14.01%. Hence, this calculus estimates that the housing bust explains around half of the national productivity gap in 201524.

### 4.3 Robustness Checks

The effect of the housing bust on the productivity slowdown passes an extensive battery of robustness checks. Appendix D details the robustness examinations; I provide an overview here.

**Trends.** Some authors (see, mainly, Gordon, 2017) have supported the view that the U.S. has undergone a change in trend, where the levels of economic growth that preceded the Great Recession are over. Under this hypothesis, the trend previous to the Great Recession would not be the right benchmark for comparison anymore since the economy would be in a 'new normal’ that oscillates around a completely different trend. To address this concern, I estimate a specificaion where the productivity gap measure is substituted by the 2010-2015 log change in productivity25. This approach has the advantage of being robust to a change in trend, as it is directly based on the productivity growth after the Great Recession. However, it is a demanding exercise for the reasons detailed in section 3. The results are qualitatively similar and quantitatively larger in terms of their economic size.

The results are robust to a variety of approaches to construct the trend: using the Hodrick-Prescott filter to get the 1969-2001 trend, then fit a polynomial and expand it to the post-2001 period26; experimenting with different historical periods to construct the trend; restrict to the 2001-2005 period to use labor productivity data only (or a neighborhood of these years); using polynomials of different degrees and other bases; extrapolate different trends from 2010 based on simple average growth rates as in figure 1; etc.

**Trade links.** A valid concern, analyzed by Adao et al. (2019), is that an MSA is not only directly affected by its local shocks but, also, it is indirectly affected by the local shocks of

---

24 See Beraja et al. (2016) and Chodorow-Reich (2019) for some caveats about this interpretation and a detailed discussion.

25 The reason to begin in 2010 is to make sure that the measure is not exposed to the contemporaneous effects of the Great Recession. Also, the results are similar when using the annualized growth rate instead.

26 The results are also robust when passing the HP filter to the whole period 1969-2015. However, this is a much less conservative approach as the measure of the productivity gap is based on a long-run trend largely influenced by the years of the boom-bust cycle.
the areas with which it has trade links. A simple way of dealing with this is to concentrate on the non-tradable sector. Given that the level of disaggregation for the MSA GDP measure is quite low, I focus on accommodation and food services\textsuperscript{27}. The measured effect is significantly greater, which implies that the benchmark estimates are a lower bound to an ideal estimate that controls for the trade links.

**Placebo: the tradable sector.** I carry out a placebo test that consist on concentrating on purely tradable industries. A purely tradable industry should be largely unexposed to local shocks, so the response of the productivity gap with respect to the local housing bust should be close to zero. I follow Mian and Sufi (2014) to the extend allowed by the level of disaggregation of the MSA GDP measure, and I concentrate on durable goods manufacturing industries. The measured effect is close to zero and statistically insignificant.

**Alternative instrument.** A different instrument for the housing boom-and-bust cycle that has been introduced in the recent literature comes from Charles et al. (2018a). It consists on the size of the structural break of house prices during the housing boom. The results are similar when restricting to the set of MSAs for which the housing supply elasticity measure is available, and slightly smaller when including the whole set of MSAs. Both instruments are uncorrelated conditional on the controls, and both pass the Hausman (1978) test for the validity of the exclusion restriction conditional on the other instrument being valid.

**Other robustness checks.** The results are robust to a variety of other concerns: measuring labor productivity as GDP over hours (both when using the CES and the ACS measures of hours); excluding MSAs that have more than half of their population in California, Florida, Arizona or Nevada; excluding finance, insurance, real estate and construction, as well as public industries; experimenting with different local deflators for the labor productivity series, and also using local deflators to get inflation-adjusted local house prices; clustering standard errors at the state level (according to the state in which each MSA has most of its population); etc.

Appendix F complements this part by carefully addressing four alternative channels that deserve a particular treatment: industry-specific shocks, the aggregate supply shock hypothesis, the business expectations hypothesis, and the business uncertainty hypothesis.

\textsuperscript{27}Previous literature (see, for example, Mian and Sufi, 2014) includes, in addition, certain subsectors inside the retail sector. However, the GDP measure is only available for the retail sector as a whole, and by including it I would be including online retail too, which is important nowadays. In any case, the results are similar when including the retail sector.
5 Mechanisms

Section 4 supports the existence of a robust, causal relationship between the housing bust and the productivity gap, and shows that the implied effect is large in the economic sense. However, there are, in principle, alternative theories that could rationalize this effect. Hence, before getting to the model, the question that follows is: what is the mechanism behind this relationship? This section approaches this question on empirical grounds in two steps.

First, I construct a capital expenditures measure at the MSA level and I provide evidence that supports that the long slump in investment that followed the Great Recession was an important pathway through which the housing bust affected the productivity slowdown.

Second, I analyze the arguably two main channels that could explain how the housing bust affected this slump in investment and, ultimately, the productivity gap. On the one hand, the housing bust led to a slump in consumption expenditures (Mian et al., 2013), which could have consistently depressed corporate investment. On the other hand, the housing bust might have led to limitations in some businesses’ access to external funding (for example, if they use housing or land as a collateral) and, hence, influence their investment decisions. I provide empirical evidence that supports a central role for the former and a limited one for the later.

5.1 The Investment Slump

Figure 6 displays the time series for the flow of total capital expenditures (Z.1. financial accounts of the United States). The figure depicts two lines, which correspond to the series deflated by two different indexes: the producer price index relative to private capital equipment and—for comparison—the GDP implicit price deflator. We can see that the series pick around 2006-2007 and then declines to the levels of the late 1990s. Moreover, the series did not came back to the levels previous to the Great Recession until around 2014-2015, and it was out of trend for years afterwards. Is this long slump in investment an important pathway through which the housing bust affected the productivity gap?

The main problem that arises when trying to incorporate a measure of capital expenditures to a cross-MSA analysis is the lack of data availability at this geography level. While there are many measures of investment at the national level—even disaggregated into a wide

\footnote{In fact, depending on how one defines and constructs the long-run trend, we could argue that—a decade after the Great Recession—it still has not caught up with the trend or is doing it at this time.}
Figure 6: Total capital expenditures, flow. Z.1. Financial Accounts of the United States. The two series use two alternative deflators: the GDP – implicit price deflator; and the Producer Price Index – Private Capital Equipment. Capital expenditures include business structures and equipment.

variety of categories—, there is, to the best of my knowledge, no publicly available and comprehensive measure of capital expenditures anywhere beyond the national level, not even at the state level. The publicly available data series that probably gets closest to it at the MSA level is the public statistics on capital expenditures coming from the Census of Manufactures. However, this data series is only available at the Economic Census years (2002, 2007 and 2012) and, more importantly, only includes the manufacturing sector. To overcome this problem, I directly construct a measure of capital expenditures at the MSA level through confidential Census Bureau microdata from the Annual Capital Expenditures Survey. All the details about this procedure are explained in Appendix A.

I begin by using the constructed series for capital expenditures at the MSA level to compute the log gap in 2015 with respect to the trend\textsuperscript{29}, which I call ‘investment gap’.

\textsuperscript{29}It is not obvious what is the optimal way to construct the trend since the series is available only from 1997 onwards. What I do, therefore, is to use MSA personal income—as I did in the case of GDP—during the years where capital expenditures are not available, which allows me to go back to 1969. The idea is that, as appendix A shows, even though capital expenditures is more volatile, the medium and long-run growth of both series is very similar at the national level, so they are expected to be similar at the MSA level too. In
Then, I examine whether the housing bust could have affected the productivity gap via the investment gap\textsuperscript{30}. The idea of this section is not to estimate a complete causal channel, as that would require an additional source of exogenous variation for the investment gap—
independent from the housing supply elasticities—, which would go beyond the scope of this paper. Instead, I focus on providing suggestive evidence to shrink the set of alternative theories and further inform the model\textsuperscript{31}. To this end, I proceed in two steps. First, I extend the IV approach developed in section 4 to examine the causal relationship between the housing bust and the investment gap. In other words, I regress the investment gap on the housing bust, instrumenting the housing bust by the housing supply elasticity measure. Second, I carry out an OLS regression of the productivity gap on the investment gap.

Table 2 reports the results. Column (1) corresponds to the IV regression of the investment gap on the housing bust, which implies that a standard deviation decline in 2006-2012 house prices translates into an increase of the investment gap on 18.8p.p. Column (2) displays the results from the OLS regression between the productivity gap and the investment gap, which implies that every percentage point increase in the investment gap is associated to a 0.623 percentage point increase in the productivity gap.

Two important links emerge from these findings. On the one hand, we have the link that connects the investment gap to the productivity gap. This empirical analysis keeps agnostic about the particular forces behind this link which, in the model, works through two channels: a slowdown in the incorporation of investment-specific technological change (or, in other words, a decline in the pace of technology diffusion) and, also, a slowdown in the accumulation of capital per worker. On the other hand, we have the link between the

\textsuperscript{30}Appendix G presents evidence suggesting a very limited role for two mechanisms alternative to the slump in investment: the R&D hypothesis and the local labor hypothesis.

\textsuperscript{31}Appendix H carries out a mediation analysis, which is a much more ambitious exercise that aims to uncover the full causal and mediation relationship but that, at the same time, relies on stronger assumptions. The results are in line with the suggestive evidence shown in this section.
Table 2: Capital Expenditures Results

<table>
<thead>
<tr>
<th></th>
<th>Investment gap</th>
<th>Productivity gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) IV</td>
<td>(2) OLS</td>
</tr>
<tr>
<td>House price 2006-2012</td>
<td>-0.188***</td>
<td>0.623***</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td>(0.134)</td>
</tr>
<tr>
<td>Investment gap</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Include baseline controls</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>First Stage F-statistic</td>
<td>104.60</td>
<td></td>
</tr>
<tr>
<td>FS (F-test) p-value</td>
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<td></td>
</tr>
<tr>
<td>Mean dependent variable</td>
<td>0.1473</td>
<td>0.1451</td>
</tr>
<tr>
<td>N (Number of MSAs)</td>
<td>243</td>
<td>243</td>
</tr>
</tbody>
</table>

NOTE.—Column 1 corresponds to the IV regression of the investment gap on the housing bust, where the housing bust is instrumented by the housing supply elasticity measure. Column 2 displays the results of the OLS regression between the productivity gap and the investment gap. House price change is standarized. Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.

housing bust and the investment gap, which I examine in the next section.

### 5.2 Consumption versus Credit Supply

This section examines the arguably two main channels that could explain how the housing bust affected the slump in investment and, ultimately, the productivity gap: the slump in consumption spending that followed the housing bust, and the credit supply hypothesis. The section provides empirical evidence that suggests a central role for the former and a limited one for the later.\(^{32}\)

**The Consumption Channel**

The housing bust led to a slump in consumption spending (Mian et al., 2013) that may have consistently discouraged corporate investment. This section analyzes whether the housing bust could have affected investment—and, through it, ultimately the productivity gap—via the slump in consumption expenditures that followed the bust.

\(^{32}\)It is important to reemphasize that, even if the empirical evidence suggests a limited role for a particular channel as a mechanism, it could still importantly affect the productivity slowdown in its own way.
First, I use the proxy for local consumption introduced in section 2 and compute the log gap in 2015 with respect to the trend\textsuperscript{33}, which I call ’consumption gap’. Then, I examine whether the housing bust could have affected the investment gap via the consumption gap. To this end, I follow the same approach of section 5.1. First, I extend the IV approach developed in section 4 to analyze the causal relationship between the housing bust and the consumption gap. In other words, I regress the consumption gap on the housing bust, instrumenting the housing bust by the housing supply elasticity measure. Second, I carry out an OLS regression of the investment gap on the consumption gap\textsuperscript{34}.

Table 3 reports the results. Column (1) corresponds to the IV regression of the consumption gap on the housing bust, which implies that a standard deviation decline in 2006-2012 house prices translates into an increase of the consumption gap on 6.1p.p. Column (2) displays the results from the OLS regression between the investment gap and the consumption gap, which implies that every percentage point increase in the consumption gap is associated to a 0.776 percentage point increase in the investment gap.

To complement these findings, the following section provides evidence that suggests a limited role for the arguably most important alternative channel: the credit supply hypothesis.

**The Credit Supply Hypothesis**

The arguably most important alternative mechanism through which the housing bust could have affected the investment gap—and, ultimately, the productivity gap—is the credit supply hypothesis. This hypothesis is based on the idea that firms in MSAs with larger house price declines might have also suffered limitations in their access to external funding, and this could have affected them in a way that influenced their productivity dynamics\textsuperscript{35}. For instance, this

\textsuperscript{33}As in the case of capital expenditures, it is not obvious what is the optimal way to construct the trend since the series is only available from 2001 onwards. Therefore, I use MSA personal income during the years prior to 2001 to construct the trend, which allows me to go back to 1969. The idea is that, as appendix A shows, the growth rates of consumption and personal income are very similar—and even track each other really well at relatively high frequencies—at the national level, so they are expected to be similar at the MSA level too. In any case, the results are similar when only using the 2001-2006 period to construct the trend. Even though it is imperfect, I believe that the former approach is more conservative since it allows us to get a trend over a much longer horizon.

\textsuperscript{34}Of course, as I mentioned in section 5.1, the idea of this section is not to uncover a complete causal channel, which would require an additional source of exogenous variation for the consumption gap—indeed from the housing supply elasticities—and would go beyond the scope of this paper. Instead, this part focuses on providing suggestive evidence to shrink the set of alternative theories and further inform the model. Therefore, in principle, the link between the consumption gap and the investment gap could technically go both ways. Section 6 builds a general equilibrium model that rationalizes these findings.

\textsuperscript{35}There is a large literature analyzing the financial factors during the Great Recession from many different contexts. See, to name just a few, Jermann and Quadrini (2012), Chodorow-Reich (2013), Greenstone et al. (2014), Christiano et al. (2015), Gertler and Gilchrist (2018).
would have happen if firms using housing as a collateral had their access to external funding limited due to the decline in house prices, or if lending standards were tightened in the areas where house prices declined more due to an increase in defaults and the overall financial risk. Any of these cases could influence investment and other decisions of firms that can ultimately affect their productivity. This section presents several results that make this hypothesis unlikely.

On the one hand, the business survey evidence from business owners introduced in figure B.5 reports that only 3% of respondents declared financing as their most important problem in 2007. Moreover, this percentage stayed this low—and stable—during the recession and after, while poor sales and regulation concerns increased substantially.

On the other hand, a credit supply shock would be expected to affect firms from both tradable and non-tradable sectors, which would be at odds with the evidence showed in section 4.3.

Furthermore, table C.5 presents the results of two exercises, analyzed below, that directly try to test the credit supply hypothesis.

The first exercise, which follows Mian and Sufi (2014), uses the summary of deposits data from the FDIC to measure the degree of geographic diversification of the banks that operate
in each MSA. First, I calculate, at the bank level, the share of deposits that each specific bank has on each MSA. After this, this statistic is averaged, at the MSA level, over the banks located on each MSA, weighting by the deposits that each bank has in the MSA. The result is called ‘average share of deposits’ and it is MSA specific. An MSA whose banks have a very low fraction of their deposits in it is considered a national banking MSA, and it should not be as sensitive to local credit supply conditions as MSAs whose banks have a large fraction of their deposits there. Column 2 of table C.5 reports the results of a specification that interacts the 2006-2012 log change in house prices with two dummies: ‘national’, which takes value one if the average share of deposits is lower than the median across MSAs and value zero otherwise; and ‘local’, that takes value one when it is greater than the median and value zero otherwise. Both estimates are remarkably similar, and also not significantly different from the baseline estimate. Column 3, instead, directly interacts the house price decline with the average share of deposits. The interaction is insignificant and the estimated elasticity is not significantly different from the baseline estimate. A caveat is that this test would, in principle, not account for credit supply shocks that originate purely from the borrower, instead of the lender, like a deterioration of the value of the collateral as hypothesized at the beginning of the section. The next exercise does.

The second exercise, which follows Stumpner (2019), uses FDIC data to construct a measure of commercial and industrial loans growth at the MSA level. The problem is that the data on loans is only available aggregated at the bank level. To extrapolate from banks to MSAs, I use the pre-recession (2007) share of deposits as a measure of the bank’s market share across MSAs (fixed throughout the exercise). Then, I add up (across banks) at the MSA level, restricting to commercial and industrial loans. After this, I calculate, for each MSA, the log gap in 2015 with respect to the 1992-2005 trend. Column 4 presents the results of a specification that includes this loans gap, as a control for credit supply shocks, instrumented by the share of long term illiquid assets in 1996. The coefficient of the loans gap is insignificant, and the estimated elasticity is not significantly different from the baseline estimate. The caveats from this exercise are, of course, that the share of deposits is an imperfect measure of the bank’s market share across MSAs for the loans market and, also, that distance between the bank branch and the firm might not play an important role.

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36 The instruments are the housing supply elasticity measure interacted with each dummy.
37 These results are based on the bank holding company instead of the bank. This is done with the aim of accounting for the potential existence of risk sharing across the banks of the same holding company. In any case, table ?? presents the results based on banks, which are very similar.
38 The idea of this instrument comes from Cornett et al. (2011), which shows that banks that held more illiquid assets in their balance sheets were more likely to reduce lending during the Great Recession. Given that this is quite of a persistent statistic, I am able to use the 1996 share—much before the beginning of the housing boom—while still getting a strong first stage.
in the matching between banks and firms, although the empirical evidence suggests that it still does\footnote{See Petersen and Rajan (2002) and Brevoort et al. (2010), which report that borrower-lender distance tends to be very low (5 miles) and has only modestly increased during the recent decades.}.

6 Model

This section builds a quantitative general equilibrium model—well disciplined and informed by the empirical evidence above—that rationalizes the empirical findings. Then, I use the model to perform some quantitative exercises to shed light on how much of the productivity slowdown is explained by the housing bust.

The model starts from a New Keynesian framework with nominal rigidities and includes three key ingredients. The first ingredient is a vintage capital structure, where each period there is a new generation of machines that is more productive than the ones built in the previous period; this ingredient directly connects investment and productivity through the pace of technology diffusion. The second ingredient is a putty-clay production function with irreversible investment, where firms can freely choose the characteristics of the capital goods at the time of construction, but these stay fixed thereafter; this ingredient makes the degree of persistence of the productivity dynamics more in line with the data. These two ingredients, together, affect the way productivity responds to changes in investment, generating more empirically plausible dynamics. The third ingredient includes housing, borrowers and lenders, and a borrowing constraint based on housing as collateral; this ingredient is a parsimonious way, standard in the literature, to generate consumption responses to large movements in house prices.

The subsections below construct the model, describe the environment of each of its agents, and introduce its key equilibrium conditions.

6.1 Households

Demographics and Preferences. There is a continuum of measure one of households, divided into two families that differ in their preferences. A family of measure $\lambda$ consists of relatively impatient households, named ”borrowers”, and denoted by subscript $b$. The other family, of measure $1 - \lambda$, contains relatively patient households, named ”savers”, and denoted by subscript $s$. Households are identical within family, and both families supply perfectly.
substitutable labor\textsuperscript{40}.

Each household of family \( j \in \{b, s\} \) seeks to maximize its expected lifetime utility

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_j^t u(c_{j,t}, h_{j,t}, n_{j,t}),
\]

where \( c_{j,t}, h_{j,t}, \) and \( n_{j,t} \) are nondurable consumption, housing services, and labor supply; and where \( \beta_j \in (0, 1) \) is the discount factor. The period utility takes the separable form:

\[
u(c, h, n) = \log(c) + \xi_t \log(h) - \psi \frac{n^{1+\eta}}{1 + \eta}.\]

The term \( \xi_t \) captures the marginal utility of housing services\textsuperscript{41}, which evolves according to

\[
\log(\xi_t) = (1 - \rho_\xi) \log(\bar{\xi}) + \rho_\xi \log(\xi_{t-1}) + \epsilon_\xi,
\]

where \( \epsilon_\xi \) is a white noise process.

All preference parameters are identical across both families with the exception that \( \beta_b < \beta_s \), so that borrowers are less patient than savers.

**Asset Structure.** Households can trade a one-period nominal bond, denoted by \( B_t \). One unit of this bond costs $1 in period \( t \) and pays $(1+i_t)$ in period \( t+1 \). There is no government in this economy, so only savers can lend to borrowers and the net supply of bonds is zero, i.e., \( \lambda B_{b,t} + (1 - \lambda) B_{s,t} = 0 \).

Both types of households can own housing, whose stock is denoted by \( h_t \) and its price by \( P_t^H \), and produces a service flow equal to its stock. Households must pay a maintenance cost consisting on a constant fraction \( \delta_h \) of the value of the house at the start of each period.

Borrowers face a constraint that limits the amount they can borrow. As in Guerrieri and Iacoviello (2017), I extend the standard Kiyotaki and Moore (1997) borrowing constraint as follows:

\[
-B_t \geq -\gamma_{BC} B_{t-1} + (1 - \gamma_{BC}) \theta P_t^H h_t,
\]

\textsuperscript{40}This structure constructs on Iacoviello (2005).

\textsuperscript{41}An increase in \( \xi_t \) shifts preferences away from consumption of nondurables and leisure towards housing services, directly affecting housing demand and, hence, house prices. Therefore, a shock to \( \xi_t \) offers a parsimonious way to model any kind of force with the potential to shift housing demand.
where $\gamma_{BC}$ governs the degree of inertia of the borrowing limit and $\theta$ is the loan-to-value ratio. This specification is a parsimonious way to incorporate that the borrowing constraint on housing is reset only for households that buy new housing or refinance their existing mortgage contracts\textsuperscript{42}.

Following Justiniano et al. (2019) and others, I make two assumptions to simplify the analysis. First, I assume that the housing stock is fixed at $\bar{H}$, so that the price of housing completely describes the aggregate state of the housing market\textsuperscript{43}. Second, I assume that the housing demand of the saver is fixed at $h_{s,t} = \bar{h}_s$, so that the borrower is the marginal buyer in the housing market, i.e., houses are priced by borrowers. The later assumption is motivated by the empirical analysis of Landvoigt et al. (2015), who supports that the housing market is highly segmented, and finds a primary role for the lower end of the house price distribution (where, presumably, houses are mostly owned by leveraged borrowers) in the house price movements during the boom-bust cycle\textsuperscript{44}.

Lastly, I assume that savers own all shares in the economy\textsuperscript{45}.

**Representative Borrower’s Problem.** The representative borrower chooses nondurable consumption $c_{b,t}$ and housing $h_{b,t}$ to maximize (1) using the utility function

$$u(c_{b,t}, h_{b,t-1}, n_{b,t}) = \log(c_{b,t}) + \xi_t \log(h_{b,t-1} - 1) - \psi \frac{n_{b,t}^{1+\eta}}{1 + \eta},$$

subject to the budget constraint

$$P_t c_{b,t} + B_{b,t} + P_t^H h_{b,t} \leq W_{t} N_{b,t} + (1 + \delta_{t-1})B_{b,t-1} + P_t^H (1 - \delta)h_{b,t-1} - F_t$$

\textsuperscript{42}Justiniano et al. (2015), which analyzes the recent process of household leveraging and deleveraging, includes a more realistic—yet less parsimonious—specification. Both are analogous for the purposes of this model.

\textsuperscript{43}The alternative is to model a sector that produces new houses. Under a flexible housing supply, a shock to $\xi_t$ would lead to a smaller movement in house prices, which would be compensated by a larger movement in the housing stock. Since, from the perspective of household consumption, the key is the effect on the borrowing limit (which depends on the product of price and quantity), I believe this simplifying assumption is reasonable.

\textsuperscript{44}In the model, without this assumption, there would be unrealistically large flows of housing—along the intensive margin, or house size—between the two groups in equilibrium. This happens because, in this model, housing is a homogeneous, perfectly divisible good. A more realistic—but also less parsimonious—alternative, would be to assume that borrowers and lenders enjoy two different types of housing that are traded in two separate markets. This alternative should be analogous for the purposes of this model.

\textsuperscript{45}In the model, this implies that the intertemporal problems of firms will be discounted by the stochastic discount factor of the savers. The issue of what discount factor to use in the intertemporal problems of firms is not obvious in a model with household heterogeneity, but it plays a minor role quantitatively. This assumption is a natural one and overcomes this problem.
and the borrowing constraint

$$- B_{b,t} \geq -\gamma B_{b,t-1} + (1 - \gamma)\theta P^H_t h_{b,t},$$

where $F_t$ is a nominal fee that covers an equal share of the adjustment costs of wages. Appendix J shows that the assumption $\beta_b < \beta_s$ guarantees that (5) always holds with equality in and around the steady state.

**Representative Saver’s Problem.** The representative saver chooses nondurable consumption $c_{s,t}$ to maximize (1) using the utility function

$$u(c_{s,t}, n_{s,t}) = \log(c_{s,t}) + \xi_t \log(\bar{h}_s) - \psi n_{s,t}^{1+\eta} / (1 + \eta),$$

and subject to the budget constraint

$$P_t c_{s,t} + B_{s,t} + P^H_t \delta \bar{h}_s \leq W_t N_{s,t} + (1 + i_{t-1})B_{s,t-1} + \frac{\Pi_{s,t}}{1 - \lambda} - F_t,$$

where $\Pi_{s,t}$ are dividends.\(^{46}\)

**Labor Market Structure.** The labor market structure constructs on Erceg et al. (2000) and Galí et al. (2007). A representative labor aggregator (or "employment agency") combines all labor in the economy and, subject to a quadratic adjustment cost of wages as in Rotemberg (1982), sets a common wage that optimizes both household types given their relative measures. Hence, firms’ demand determines hours worked, and households are willing to meet that demand as long as the wage remains above their marginal rate of substitution. Wage adjustment costs are ultimately transmitted to households through the fee $F_t$. See appendix I for a detailed description of the labor market structure.

**6.2 Final Good Firm**

There is a continuum of monopolistically competitive intermediate goods firms, indexed by $i \in [0,1]$, that produce differentiated output $Y_t(i)$ and sell it at price $P_t(i)$. Each intermediate firm supplies its differentiated output to the final good firm. The final good firm then

\(^{46}\text{In principle, savers are subject to the same borrowing constraint than borrowers. However, appendix J shows that this constraint is redundant for savers, so I directly skip it.}\)
bundles the differentiated outputs into a homogeneous output available for consumption and investment, denoted by \( Y_t \). The bundling technology is

\[
Y_t = \left( \int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{9}
\]

where the parameter \( \epsilon > 1 \) is the elasticity of substitution among intermediate goods.

The profit maximization problem of the final good firm is

\[
\max_{Y_t(i)} P_t \left( \int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 P_t(i)Y_t(i)di.
\]

The resulting demand schedule for intermediate firm \( i \) and price index are \( Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\epsilon} Y_t \) and \( P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}} \).

### 6.3 Intermediate Goods Firms

**Environment.** As stated above, there is a continuum of monopolistically competitive intermediate goods firms, indexed by \( i \in [0, 1] \). Intermediate firm \( i \) produces a differentiated output \( Y_t(i) \), whose price is denoted by \( P_t(i) \).
Intermediate firms combine machines and workers as inputs to produce a single good, \( Y_t(i) \). Each period \( t \), there is a distribution of heterogeneous machines \( H_t(x) \), where \( x \in [0, \bar{x}] \) stands for the efficiency level of a given machine. The labor productivity of a machine with efficiency level \( x \) is \( A_t x \), where the term \( A_t \) represents the disembodied aggregate technological change and grows at a gross rate \( \gamma_A \).

There is a rental market for machines, where intermediate firms rent machines from the capital owners. Let \( h_t^i(x) \) be the number of machines with efficiency level \( x \) rented by firm \( i \) in period \( t \), and \( R_t(x) \) be the rental cost of each of such machines. Notice that both firm \( i \)'s demand for machines and the rental price of machines are not scalars, but functions of the efficiency level \( x \).

I assume, without loss of generality, that each machine takes one worker to operate, which costs \( W_t \) to the intermediate firm\(^{47}\).

**Costs minimization.** Intermediate firms take input prices as given and set their output price optimally, which is subject to nominal rigidities. Intermediate firms will always choose inputs to minimize costs each period. Firm \( i \)'s cost minimization problem is:

\[
\begin{align*}
\min_{h_t^i(x)} \quad & \int_0^\infty [R_t(x) + W_t]h_t^i(x)dx \\
\text{st.} \quad & \int_0^\infty A_t x h_t^i(x)dx \geq Y_t(i).
\end{align*}
\]

The normalization assumption that each machine takes one worker to operate implies that the cost of operating a machine with efficiency level \( x \) is \( R_t(x) + W_t \). This term, multiplied by the total number of machines of such efficiency level that the firm rents, and summed across efficiency levels, equals the total cost\(^{48}\). On the other side, \( A_t x \) is the labor productivity of a machine with efficiency level \( x \), so \( A_t x h_t^i(x) \) is the total production from all machines of such efficiency level that the firm operates. Summing this across efficiency levels yields the total production of firm \( i \). The expressions for output and labor at the firm level can be written as

\[
Y_t(i) = \int_0^\infty A_t x h_t^i(x)dx
\]

\(^{47}\)The reason why this assumption involves no loss of generality will become clear in next section, at the introduction of the production function.

\(^{48}\)The assumptions below will imply that \( H_t(x) \) and \( h_t(x) \) are both continuous. This is why I use the integral to sum over efficiency levels. In a more general setting, this notation can be adapted to accommodate any type of \( h_t^i(x) \) function.
and

\[ N_t(i) = \int_0^\infty h_t^i(x)dx. \] (12)

**Profits maximization.** Firm \( i \) sets its price \( P_t(i) \) to maximize its discounted sum of future profits

\[
\mathbb{E}_0 \sum_{i=0}^{\infty} \beta_s^{c_s,i} \left\{ P_t(i)Y_t(i) - \int_0^\infty \left[ R_t(x) + W_t(h_t^i(x)) \right]dx - \frac{\phi_P}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} \right)^2 - \gamma_P P_t(i)Y_t(i) \right\},
\]

subject to its demand schedule

\[ Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\epsilon} Y_t. \]

Profits are discounted by the stochastic discount factor of the representative saver, as savers own the intermediate firms. Also, prices are subject to a quadratic adjustment cost, as in Rotemberg (1982). The term \( \gamma_P \) stands for gross inflation along the balance growth path, so the adjustment cost term is a quadratic function of the difference between the balance growth path inflation and the growth rate of firm \( i \)'s price. The adjustment cost is governed by \( \phi_P \), which controls the size of the adjustment cost and is proportional to firm \( i \)'s nominal revenues.

### 6.4 Capital Owners

There is a continuum of competitive capital owners, indexed by \( \nu \in [0, 1] \). They make the investment decisions, own the capital, and rent it to the intermediate firms.

**Technology.** The technology side builds on Gilchrist and Williams (2000) and has three key ingredients: vintage capital, embodied technical change, and a putty-clay production function.

Capital goods take the form of machines. A new machine constructed in period \( t \), and denoted by \( Q_t \), is characterized by three components:

1. The vintage level of embodied technology, \( \theta_t \). A new vintage is available each period,
and it is associated to an economywide level of embodied technology, \( \theta_t \), which grows at rate \( (1 + g) \). The term \( \theta_t \) is vintage specific and, hence, common across all machines constructed in the same period.

2. The capital-labor ratio of the machine, \( k_t \), which is machine specific, and chosen by the capital owners at the time of the investment decision.

3. An idiosyncratic shock to productivity, \( \mu_t \), which is machine specific, and iid drawn from a continuous distribution with pdf \( f_{\mu}(\cdot) \), positive support \( \mu_t > 0 \), and unit mean \( \mathbb{E}[\mu_t] = 1 \).

Each period \( t \), the investment decision of the capital owner has, hence, two dimensions: the number of new machines to buy, \( Q_t \), and the capital-labor ratio per machine, \( k_t \). Let investment be defined as \( I_t := Q_t k_t \).

Investment is irreversible, in the sense that, once in place, it cannot be converted into consumption goods or into other capital goods with different embodied characteristics. Each period, a machine can be operated or not, and there is no cost of leaving it idle. Lastly, machines have zero scrap value.

Capital goods take one period to be ready to operate, and they fail exogenously at rate \( \delta \). As stated above, each machine takes one worker to operate. Output produced at time \( t \) by a machine built in period \( t - j \) is

\[ \text{Output} = \mu_t \theta_t^{j-1} (k_t)^{j-1} \]

49The existence of an idiosyncratic shock to productivity drawn from a continuous distribution is very convenient for two reasons. First, it greatly simplifies the computation of the equilibrium, as the investment decision will be characterized by a smooth and well behaved optimization problem. Second, it smooths the aggregate allocation and leads to a well behaved aggregate production function, despite the putty-clay structure at the microeconomic level. The problem can be solved without invoking this assumption, but it would lead to a much more cumbersome and demanding computation, together with a more convoluted algebra, without tangible conceptual gains.

50This means that, each period, a machine has a constant probability of exogenous failure \( \delta \). Exogenous machine failure captures wear and tear, which is a major portion of measured rates of depreciation. In the model, as will become clear below, this assumption greatly simplifies the analysis. The model, of course, allows for endogenous depreciation, but it would lead to a much more complicated and crowded analysis without clear conceptual gains. Setting \( \delta = 0 \) would have no qualitative consequences as long as economic growth is positive and some form of machine retirement occurs along the balanced growth path.

51This assumption can be rationalized in a setup where, \textit{ex ante}, before the machine has been ordered, the production function is Cobb-Douglas with constant returns to scale; but, \textit{ex post}, once the machine has been produced, the production function has a putty-clay structure, meaning that the capital-labor ratio of such machine stays fixed at the point decided ex ante. In such a setup, constant returns to scale imply an indeterminacy of scale at the machine level. To see why, notice that, for a given capital-labor ratio, constant returns to scale make a producer ex ante indifferent between a machine with, say, \( N \) workers, and \( N \) machines with one worker. To deal with this indeterminacy, machines are normalized to employ one unit of labor in full capacity (and this is why this assumption implies no loss of generality). This is the underlying setup that I have in mind in this model, but I believe that these details are second order and I am simplifying as much as possible for the sake of clarity.
\[
Y_t(A_t\mu_{t-j}\theta_{t-j}k_{t-j}^\alpha) = A_t\mu_{t-j}\theta_{t-j}k_{t-j}^\alpha 1[L_t(A_t\mu_{t-j}\theta_{t-j}k_{t-j}^\alpha) = 1],
\]

(13)

where \( \alpha \in (0, 1) \), \( L_t(A_t\mu_{t-j}\theta_{t-j}k_{t-j}^\alpha) \) is the labor employed in the machine, and the term \( 1[L_t(A_t\mu_{t-j}\theta_{t-j}k_{t-j}^\alpha) = 1] \) is an indicator function that captures the putty-clay structure of the production function. This indicator function means that, whereas a machine can be operated or left idle, it must keep the ex-ante fixed capital-labor proportion if operated. Notice that all terms have subscript \( t-j \) except \( A_t \), which is the disembodied aggregate technological change term, introduced above. While \( A_t \) changes every year, the terms \( \mu_{t-j}, \theta_{t-j}, \) and \( k_{t-j}^\alpha \) are embodied in the machine and fixed for the machine’s lifetime.

Now, let
\[
X_t := \mu_t\theta_t k_t^\alpha,
\]

(14)

which represents the efficiency level of a particular machine built at time \( t \), a term that was already introduced in the previous section. The exogenous fail rate \( \delta \) assumption implies that, once produced, machines are only distinguishable by their efficiency level \( X \), regardless of the period where they were built. Let \( H_t(X) \) denote the quantity of machines with efficiency level \( X \) available for production at time \( t \). \( H_t(X) \) evolves according to
\[
H_{t+1}(X) = (1 - \delta)H_t(X) + f_X(X; \bar{X}_t)Q_t,
\]

(15)

where \( f_X(X; \bar{X}_t) = \frac{1}{\bar{X}_t}f_{\mu}\left(\frac{X}{\bar{X}_t}\right) \) is the pdf of \( X_t \), and where \( \bar{X}_t = \mathbb{E}[X_t] = \theta_t k_t^\alpha \) is the mean efficiency level of machines built in period \( t \). That is, the quantity of machines with efficiency level \( X \) available for production at period \( t + 1 \) equals the quantity of machines with such efficiency level that survived from last period, plus the quantity of new machines with such efficiency level that were built at time \( t \) and will be available for production, for the first time, at period \( t + 1 \). Given the assumptions on \( f_{\mu}(\cdot) \), the realized efficiency level of a new machine, \( X_t \), is distributed around \( \bar{X}_t \). \( Q_t \) is the aggregate quantity of machines constructed at time \( t \).

\[\text{As I will show below, if a machine is used in equilibrium, it will be used in full capacity. So equation (13) can be equivalently written using the more traditional Leontief notation:}\]
\[
Y_t(A_t\mu_{t-j}\theta_{t-j}k_{t-j}^\alpha) = A_t\mu_{t-j}\theta_{t-j}k_{t-j}^\alpha \min[L_t(A_t\mu_{t-j}\theta_{t-j}k_{t-j}^\alpha), 1].
\]

\[\text{This is the case because, under this assumption, all machines have a constant probability of exogenous failure \( \delta \) each period, regardless of their age. So we can eliminate the time subscript.}\]
The continuity assumption on \( f_\mu(\cdot) \) implies that \( H_t(X) \) is a continuous distribution, which fully characterizes, together with \( A_t \) and the total number of workers, the production possibilities of the economy at period \( t \). Notice that the dynamics of the distribution \( H_t(X) \) will be governed by the term \( f_X(X; \bar{X}_t)Q_t \) which, at the same time, depends on \( \theta_t, \_k_t, \) and \( Q_t \). Hence, for a given path of \( k \)'s and \( Q \)'s, \( H_t(X) \) would tend to shift to the right, every period, along with the vintage component \( \theta_t \). In addition, \( H_t(X) \) would tend to shift more to the right the larger the growth of \( k_t \) is, as new machines would be more capital intensive; and the larger the growth of \( Q_t \) is, as it would lead to a larger proportion of more modern machines, which are associated to a higher vintage term \( \theta_t \).

**Investment Decision.** Each capital owner \( \nu \) chooses how many machines to buy, \( Q_t(\nu) \), and the capital-labor ratio, \( k_t(\nu) \), per machine. Capital owners make the investment decision after observing the economywide vintage technology term \( \theta_t \). However, the idiosyncratic shock \( \mu_t \) is only revealed after the investment decision has been made. At time \( t \), capital owner \( \nu \) chooses the quantity of machines \( Q_t(\nu) \) and the capital-labor ratio \( k_t(\nu) \) per machine to maximize

\[
E_t \left[ -P^I_t Q_t(\nu)k_t(\nu) + \sum_{j=1}^{\infty} \beta^j \frac{C_{s,t}}{C_{s,t+j}} R_{t+j}(\mu_t \theta_t k_t^\alpha(\nu)) \right] \tilde{Q}_t(\nu),
\]

where

\[
\tilde{Q}_t(\nu) = \left[ 1 - S_Q \left( \frac{Q_t(\nu)}{Q_{t-1}(\nu)} \right) \right] Q_t(\nu).
\]

The expression (16) characterizes the present discounted value of profits. The term \( P^I_t \) is the price of investment goods, so the left hand side term of equation (16) captures the total cost of investment at time \( t \). The term \( \beta^j \frac{C_{s,t}}{C_{s,t+j}} \) is the stochastic discount factor of the representative saver. Capital owners rent their machines to the intermediate firms, and the term \( R_{t+j}(X) \) stands for the rental price of a machine with efficiency level \( X \) at time \( t+j \).

Notice that \( t \) (and not \( t+j \)) is the subscript of terms \( \mu_t, \theta_t, \) and \( k_t \), since these characteristics are not a probability density function, though. The distribution \( H_t(X) \) aggregates to the total number of machines available in the economy at period \( t \).

Of course capital owners are allowed to choose whether or not they want to rent their machines. However, it will always be optimal to rent if the rental cost is positive. A more precise way to write this expression would include a dummy variable that takes value one if the capital owner rents the machine and zero otherwise; or even directly the term \( \max \{ R_{t+j}(X), 0 \} \), which already incorporates the optimal choice strategy and which—given that \( R_{t+j}(X) \geq 0 \)—is, at the end, analogous to \( R_{t+j}(X) \). I directly omit this choice dimension in the text for the sake of clarity and simplicity.
are embodied in the machine and fixed for the machine’s lifetime. The function $S_Q$ captures the presence of adjustment costs in investment, as in Christiano et al. (2005), and it is parameterized as

$$S_Q(x) = \frac{\phi_Q}{2} (x - 1)^2,$$  \hspace{1cm} (18)$$
so that, in steady state, $S_Q = S'_Q = 0$ and $S''_Q = \phi_Q > 0$.\(^{56}\)

The timing assumption for the idiosyncratic shock $\mu_t$ implies that the investment problem is symmetric across capital owners so, in equilibrium, $Q_t(\nu) = Q_t$ and $k_t(\nu) = k_t$ for all $\nu \in [0, 1]$.\(^{57}\) The expectation in equation (16) is, hence, taken over the time $t$ idiosyncratic shock $\mu_t$ and the future values of the discount rate components and the rental price function.

### 6.5 Capital Goods Producers

As in Gertler et al. (2017) there is a continuum of competitive capital goods producers, indexed by $\xi \in [0, 1]$. They produce capital goods and sell them to the capital owners at price $P_t^I$.

Capital goods producer $\xi$ takes $I_t(\xi)$ units of final good output, and transform them into $B_t \Gamma[I_t(\xi)]$ units of new capital goods, $Q_k$, where $\Gamma' > 0$ and $\Gamma'' < 0$, and where the neutral productivity term $B_t$ grows at a gross rate $\gamma_B$.\(^{58}\) Capital goods producer $\xi$ chooses $I_t(\xi)$ to maximize

$$P_t^I B_t \Gamma[I_t(\xi)] - P_t I_t(\xi).$$ \hspace{1cm} (19)$$

Symmetry across capital goods producers implies that $I_t(\xi) = I_t$, so the first order condition can be expressed as

\(^{56}\)It is relevant to point out two things regarding the presence of adjustment costs in investment. First, of course, the capital owners will internalize that the choice of $Q_t$ at period $t$ will have an effect on the future discounted profits of $Q_{t+1}$ through the adjustment cost term. Hence, the firm is actually maximizing an infinite discounted sum of present discounted profits of the machines chosen each period. This fact is purposefully omitted in equation (16) for the sake of clarity. Second, there will be a positive economic growth along the balanced growth path in this economy, but $Q_t$ does not growth (the same happens with the number of workers). This is why I directly do not correct for the growth rate of $Q$ in equation (18).

\(^{57}\)It also implies that it will never be optimal to choose different capital-labor ratios across machines or, in other words, that $k_t(\nu)$ will be the same for all machines $Q_t(\nu)$ bought by capital owner $\nu$.

\(^{58}\)In the model, this is technically necessary for the existence of a balanced growth path.
\[
\frac{P^I_t}{P_t} = \frac{1}{B_t \Gamma'[I_t]},
\]

which, given the assumptions on \( \Gamma[.] \), implies a positive relation between the real price of investment and total investment\(^{59}\).

### 6.6 Resource Constraint and Monetary Policy

**Resource Constraint.** The economy’s resource constraint is

\[
C_t + I_t + \frac{\phi_w}{2} \left( \frac{W_t}{W_{t-1}} - \gamma_{w} \gamma_{p} \right)^2 W_t N_t + \frac{\phi_p}{2} \left( (1 + \pi_t) - \gamma_{p} \right)^2 Y_t = Y_t,
\]

where \( C_t = (1 - \lambda)c_{s,t} + \lambda c_{b,t} \) and \( (1 + \pi_t) = \frac{P_t}{P_{t-1}} \). This expression is obtained by aggregating the budget constraint of savers (multiplied by \( 1 - \lambda \)) and borrowers (multiplied by \( \lambda \)), and by aggregating profits and wage adjustment fees (see appendix J for the formal derivation).

**Monetary Policy.** The monetary authority follows a Taylor rule

\[
i_t = (1 - \rho_i)i + \rho_i i_{t-1} + (1 - \rho_i) \left[ \phi_{\pi}(\pi_t - \pi) + \phi_{x}(\log X^{gap}_t - \log X^{gap}) \right] + \epsilon_{i,t},
\]

where \( \rho_i \in [0,1] \); \( i \) and \( \pi \) are the steady state interest rate and inflation rate, and \( X^{gap} \) is output gap.

### 6.7 Equilibrium

I adopt a standard sequence-of-markets equilibrium concept. An equilibrium is a sequence of prices \( \hat{P} = \{\hat{i}_t, \hat{P}^H_t, \hat{P}_t, \hat{W}_t, [\hat{P}_t(i)]_{i\in[0,1]}, \hat{R}_t(\cdot), \hat{P}^I_t\}_{t=0}^{\infty} \) and allocations \( \hat{A} = \{[\hat{c}_j,t, \hat{h}_{j,t, \hat{N}_{j,t}, \hat{B}_{j,t}]}_{j\in\{b,s\}}, \hat{H}_s,t, \hat{H}_t, [\hat{Y}_t(i), \hat{h}_t(\cdot), \hat{N}_t(i)]_{i\in[0,1]}, \hat{H}_t(\cdot), \hat{Q}_t, \hat{k}_t, [\hat{Q}_t(\nu), \hat{k}_t(\nu)]_{\nu\in[0,1]}, [\hat{I}_t(\xi)]_{\xi\in[0,1]}\}_{t=0}^{\infty}, \) such that:

1. Given \( \hat{P}, \hat{A} \) solves the problems of the households, the final good firm, the intermediate goods firms, the capital owners, and the capital producers.
3. Aggregation conditions hold.

\(^{59}\)In the model, this works as a smoothing mechanism that makes the dynamics of investment closer to the data.
4. Price indexes equations hold.

5. Dynamic conditions hold.

Appendix J contains a variety of equilibrium results.

### 6.8 Calibration

Following Gilchrist and Williams (2000), the idiosyncratic productivity shock $\mu_t$ is lognormally distributed:

$$\log \mu_t \sim N\left( -\frac{1}{2} \sigma_{\mu}^2, \sigma_{\mu}^2 \right),$$

which is a natural choice (given that $\mu_t$ is positive) and, as appendix J shows, it greatly simplifies the analysis. Following Gertler et al. (2017), I set $\Gamma[x] = x^{1-m}$.

Table ?? presents the calibrated parameters. Some of them are conventional and set to standard values. The rest are set so that the steady state of the model matches some key statistics of the relatively stable period of the 1990s.

Time is in years to be consistent with the empirical part. I set the inflation target ($\pi$) to 2%, which is the average annual growth of the Personal Consumption Expenditures (PCE) index. In steady state, $1 + i = \gamma^*(1+\pi)$, where $\gamma^*$ is the gross rate of real GDP per capita (2% on average). Hence, I set $\beta_s = 0.99$ so that the nominal interest rate is close to the average Federal Funds Rate (5.15%). As in Justiniano et al. (2015), I set the borrowers’s discount factor ($\beta_b$) so that the relative impatience of the two groups is similar to Campbell and Hercowitz (2009) and Krusell and Smith (1998), which results in $\beta_b = 0.959$; and, also, I pick $\lambda = 0.61$, which is the average share of liquidity constrained households from the 1992, 1995 and 1998 Survey of Consumer Finances (SCF)—calculated as the share of households in the data whose liquid assets are less than two months of their income. Next, I set the labor disutility term $\psi = 7.1870$ so that total labor is normalized to 1/3 in steady state. As standard in the literature, I pick an inverse Frisch elasticity ($\eta$) equal to 1. As for the process of the marginal utility of housing services term ($\xi_t$), I choose $\rho_{\xi} = 0.88$ and $\sigma_{\xi} = 0.163$ to match the dynamics of real house prices after 2007. Lastly, and following Guerrieri and Iacoviello (2017), I choose $\bar{\xi}$ so that the steady state ratio of housing wealth to annual output is 1.5, so $\bar{\xi} = 0.14$.

I choose the collateral constraint parameter $\theta = 0.85$ following the standard practice in the literature, and I set the depreciation rate of housing $\delta_h = 0.022$ to the depreciation
rate in BEA data from 1960 to 2014. As in Greenwald (2018), I calibrate the housing stock \( \bar{H} \) and the saver housing demand \( \bar{h}_s \) so that the real price of housing equals one in steady state and the steady state ratio of saver house value to income is the same than the average value of 1992, 1995 and 1998 SCF, resulting in \( \bar{H} = 4.184 \) and \( \bar{h}_s = 4.942 \). Next, I set the degree of inertia of the borrowing limit \( \gamma_{BC} = 0.72 \), which is close to the estimates in Guerrieri and Iacoviello (2017) and makes the dynamics of the stock of debt of borrowers match the evolution of household debt after 2007. As for the second derivative of the investment cost function (\( \phi_Q \)), I choose a value of 2, which is in line with the estimates of Eberly et al. (2012). Lastly, I calibrate the price and wage Rotemberg adjustment costs (\( \phi_P \) and \( \phi_w \)) so that, to a first order approximation, they are equivalent to the conventional quarterly Calvo parameter of 0.75—0.32 in annual frequency—for prices (consistent with the evidence in Nakamura and Steinsson, 2008) and wages (as in Erceg et al., 2000), resulting in \( \phi_P = 128.34 \) and \( \phi_w = 112.41 \).

On the production side, I also follow the standard practice in the literature and set the capital share \( \alpha = 0.3 \), the annual capital depreciation rate \( \delta = 0.1 \), and the CES aggregation parameters \( \epsilon = \epsilon_w = 6 \). As in Gertler et al. (2017), I set the elasticity of the price of capital with respect to investment rate \( \eta_I = 0.25 \), which is within the rage of panel data estimates from Gilchrist and Himmelberg (1995), Eberly (1997) and Hall (2004). Next, I set the growth rate of the vintage level of embodied technology \( g = 0.009 \), to match the average growth rate of the inverse real price of investment during the 1990s\(^{60} \). To calibrate the growth rate of the disembodied aggregate technological change term (\( \gamma_A \)), I use the fact that the growth rate of real GDP per capita along the BGP is \( (1 + \gamma_A)^{\frac{1}{1-\alpha}} (1 + g)^{\frac{1}{1-\alpha}} = 1.02 \), so I set \( \gamma = 0.005 \). Finally, I set the standard deviation of the idiosyncratic shock to productivity \( \sigma_{\mu} = 0.2834 \), so that the steady state capital utilization ratio is close to the average capacity utilization during the 1990s (82.39%).

Finally, to parameterize the Taylor rule, I set \( \rho_i = 0.8 \), \( \phi_{\pi} = 1.5 \) and \( \phi_x = 0.1 \); which is in line with both the available empirical evidence of the Taylor rule in the post-1984 period and the previous literature.

### 6.9 Results

I use the calibrated model to measure the quantitative importance of the mechanism documented in the empirical part. For this, I implement an experiment where I shock the marginal

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\(^{60}\)I measure the real price of investment as the ratio between Producer Price Index: Private Capital Equipment—as investment deflator—over the Personal Consumption Expenditures Index—as nondurable consumption deflator.
utility of housing services term \((\xi_t)\) so that the dynamics of real house prices mimics the one observed in the data. Although I do not regard taste shocks as the primary driver of the housing bust, this is a parsimonious way to model any kind of force that could affect housing demand in general and, moreover, a conventional device to generate empirically plausible house price movements in DSGE models\(^61\).

Figure 9 presents the main results of this experiment, all corresponding to *detrended* impulse response functions—so all declines are measured with respect to the trend and not in absolute terms—, where time zero is 2008. First, the decline in house prices generated by the shock to \(\xi_t\) leads to a decline in consumption through the collateral channel. At the same time, this decline in consumption is followed by declines in real GDP and labor. Even though the price of capital goods decreases\(^62\), there is a pronounced decline in the expected returns to investment that leads to a drop in investment and a slow recovery after that. The dynamics of productivity are affected, then, by three forces:

1. Even though the labor force is homogeneous in the model, capital goods are heterogeneous. The decline in the economic activity generates a decrease of the capacity utilization. This results in a composition change where the least productive machines—up to a certain threshold—stop being used, increasing the average machine productivity. This generates an increase in productivity at the time of the shock and shortly after, which closely mimics the pattern of the data\(^63\). After that, this effect is soon dominated by other forces that shift productivity downwards.

2. The decline in the capital-labor ratio makes all new machines less productive, reducing the average machine productivity over time—more as the share of the machines that incorporated after the shock gets larger—and affecting negatively the productivity dynamics. This effect tends to vanish as the capital-labor ratio comes back to the steady state levels, but presents a substantial lag coming from the putty-clay structure of the production function.

3. There is a new vintage every period, which is more productive than the previous ones and, hence, shifts the productivity distribution to the right. The drop and slow recovery of the flow of new machines significantly slows down the pace at which this distribution

---

\(^{61}\)See, for example, Iacoviello and Neri (2010); and Guerrieri and Iacoviello (2017)

\(^{62}\)This causes the increase in capital-labor ratio at the time of the shock and shortly after, which has a marginal impact in productivity.

\(^{63}\)In reality, the heterogeneity of the labor force likely plays an important role as suggested above (Charles et al. (2018b)). In the model, for simplicity, I abstract from such a source of heterogeneity which, in every practical sense, is incorporated into the capital heterogeneity.
shifts to the right every period or, in other words, reduces the pace of technological diffusion. This tends to generate a stronger effect over time, as the counterfactual number of new machines that would have been incorporated over the years in the absence of the shock becomes a larger share of the whole distribution of machines. At the same time, this effects tends to vanish as the flow of new machines reaches back its steady state level, but has a substantial lag with respect to this, as it continues affecting the productivity distribution for as long as these machines are in use.

Figure 9: Impulse response functions from the $\xi_t$ shock experiment.

The model does reasonably well at matching a number of key moments in the data that have not been targeted in the calibration. Table 4 compares the model and data values of the 2018 percentage gap with respect to the trend for a number of key macroeconomic variables. The numbers stand for how much the series should grow in 2018 to catch up with the trend (extrapolated from 2008 to 2018). The model comes reasonably close for consumption, GDP and labor units, despite not having been calibrated to match those. It

64 Notice that the way of computing this gap is different from the ones in the empirical analysis—and this makes the levels to slightly differ. In figure 1, the national trend is extrapolated from 2010, to see how the evolution of the productivity series after the Great Recession differs from the long-run trend. In the main empirical analysis, the MSA trends are extrapolated from 2001 to be conservative and abstract from the years of the housing boom and bust. The model takes 2008 as the year of the shock, so the moment comparison extrapolates the trend from that year. The difference in levels comes mainly from the sharp increase in productivity that takes place around 2009.
Table 4: Consumption is real personal consumption expenditures per capita: goods: nondurable goods (chained 2012 dollars); GDP is real gross domestic product per capita (chained 2012 dollars); labor units is hours worked by full-time and part-time employees; investment is the deflated flow of total capital expenditures for all sectors (Z.1. financial accounts of the United States); labor productivity is nonfarm business sector: real output per hour of all persons (chained 2012 dollars). The numbers come from computing the pre-2008 average growth rates, extrapolating that growth from 2008 onwards, and taking the difference of the logs of this extrapolated trend component and the actual series in 2018. Each number is a unit percentage point, and stands for how much the series should grow to catch up with the trend in 2018.

does, however, fall rather short for investment, which implies that the quantitative measure of the productivity gap is probably a lower bound. The labor productivity numbers suggest that the model is able to explain roughly 50% of the productivity gap. These numbers are roughly similar independently on the benchmark year where the gaps are computed.

### 7 Conclusion

The United States has been experiencing a slowdown in labor productivity for more than a decade, which could be behind an accumulated 16% gap in GDP (Syverson, 2017). This well-known fact constitutes a puzzle, as the causes have been unclear and the debate is still ongoing. What is not well-known is that the productivity slowdown is not homogeneous across the U.S., but there is a large heterogeneity across regions: from MSAs with very large and dramatic slowdowns (e.g. Los Angeles, CA, and Miami, FL) to MSAs that had very mild slowdowns or even grew over the trend (e.g. Pittsburgh, PA, and San Antonio, TX). This rich spatial variation allows me to investigate the link between the 2006-2012 decline in house prices (the housing bust) and the productivity slowdown using the U.S. MSAs as a laboratory.

Exploiting this geographic variation through an IV approach, I find evidence to support a causal relationship between the housing bust and the productivity slowdown. In particular, my estimates suggest that a standard deviation decline in 2006-2012 house prices translates
into an increment of the productivity gap—i.e. how much an MSA would have to grow to catch up with the trend—of 6.9p.p., where the average productivity gap is 14.51%.

A mediation analysis supports that the housing bust impacted the productivity gap mostly through the long collapse in consumption spending that followed the bust which, at the same time, generated a long slump in investment that ultimately had a large and long impact on productivity. I also document the limited role of the credit supply hypothesis at this horizon, as well as several other factors, as a competing mechanism.

Lastly, I construct a general equilibrium model that accounts for my empirical findings and is disciplined by them. The model generates a slump in productivity in a setup where the technology frontier keeps growing as usual, which connects with the evidence that shows no significant contraction in R&D expenditures and patents during this period, shedding light on the productivity puzzle. The model also suggests that roughly half of the productivity slowdown has its origin on the housing bust. This number is roughly similar to the partial equilibrium back-on-the-envelope calculation and, together with the empirical evidence, contrasts with the common argument that attributes the productivity slowdown solely to secular forces.

The documented mechanism exemplifies the so-called ‘inverse Say’s Law’, or the idea that weak demand—when sustained over a period of time—, can affect potential output. This may have important policy implications at times when a fundamental problem of demand of this sort is perceived, instead, as a situation where output is close to potential, leading to policies that could even aggravate the problem of origin.

Furthermore, this paper contributes to the already well-known concern about the need to develop policies to prevent housing bubbles, as they can generate recessions that go hand in hand with a massive stock of household debt and a severe depreciation of real assets. These can generate a sustained period of weak consumption and inflation, and lead to weak investment growth, weak productivity growth, and an even weaker growth of wages that could revert back to demand and contribute to the negative cycle.
References


Appendices

A Data

This section elaborates on the data sources used in this paper. The unit of observation is the MSA, which is defined by the U.S. Office of Management and Budget as a contiguous area of at least 50,000 people, delineated on the basis of a central urban cluster. According to the 2010 census, 83.6% of Americans live in MSAs. The rationale to choose MSAs is two-fold: they constitute local labor markets (Moretti, 2010)—unlike counties—, and they are the smallest geographic delineation for which one can measure labor productivity at this time.

Labor Productivity

Labor productivity is measured as real GDP per worker, and it comes from the Bureau of Economics Analysis (BEA). For some robustness checks I use hours worked from the Current Employment Statistics (CES) of the Bureau of Labor Statistics (BLS) and from the American Community Survey (ACS)\textsuperscript{65}.

Given that the GDP data at the MSA level is only available from 2001, I also use personal income (also from the BEA) to construct the trend during the years where GDP is not available, which allows me to go back to 1969\textsuperscript{66}. I deflate personal income with the local CPI price indexes.

Table A.1 shows that the annual growth of personal income responds nearly one-for-one with the annual growth of GDP: at the state level for the years 1963-2017, and the the MSA level for the years that both series overlap (2001-2017).

\textsuperscript{65}There are, to my knowledge, only two sources of hours worked data at the MSA level: the American Community Survey (ACS) and the Current Employment Statistics (CES). The problem with the ACS's measure of hours is that it is only available for a subset of MSAs, as the smallest geographic area is the Public Use Microdata Area (PUMA), which is a geographic unit that contains at least 100000 people and only overlaps with MSAs with 100000 people or more (hence, the PUMA areas do not encompass those MSAs with 50000 to 100000 people). The problem with the CES's measure of hours is that it is only available after 2006. These problems with the existing measures of hours worked are the motivation to take output per worker as the baseline definition. However, as shown in section D, the results are robust to measure labor productivity using any of these two measures of hours worked. Also, there is no measure of GNP beyond the national level.

\textsuperscript{66}Section D shows that the results are robust to a variety of alternative approaches and sources used to construct the trend.
State MSA

<table>
<thead>
<tr>
<th></th>
<th>State</th>
<th>MSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.9532</td>
<td>0.9641</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.0521</td>
<td>0.0487</td>
</tr>
<tr>
<td>Median</td>
<td>0.9771</td>
<td>0.9814</td>
</tr>
<tr>
<td>25th percentile</td>
<td>0.9263</td>
<td>0.9476</td>
</tr>
<tr>
<td>10th percentile</td>
<td>0.9078</td>
<td>0.9283</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.7705</td>
<td>0.7277</td>
</tr>
</tbody>
</table>

Table A.1: Column 'State' includes descriptive statistics across states of the R-squared from the within state and across years (1963-2017) regressions of the annual growth rate of GDP on the annual growth rate of personal income (one regression per state). Column ‘MSA’ does the same for MSAs, spanning the years 2001-2017 (one regression per MSA).

House Prices

The house price indexes come from the Federal Housing Finance Agency (FHFA). I compute the log difference between the years 2006 and 2012. I standardize to facilitate the interpretation of the results: the cross-MSA standard deviation is 19.90, whereas the average is -18.45. Due to the lack of an appropriate local deflator of house prices (such as the national CPI: all items less shelter), I use the nominal local house prices to compute the housing bust; section D performs a robustness check where I compute the inflation-adjusted housing bust using local house prices deflated by the local CPI indexes, which yields virtually the same results.

Consumption

There is no direct measure of consumption expenditures at the MSA level (the finest level of geography aggregation is the state). Some of the regional measures for local consumption expenditures previously used in the literature are car purchases and credit card data (for both of them see, for example, Mian et al., 2013), but they are both proprietary and not available to me at this time. Publicly available measures include retail sales (Fishback et al., 2005), which are only available at a geographically disaggregated level during the Economic Census years (every five years); and retail sales tax data, which are noisy and only available for a subset of regions (Garrett et al., 2005). My main measure of consumption expenditures at the MSA level extrapolates national retail sales to MSAs with the use of MSA retail GDP from the BEA as weight.

Table A.2 shows that this measure, when aggregated to the state level to compare it with direct consumption expenditures measures, responds nearly one-for-one with changes in consumption expenditures for each state. In table A.3, I substitute the benchmark

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67 The rationale for proxying local consumption expenditures with retail sales, gdp or employment, and
consumption proxy by retail employment per capita from the quarterly census of employment and wages following Guren et al. (2018), the results are qualitatively similar.

<table>
<thead>
<tr>
<th>State</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.9359</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.0595</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>0.9490</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25th percentile</td>
<td>0.9233</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10th percentile</td>
<td>0.8925</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>0.6073</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A.2: Descriptive statistics across states of the R-squared from the within state and across years (1997-2017) regressions of the annual growth of the state-level direct measure of consumption expenditures from the BEA on the annual growth of the consumption proxy aggregated at the state level (one regression per state).

<table>
<thead>
<tr>
<th>Consumption gap</th>
<th>Investment gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Baseline</td>
<td>(2) alternative</td>
</tr>
<tr>
<td>House price 2006-2012</td>
<td>-0.061*** (0.020)</td>
</tr>
<tr>
<td></td>
<td>0.776*** (0.181)</td>
</tr>
</tbody>
</table>

| (3) Baseline     | (4) alternative |
| Include baseline controls | Yes | Yes |
| Mean dependent variable | 0.0970 | 0.0943 |
| N (Number of MSAs) | 243 | 243 |

Table A.3: Columns (1) and (3) corresponds to the results in table 3. Columns (2) and (4) correspond to the same exercises but substituting the consumption proxy described in the text by retail employment per capita from the quarterly census of employment and wages as in Guren et al. (2018). Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.

**Capital Expenditures**

There is, to the best of my knowledge, no publicly available (I believe not even proprietary), comprehensive measure of capital expenditures at any geographic aggregation finer than the nation as a whole. The data series that probably gets closest to it at the MSA level is the public statistics on capital expenditures from the Census of Manufacturers, which is only why they track consumption so closely across many independent studies (apart from table A.2 see, for example, Kaplan et al., 2016; and Guren et al., 2018), is that retail is an intermediate input for household consumption, since one has to purchase something to be able to consume it.
available for the Economic Census years (every five years) and is restricted to manufacturing.
I directly construct a new capital expenditures measure at the MSA level with the use
of confidential Census Bureau microdata from the Annual Capital Expenditures Survey
(ACES).

The main data source is the Annual Capital Expenditures Survey of the Census Bureau.
This is an annual survey of 50,000 firms, covering all domestic non-farm businesses and
detailing investment by type (new and used equipment and structures expenditures, among
others) and industries. My benchmark measure of capital expenditures includes expenditures
in both new and used equipment and structures, although the results are robust to restricting
to only new equipment and structures, or just equipment.

Once we have a measure of capital expenditures at the firm level, the next steps consist on
extrapolating it to the establishment level to eventually assign it to its corresponding MSA,
and then aggregate it up at the MSA level. I merge the ACES—where the unit of observation
is the firm— to the Longitudinal Business Database, which contains the whole universe of
establishments, including data on the firm that owns each establishment, the location of the
establishment, the industry classification of the establishment, and economic information
such as the number of workers and the payroll of the establishment. Once I match each firm
to its establishments (and, at the same time, each establishment to its MSA—if it actually
lies in an MSA), I distinguish between: i) single-unit firms and firms that have all their
establishments within the same MSA; and ii) firms that have establishments in different
MSAs (or in areas different from MSAs). The former group can be directly assigned to its
corresponding MSA, whereas the later group is more challenging, and the rest of this section
elaborates on this.

To extrapolate capital expenditures from the firm level to the establishment level for all
firms that have multiple establishments in different MSAs, I follow the following procedure.
First, I restrict to:

- Firms with less than 20 establishments (restricting to 10 or increasing the bar to 30,
  and even 50, produces a very highly correlated measure). This drops large firms with
  many establishments.

- Firms whose establishments all share the same NAICS classification. This drops firms
  with establishments that attend different lines of business and, hence, are hard to
  compare.

- Firms with establishments at least three years old. This drops firms with new es-
  tablishments, or establishments recently created that may have capital expenditures
requirements very different to the rest of the establishments of the firm.

Then, I distribute the total capital expenditures at the firm level across the establishments of the firm according to the relative weight of the economic activity of each establishment within the firm. The ideal weight would be some measure of value added, or even revenues, but these are only available at the firm level from the revenue enhanced LBD and not at the establishment level. Therefore, I use total payroll at the establishment level (using the number of workers produces a very similar measure). Then, I aggregate up at the MSA level by calculating the average (across establishments) amount spent on capital goods per each dollar spent on labor (the average amount spent on capital goods per each worker produces a highly comparable measure). I restrict to MSAs with at least 250 observations (this includes all MSAs for which the housing supply elasticity measure is available). The resulting measure is highly correlated to the capital expenditure measure at the MSA level from the Census of Manufacturers for the Economic Census years, and to an alternative measure that restricts to single-units firms (this, however, has much less observations per MSA, which makes me restrict the sample importantly to keep the requirement of at least 250 observations per MSA). I deflate with the regional CPIs (deflating at the establishment level with the industry disaggregated PPIs produces a very similar measure and virtually similar results).

There are, of course, two main concerns to this exercise. First, some capital expenditures are naturally related to the firm as a whole and serve all establishments at the same time. However, even if such investments are not physically placed in any particular establishment, they contribute to the economic activity of all establishments and they can be related to them from an economic point of view. Second, it can be that payroll is not the appropriate weight to distribute capital expenditures from firms to establishments within a given year as some firms could have establishments with different capital-labor ratios. While this is certainly likely in general, it is much less likely when comparing establishments within the same firm and within the same NAICS classification.

**Other variables**

Other variables include population and the share of workers with a college degree from the decennial census; the employment share of different sectors from the BLS; the GDP share of

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68The aggregate amount spent on capital goods at the MSA level per dollar spent on labor, or per worker, produces a very similar measure. Notice that we need a relative measure to aggregate, as we have a sample of observations within MSA and not the universe. The average amount spent on capital goods also produces a very similar measure.
different sectors, income per capita and wages from the BEA; the housing supply elasticity measures of Saiz (2010); and summary of deposits and commercial and industrial loans data from the Federal Deposit Insurance Corporation (FDIC).
Figure B.1: Labor productivity (real GDP over hours worked) from the Bureau of Labor Statistics, and some projections on labor productivity from the Congressional Budget Office.
Figure B.2: Nonparametric kernel density estimate of the average growth rate (in percentage units) of labor productivity 2001-2005 across MSAs. Kernel=epanechnikov, bandwidth=0.2742. Average is 1.80, standard deviation is 1.68. The figure is truncated at the 1st and 99th percentiles for clarity.

Figure B.3: Nonparametric kernel density estimate of the productivity gap across MSAs. Kernel=epanechnikov, bandwidth=0.0283.
Figure B.4: First stage regression. Corresponds to equation 2 with no controls. See table C.2.

Figure B.5: Taken from Mian and Sufi (2014). National Federation of Independent Business survey. The survey asks small business owners the question: "What is the single most important problem facing your business today?" Fraction of responses stating five of the ten possible concerns, with "regulation" and "taxes" combined into one category.
## C Additional Tables

<table>
<thead>
<tr>
<th></th>
<th>Labor Productivity gap</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OLS</td>
<td>OLS</td>
</tr>
<tr>
<td>House price 2006-2012</td>
<td>-0.057***</td>
<td>-0.053***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
<td></td>
</tr>
<tr>
<td>Include baseline controls</td>
<td>Yes</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.2364</td>
<td>0.1957</td>
<td></td>
</tr>
<tr>
<td>Mean dependent variable</td>
<td>0.1451</td>
<td>0.1451</td>
<td></td>
</tr>
<tr>
<td>N (Number of MSAs)</td>
<td>382</td>
<td>382</td>
<td></td>
</tr>
</tbody>
</table>

Table C.1: Benchmark OLS estimates. House price change is standarized. Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>House price 2006-2012</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OLS</td>
</tr>
<tr>
<td>Housing supply elasticity</td>
<td>0.528***</td>
<td>0.444***</td>
</tr>
<tr>
<td></td>
<td>(0.082)</td>
<td>(0.071)</td>
</tr>
<tr>
<td>Include baseline controls</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.3082</td>
<td>0.2285</td>
</tr>
<tr>
<td>N (Number of MSAs)</td>
<td>243</td>
<td>243</td>
</tr>
</tbody>
</table>

Table C.2: First stage regression. Corresponds to equation 2. Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>OLS</td>
<td>OLS</td>
</tr>
<tr>
<td>Housing supply elasticity</td>
<td>0.005</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Include baseline controls</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.7698</td>
<td>0.1632</td>
</tr>
<tr>
<td>N (Number of MSAs)</td>
<td>243</td>
<td>243</td>
</tr>
</tbody>
</table>

Table C.3: Two additional regressions in support of the exclusion restriction. The dependent variables are the log of the labor productivity levels in 2001, and the change in labor productivity growth (2002-2006 with respect to 1998-2002). Baseline controls are enumerated in the text, I exclude the labor productivity levels in 2001. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.

|                                | Labor Productivity gap |
|                                | (1)                     | (2)                             |
|                                | IV                      | IV                              |
| House price 2001-2006          | 0.062***                | 0.060***                        |
|                                | (0.019)                 | (0.018)                         |
| Include baseline controls      | Yes                     | No                              |
| First Stage F-statistic        | 39.24                   | 42.23                           |
| First Stage (F-test) p-value   | 0.0000                  | 0.0000                          |
| Mean dependent variable        | 0.1451                  | 0.1451                          |
| N (Number of MSAs)             | 243                     | 243                             |

Table C.4: Second stage regression with the housing boom as the main independent variable. Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.
<table>
<thead>
<tr>
<th>Model</th>
<th>Baseline</th>
<th>Split</th>
<th>Interaction</th>
<th>Loans</th>
</tr>
</thead>
<tbody>
<tr>
<td>House price 2006-2012</td>
<td>-0.069***</td>
<td>-0.068***</td>
<td>-0.064***</td>
<td></td>
</tr>
<tr>
<td>House price x National</td>
<td>-0.066***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>House price x Local</td>
<td>-0.068***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>House price x av. share</td>
<td></td>
<td>-0.015</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loans gap</td>
<td></td>
<td></td>
<td></td>
<td>0.014</td>
</tr>
</tbody>
</table>

| Incl. baseline controls | Yes | Yes | Yes | Yes |
| FS F-statistic: house prices | 137.60 |       |       | 70.07 |
| F-test p-value: house prices | 0.0000 |       |       | 0.0000 |
| FS F-statistic: loans |       | 8.82 |       |       |
| F-test p-value: loans |       | 0.0002 |       |       |
| Wald chi2 |       | 0.02 |       |       |
| N (Number of MSAs) | 243 | 243 | 243 | 243 |

Table C.5: Credit supply hypothesis exercises (see text). In column 2, the instrument is the housing supply elasticity interacted with each dummy. In column 3, the instrument for the interaction term is housing supply elasticity interacted with the average share of deposits. In column 4, loans are instrumented by the share of long term illiquid assets in 1996. House price change is standarized. Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.
D Robustness examinations

This section describes and discusses a series of robustness and validity exercises on the main results.

No trend. Some authors (see, mainly, Gordon, 2017) have supported the view that the U.S. has undergone a change in trend, where the levels of economic growth that preceded the Great Recession are over. Under this hypothesis, the trend previous to the Great Recession would not be the right benchmark for comparison anymore since the economy would be in a 'new normal' that oscillates around a completely different, new trend. To address this concern, I estimate a specification where the productivity gap measure is substituted by the 2010-2015 log change in productivity. This approach has the advantage of being robust to a change in trend, as it is directly based on the productivity growth after the Great Recession. However, it is a demanding exercise for the reasons detailed in section 3. Table E.1, column (2), displays the results, which imply that a standard deviation decline in house prices decreases the 2010-2015 growth in productivity by 2 percentage points, where the average is -1.35%. The coefficient is different because the dependent variable is a different object—in particular, a growth rate instead of a log deviation with respect to the trend—but the economic size is even larger than the baseline estimates (in the sense that the housing bust predicts a larger fraction of the dependent variable).

Alternative trends. The results are robust to a variety of approaches to construct the trend: using the Hodrick-Prescott filter to get the 1969-2001 trend, then fit a polynomial and expand it to the post-2001 period; experimenting with different historical periods to construct the trend; restrict to the 2001-2005 period to use labor productivity data only (or a neighborhood of these years); using polynomials of different degrees and other bases; extrapolate different trends from 2010 based on simple average growth rates as in figure 1; etc. The results are displayed in tables E.1, E.2, and E.3.

Trade links. A valid concern, analyzed by Adao et al. (2019), is that an MSA is not only directly affected by its local shocks but, also, it is indirectly affected by the local shocks of the areas with which it has trade links. A simple way of dealing with this is to concentrate on the non-tradable sector. Given that the level of disaggregation for the MSA GDP measure is

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69 The reason to begin in 2010 is to make sure that the measure is not exposed to the contemporaneous effects of the Great Recession. Also, the results are similar when using the annualized growth rate instead.

70 The average growth rate 2010-2015 is slightly negative when productivity is measured as GDP over workers, and slightly positive when using hours worked.

71 The results are also robust when passing the HP filter to the whole period 1969-2015. However, this is a much less conservative approach as the measure of the productivity gap is based on a long-run trend largely influenced by the years of the boom-bust cycle.
quite low, I focus on accommodation and food services\textsuperscript{72}. The measured effect is significantly greater, which implies that the benchmark estimates are a lower bound to an ideal estimate that controls for the trade links. Table E.4 presents the results.

**Placebo: the tradable sector.** I carry out a placebo test that consist on concentrating on purely tradable industries. A purely tradable industry should be largely unexposed to local shocks, so the response of the productivity gap with respect to the local housing bust should be close to zero. I follow Mian and Sufi (2014) to the extend allowed by the level of disaggregation of the MSA GDP measure, and I concentrate on durable goods manufacturing industries. The measured effect is close to zero and statistically insignificant. Table E.4 presents the results.

**Alternative instrument.** A different instrument for the housing boom-and-bust cycle that has been introduced in the recent literature comes from Charles et al. (2018a). It consists on the size of the structural break of house prices during the housing boom. The results are similar when restricting to the set of MSAs for which the housing supply elasticity measure is available, and slightly smaller when including the whole set of MSAs. Both instruments are uncorrelated conditional on the controls, and both pass the Hausman (1978) test for the validity of the exclusion restriction conditional on the other instrument being valid. Table E.4 presents the results.

**Other robustness checks.** The results are robust to a variety of other concerns: measuring labor productivity as GDP over hours (both when using the CES and the ACS measures of hours); excluding MSAs that have more than half of their population in California, Florida, Arizona or Nevada; excluding finance, insurance, real estate and construction, as well as public industries; experimenting with different local deflators for the labor productivity series, and also using local deflators to get inflation-adjusted local house prices; clustering standard errors at the state level (according to the state in which each MSA has most of its population)\textsuperscript{73}; etc. Tables E.5 and E.6 present the results.

Appendix F complements this part by carefully addressing four alternative channels that deserve a particular treatment: industry-specific shocks, the aggregate supply shock hypothesis, the business expectations hypothesis, and the business uncertainty hypothesis.

\textsuperscript{72}Previous literature (see, for example, Mian and Sufi, 2014) includes, in addition, certain subsectors inside the retail sector. However, the GDP measure is only available for the retail sector as a whole, and by including it I would be including online retail too, which is important nowadays. In any case, the results are similar when including the retail sector.

\textsuperscript{73}The reason why I do not do this in the main specification is because a large fraction of MSAs cross state borders. This exercise just assigns each MSA to the state in which it has most of its population.
### E Robustness tables

<table>
<thead>
<tr>
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<th>labor productivity gap</th>
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Table E.1: Each column includes an alternative measure of the productivity gap. Column (1) presents the baseline results. Column (2) measures the productivity gap as the 2010-2015 log change in productivity (the average—across MSAs—is -0.0135). Column (3) uses the Hodrick-Prescott filter to get the 1969-2001 trend, and then fits a fifth degree polynomial and expands it to the post-2001 period. Column (4) restricts to the period 2001-2005 to construct the trend. Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.

<table>
<thead>
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<td>-0.069***</td>
<td>-0.062***</td>
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<tr>
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Table E.2: Each column includes an alternative measure of the productivity gap. Column (1) restricts to the period 1948-1994 to construct the trend. Column (2) restricts to the period 1980-2007 to construct the trend. Column (3) restricts to the period 1980-1994 to construct the trend. Column (4) restricts to the period 1995-2005 to construct the trend. Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.
Table E.3: Each column includes an alternative measure of the productivity gap. Column (1) uses a polynomial of degree 3 in the construction of the trend. Column (2) uses a polynomial of degree 7 in the construction of the trend. Column (3) constructs the trend by extrapolating the 1948-2001 average growth rate from 2010. Column (4) constructs the trend by extrapolating the 1980-1994 average growth rate from 2010. Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.

<table>
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Table E.4: Column (1) includes the baseline results. Column (2) restricts the productivity measure to purely non-tradables: accommodation and food services. Column (3) restricts the productivity measure to purely tradables: durable goods manufacturing. Column (4) uses an alternative instrument consisting on the size of the structural break of house prices during the housing boom (Charles et al., 2018b). Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.

<table>
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<tr>
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<th>(1)</th>
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<tr>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Table E.5: Column (1) measures productivity as GDP over hours worked, with the use of the CES measure of hours worked. Column (2) measures productivity as GDP over hours worked, with the use of the ACES measure of hours worked. Column (3) excludes the MSAs that have more than half of their population in California, Florida, Arizona or Nevada. Column (4) excludes finance, insurance, real estate and construction from the productivity measure. Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.

<table>
<thead>
<tr>
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<tr>
<td></td>
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<td>(3) adjusted HP</td>
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</table>

Table E.6: Column (1) excludes finance, insurance, real estate, construction, and public industries from the productivity measure. Column (2) substitutes the real GDP measure of the BEA by nominal GDP deflated by an alternative price index: the regional PPI indexes. Column (3) uses the local CPI indexes to get an inflation-adjusted local house price index to measure the housing bust. Column (4) clusters the standard errors at the state level: each MSA is included in the state in which it has more than half of its population. Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.
F Potential concerns

This appendix complements section 4.3 by analyzing potential alternative factors that could be behind the measured effect. While some of these factors could affect the productivity slowdown in its own way, this appendix focuses on the extent to which they could bias the results.

Industry-Specific Shocks

A potential concern, already introduced in subsection 4.1, is that industry-specific supply shocks might have simultaneously influenced the housing bust and the productivity gap. This would happen if supply shocks affect some industries more than others during the recession and, hence, the MSAs more exposed to these industries suffered both a decline in house prices and persistent detrimental effects on productivity.

Following Mian and Sufi (2014), table F.1 controls for separate industry-specific effects by including the employment share for each of the 23 two-digit industries. Column (1) are the benchmark estimates, and column (2) includes the employment shares in 2001, at the beginning of the sample; while column (3) includes the 2006 ones, before the beginning of the recession. The results are similar except from a substantial increase in the R-squared.

<table>
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<tbody>
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<td>-0.069***</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
</tr>
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<td>Include industry controls</td>
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<tr>
<td>R-squared</td>
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<td>First Stage F-statistic</td>
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<tr>
<td>Mean dependent variable</td>
<td>0.1451</td>
</tr>
<tr>
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<td>243</td>
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</tbody>
</table>

Table F.1: Second stage regression with additional industry controls. Baseline controls are enumerated in the text. Additional industry controls are the employment shares of each of the 23 two-digit industries. The employment shares correspond to year 2001 in column (2), and year 2006 in column (3). Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.
The Aggregate Supply Shock Hypothesis

An additional concern is the hypothesis that a persistent aggregate supply shock could be behind both the housing bust and the productivity gap. This possibility is unlikely to bias the results because of two main reasons.

First, an aggregate supply shock would be expected to affect roughly all industries. However, section 4.3 shows no correlation between the housing bust and the productivity gap for purely tradable industries. If an aggregate supply shock were the reason behind the relationship between the housing bust and the productivity gap, it would be expected to affect the tradable sector too. There is no clear reason why it would affect the non-tradable sector and not the tradable one, specially taking into account that the productivity slowdown extends widely across industries, including the tradable ones (see, for example, Manyika et al., 2017).

Second, an aggregate supply shock would not explain the clear geographic patterns documented above—which is essential to qualify as an alternative explanation for the results—unless it disproportionately affected the industries that the MSAs with the larger declines in house prices were more exposed to. Even in such a case, this effect would have been absorbed, to a great extent, by the industry-specific effects included in the previous exercise, which do not change the estimates in any statistically significant way.

The Business Expectations/Uncertainty Hypothesis

Finally, this section analyzes together the hypotheses that either a shift in business expectations or a business uncertainty shock could have been behind both the housing bust and the productivity gap.

As in the previous case, any of these shocks is also expected to have a broad effect across sectors, which would be at odds with the finding in subsection 4.3 that shows no correlation between the housing bust and the productivity gap for tradable industries. If any of these shocks were behind the results, there is no obvious reason why it would not affect the tradable sector which, as indicated above, was indeed also impacted by the productivity slowdown.

Even if either an expectation shock or an uncertainty shock hit the economy but had no effect on the tradable industries, to qualify as an alternative explanation for the documented results it would have to be stronger in those MSAs that experienced the larger declines in house prices. This could happen because of three reasons. First, that any of these shocks hit certain industries disproportionally harder, and those industries are precisely the ones that
the MSAs with the larger declines in house prices are more exposed to; in this case, the effect would have been absorbed by the industry-specific effects introduced above. Second, that any of these shocks happens as a consequence of the housing bust as, for example, a negative shift in business expectations after a decline in the local demand following the housing bust; in this case, this would reinforce the results, as this would just be part of the mechanism that explains the estimated elasticity. Third, that some of these shocks directly impacts the decline in house prices; I assess this below.

Even though it is likely that such a shock would affect the tradable sector, the third possibility may be problematic since it could bias the results. However, a necessary condition for this is that the shock should have hit harder those MSAs where house prices eventually declined more. While an expectations or uncertainty shock of this type is likely to be national, there are two plausible reasons why it could have affected some MSAs more than others. On the one hand, it may be that a national shock of this type affected more those MSAs where housing supply is more inelastic; an example of this is a national housing demand shock driven by expectations/sentiment (see Soo, 2018; and Shiller, 2015), or even driven by changes in fundamentals like a decline in the interest rates. In this case, the differential local effects would not come from the shock itself but from the interaction between the shock and the local housing supply elasticity; thus, the shock would differentially affect the productivity gap only through the house prices decline and, hence, it would not bias the results. On the other hand, either an expectation or an uncertainty shock of this type could be local in nature if it is directly derived from the boom. In such a case, it would just illustrate an alternative mechanism that connects the boom with the productivity gap. The housing boom is identified in the same way than the housing bust, and it has similar implications, although it is difficult to separate the consequences of one versus the other empirically since the bust manifests the ex-post unraveling of the boom. Table C.4 reports the main results with the housing boom as the main right hand side variable.

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74 In other words, while such a shock may affect the productivity gap in its own way, it would not bias the results since it would be ‘differenced out’ in the regressions.

75 The only reason why the main specification is based on the housing bust instead of the boom is that, as section 5 makes clear, the bust is the most plausible source for the mechanisms analyzed in this paper.
G Alternative Mechanisms

The R&D Hypothesis

An alternative mechanism is the hypothesis that R&D declined more in those MSAs that experienced larger declines in house prices, and this ultimately affected productivity. This would have happened if, for example, the local demand effects from the housing bust had contracted the economic returns to R&D activities.

The pattern of the aggregate behavior of R&D does not seem to make it likely to play an important role. First, figure G.1 shows that aggregate R&D expenditures as a percentage of GDP not only did not decrease during the Great Recession but it increased considerably since 2005 and, more importantly, stayed at historically high levels during the years that followed. On the other hand, Anzoategui et al. (2019) shows that R&D expenditures—in levels—display a cyclical pattern and that, while there is a noticeable decline in R&D expenditures (with respect to the trend) of the US corporations during the Great Recession, it is substantially smaller than the decline that took place during the 2001 recession, even though the contraction of the economic activity was much smaller in 2001.

![Figure G.1: Total R&D expenditures as a percentage of GDP. Source is U.S. Bureau of Economic Analysis. The shade area corresponds to 2008-2009.](image)

Table G.1 presents the results of an exercise that directly tries to test this hypothesis using
data on the count of patent grants (USPTO) as a measure of R&D and innovation. I take the count of patent grants per 10,000 population and calculate the 2015 log gap with respect to the pre-2001 trend. Column 2 reports the results of a specification that incorporates this patents gap, instrumented with the share of college graduates in 2000. The patents gap coefficient is insignificant, and the benchmark estimate is not significantly different from the baseline estimate\footnote{Alternatively, a mediation analysis following the methodology of Dippel et al. (2019) as in section 5 confirms the same conclusion.}.

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<tr>
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Table G.1: See text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.

The Local Labor Hypothesis

It is also important to consider the hypothesis that the house price decline affected the composition of local labor markets and this had an impact on aggregate productivity. This might have happened through the decline in employment that followed the housing bust (Mian and Sufi, 2014), which might have affected the local employment composition of the MSAs that experienced larger declines in house prices.

This hypothesis is unlikely to be a mechanism because it has been extensively documented that the employment decline was concentrated among the low-skilled workers (see, for example, Charles et al., 2018b). Given that those workers are expected to be associated with a relatively lower productivity, the effect would go the opposite direction, i.e., the
composition change would tend to increase productivity in those MSAs where house prices declined more. In addition, figure G.2 shows a sharp increment in real wages during the recession, which reinforces the interpretation that the composition effect should go in the direction of disproportionately losing low-productivity workers\textsuperscript{77}.

![Figure G.2: Average hourly earnings for production and nonsupervisory employees: total private. "cpi deflator" is the series deflated with the consumer price index. "pce deflator" is the series deflated with the personal consumption expenditure index. The shaded area corresponds to 2008-2009.](image)

An alternative hypothesis is that the boom that preceded the decline in house prices presumably attracted low- and middle-skill construction labor to the MSAs where house prices declined more, and this generated a labor force selection that could have negatively affected productivity in those areas. However, even though this selection could potentially have influenced the productivity dynamics during the housing boom, the fact that the employment loses were concentrated among the less skill workers makes likely that it was, to a great extent, undone during the Great Recession\textsuperscript{78}. Moreover, if this composition change were concentrated in particular sectors like construction and related industries, a substantial part of the baseline estimate would have been absorbed by the industry effects of section F and construction controls of section 4.3.

\textsuperscript{77}Part of this sharp increment may come from changes in the deflator series derived from the decrease in oil prices during the Great Recession.

\textsuperscript{78}See Charles et al., 2018b and Yagan, 2019.
H Mediation Analysis

Sections H.1 and H.2 address the questions posted in section 5 with the use of a different tool: mediation analysis. This is a more ambitious exercise that, at the same time, relies on stronger assumptions. In particular, I follow the methodology in Dippel et al. (2019), which allows to estimate how much of the effect of $X$ on $Y$ goes via a third variable (the mediator) $M$, in a setup where both $X$ and $M$ are potentially endogenous. The strength of the methodology is that it allows for any other competing channel, even potentially unobservable, and requires only one instrument. However, apart from the standard exclusion restriction, it requires that any potential confounder in the regression of $Y$ on $X$ goes through variable $M$. Sections H.1 and H.2 present the results, which are in line with the suggestive evidence introduced in section 5.

H.1 Capital Expenditures

Is the long slump in investment an important pathway through which the housing bust affected the productivity gap?

First, I use the constructed series for capital expenditures at the MSA level to compute the log gap in 2015 with respect to the trend\textsuperscript{79}, which I call ‘investment gap’. Then, I examine the extend to which the housing bust affects the productivity gap via the investment gap. To this end, I follow the methodology in Dippel et al. (2019), which allows to identify and estimate the part of the total effect ($\beta_1$ in equation 3) that goes through the investment gap (called ‘indirect effect’ in the mediation analysis literature), and the part that goes through alternative—potentially unobservable—channels (also called ‘direct effect’), taking care of the fact that both the housing bust and the investment gap are potentially endogenous. Figure H.1 displays an illustration. The main intuition is that, if the slump in investment is an important pathway through which the housing bust influences the productivity gap, two conditions should apply: first, the housing bust has to strongly predict the investment gap; and second, including the investment gap in the main specification (equation 3) should

\textsuperscript{79}It is not obvious what is the optimal way to construct the trend since the series is available only from 1997 onwards. What I do, therefore, is to use MSA personal income—as I did in the case of GDP—during the years where capital expenditures are not available, which allows me to go back to 1969. The idea is that, as appendix A shows, even though capital expenditures is more volatile, the medium and long-run growth of both series is very similar at the national level, so they are expected to be similar at the MSA level too. In any case, the results are similar when only using capital expenditures over the period 1997-2001 to construct the trend, and even when using the 1997-2006 period. Even though it is imperfect, I believe that the former approach is more conservative since it allows us to get a trend over a much longer horizon.
significantly reduce the coefficient on the housing bust. Moreover, the extend of that reduction is informative on the relative importance of the investment channel with respect to alternative channels.\footnote{Appendix G presents evidence suggesting a very limited role for two mechanisms alternative to the slump in investment: the R&D hypothesis and the local labor hypothesis.}

Table H.1 reports the results. Column (1) corresponds to the IV regression of the investment gap on the housing bust, which implies that a standard deviation decline in 2006-2012 house prices translates into an increase of the investment gap on 18.8 p.p. Column (2) applies the mediation analysis methodology described above. The coefficient of the total effect is the main elasticity estimate of the productivity gap with respect to the housing bust, and it is equal to the result displayed in table 1, column (1). The reminder are the coefficients of the direct effect (the part not mediated by the investment gap) and the indirect effect (the part mediated by the investment gap). Dividing the indirect effect coefficient by the total effect coefficient we get \(-\frac{0.039}{0.069} = 0.5652\), which suggests that more than half of the effect of the housing bust on the productivity gap works through the investment gap.

H.2 Consumption Expenditures

The housing bust led to a slump in consumption spending (Mian et al., 2013) that may have consistently discouraged corporate investment. This section examines the extend to which the housing bust affected investment—and, through it, ultimately the productivity gap—via the slump in consumption expenditures that followed the bust.

First, I use the proxy for local consumption introduced in section 2 and compute the log gap in 2015 with respect to the trend,\footnote{As in the case of capital expenditures, it is not obvious what is the optimal way to construct the trend since the series is only available from 2001 onwards. Therefore, I use MSA personal income during the years prior to 2001 to construct the trend, which allows me to go back to 1969. The idea is that, as appendix A} which I call 'consumption gap'. Then, I examine

\begin{figure}
\centering
\includegraphics[width=\textwidth]{mediation_diagram.png}
\caption{Illustration of the mediation analysis.}
\end{figure}
Table H.1: Capital Expenditures: Mediation analysis

<table>
<thead>
<tr>
<th>Investment gap</th>
<th>Productivity gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>IV</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>House price 2006-2012</td>
<td>-0.188***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td></td>
</tr>
<tr>
<td>total effect</td>
<td></td>
<td>-0.069***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.009)</td>
</tr>
<tr>
<td>direct effect</td>
<td></td>
<td>-0.034***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.005)</td>
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<tr>
<td>indirect effect</td>
<td></td>
<td>-0.039***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.012)</td>
</tr>
</tbody>
</table>

Include baseline controls: Yes
First Stage F-statistic: 76.53
First Stage (F-test) p-value: 0.0000
First stage one (T on Z) F-statistic: 137.60
First stage one (F-test) p-value: 0.0000
First stage two (M on Z|T) F-statistic: 108.2
First stage two (F-test) p-value: 0.0000
Mean dependent variable: 0.1473 0.1451
N (Number of MSAs): 243 243

NOTE.—Column 1 corresponds to the IV regression of the investment gap on the housing bust. Column 2 displays the results of the Dippel et al. (2019) mediation methodology of the productivity gap on the housing bust via the investment gap (see text). House price change is standarized. Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.
the extend to which the housing bust affected the investment gap via the consumption gap. To this end, I follow the methodology already introduced the previous section. The main intuition is that, if the consumption gap is an important pathway through which the housing bust influences the investment gap, two conditions should apply: first, the housing bust must strongly predict the consumption gap; and second, including the consumption gap in the regression of the investment gap on the housing bust should significantly reduce the coefficient of the housing bust. In addition, the extend of this reduction is informative about the relative importance of the consumption channel with respect to alternative channels.

Table H.2 reports the results. Column (1) corresponds to the IV regression of the consumption gap on the housing bust, which implies that a standard deviation decline in 2006-2012 house prices translates into an increase of the consumption gap on 6.1p.p. Column (2) applies the mediation analysis methodology described above. The coefficient of the total effect is the elasticity estimate of the investment gap with respect to the housing bust, and it is equal to the result displayed in table H.1, column (1). The reminder are the coefficients of the direct effect (the part not mediated by the consumption gap) and the indirect effect (the part mediated by the consumption gap). The direct effect coefficient is close to zero and insignificant, whereas the coefficient of the indirect effect is close to the coefficient of the total effect. This suggests that most of the effect of the housing bust on the investment gap works through the consumption gap.

shows, the growth rates of consumption and personal income are very similar—and even track each other really well at relatively high frequencies—at the national level, so they are expected to be similar at the MSA level too. In any case, the results are similar when only using the 2001-2006 period to construct the trend. Even though it is imperfect, I believe that the former approach is more conservative since it allows us to get a trend over a much longer horizon.

A complementary exercise in appendix ?? also analyzes the consumption gap as a direct mediator between the housing bust and the productivity gap, which is a necessary condition for the consumption gap to play a central role in this context. These two exercises together let us understand the extent of the role of the consumption gap as the pathway through which the housing bust affected the investment gap and—ultimately—the productivity gap. However, from a rigorous point of view, the complete causal line that follows as housing bust, consumption gap, investment gap, and productivity gap, would require an econometric causal mediation model that incorporates all these variables together. This paper does not go that far because such a model would require additional instruments and, moreover, an econometric methodology that—to the best of my knowledge—is still undeveloped.
<table>
<thead>
<tr>
<th></th>
<th>Consumption gap</th>
<th>Investment gap</th>
</tr>
</thead>
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<tr>
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<td>(2)</td>
</tr>
<tr>
<td>IV</td>
<td></td>
<td>DGHP mediation</td>
</tr>
<tr>
<td>House price 2006-2012</td>
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<td>-0.188***</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.066)</td>
</tr>
<tr>
<td>total effect</td>
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</tr>
<tr>
<td>direct effect</td>
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<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.046)</td>
</tr>
<tr>
<td>indirect effect</td>
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<td>-0.181***</td>
</tr>
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<td></td>
<td>(0.064)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>Include baseline controls</td>
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<td>Yes</td>
</tr>
<tr>
<td>First Stage F-statistic</td>
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<td></td>
</tr>
<tr>
<td>First Stage (F-test) p-value</td>
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<td></td>
</tr>
<tr>
<td>First stage one (T on Z) F-statistic</td>
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<td></td>
</tr>
<tr>
<td>First stage one (F-test) p-value</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>First stage two (M on Z</td>
<td>T) F-statistic</td>
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</tr>
<tr>
<td>First stage two (F-test) p-value</td>
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<td></td>
</tr>
<tr>
<td>Mean dependent variable</td>
<td>0.0970</td>
<td>0.1473</td>
</tr>
<tr>
<td>N (Number of MSAs)</td>
<td>243</td>
<td>243</td>
</tr>
</tbody>
</table>

NOTE.—Column 1 corresponds to the IV regression of the consumption gap on the housing bust. Column 2 displays the results of the Dippel et al. (2019) mediation methodology of the investment gap on the housing bust via the consumption gap (see text). House price change is standarized. Baseline controls are enumerated in the text. Robust standard errors in parenthesis. Significance levels: * 10%, ** 5%, *** 1%.
I Labor Market Structure

As in Erceg et al. (2000), there is a continuum of competitive employment agencies, indexed by \( l \in [0, 1] \). Each of them supplies a differentiated labor input, denoted by \( N_t(l) \), at price \( W_t(l) \), to a labor aggregator. The labor aggregator then bundles the differentiated labor inputs into a homogeneous labor input available for production, denoted by \( N_{d,t} \). The bundling technology is

\[
N_{d,t} = \left( \int_0^1 N_t(l) \frac{\epsilon_w - 1}{\epsilon_w} dl \right)^{\frac{\epsilon_w}{\epsilon_w - 1}},
\]

(22)

where the parameter \( \epsilon_w \) captures the elasticity of substitution among different types of labor, and \( \epsilon_w > 1 \) so that different types of labor are substitutes.

The profit maximization problem of the labor aggregator is

\[
\text{Max}_{N_t(l)} W_t \left( \int_0^1 N_t(l) \frac{\epsilon_w - 1}{\epsilon_w} dl \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} - \int_0^1 W_t(l)N_t(l)dl,
\]

where \( W_t(l) \) is the aggregate nominal wage, while \( W_t(l) \) stands for the nominal wage of labor type \( l \). The resulting labor demand and wage index are \( N_t(l) = \left[ \frac{W_t(l)}{W_t} \right]^{-\epsilon_w} N_{d,t} \) and \( W_t = \left( \int_0^1 W_t(l)^{1-\epsilon_w} \right)^{1-\epsilon_w} \).

Employment agencies. Following Gali et al. (2007), there is a continuum of competitive employment agencies, indexed by \( l \in [0, 1] \), each of which represents a particular labor type. Each agency integrates a continuum of households, and the fraction of savers and borrowers are uniformly distributed across agencies, i.e., each of them combines a fraction \( 1 - \lambda \) of savers and a fraction \( \lambda \) of borrowers. Labor demand for each agency is allocated uniformly across the different households that integrate them, independently of their household type, which implies that, in the aggregate, \( N_{b,t} = N_{s,t} = N_t \). Agency \( l \) sets the wage \( W_t(l) \) that maximizes the objective function

\[
\mathbb{E}_0 \sum_{t=0}^\infty \tilde{\beta} \left\{ (1 - \lambda) \left[ -\psi \frac{N_t^{1+\eta(l)}}{1+\eta(l)} + \frac{1}{P_t c_{s,t}(l)} \left[ W_t(l)N_t(l) - F_t(l) \right] \right] + \lambda \left[ -\psi \frac{N_t^{1+\eta(l)}}{1+\eta(l)} + \frac{1}{P_t c_{b,t}(l)} \left[ W_t(l)N_t(l) - F_t(l) \right] \right] \right\},
\]

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subject to the labor demand function of labor type $l$

$$N_t(l) = \left[ \frac{W_t(l)}{W_t} \right]^{-\epsilon_w} N_{d,t},$$

where

$$F_t(l) = \frac{\phi_w}{2} \left( \frac{W_t(l)}{W_{t-1}(l)} - \gamma_w \gamma_p \right)^2 W_t(l) N_t(l).$$

The optimality condition is discounted by $\tilde{\beta} = (1 - \lambda) \beta_s + \lambda \beta_b$. The fact that the optimality condition depends on $N_t(l)$ follows from the assumption that labor demand is allocated uniformly across the households that integrate the employment agency, regardless of their household type, which implies that $N_{b,t}(l) = N_{s,t}(l) = N_t(l)$. Lastly, the terms from the budget constraint are multiplied by their own lagrange multiplier, which will generally differ between savers and borrowers as their consumption generally differs.

Wages are subject to a quadratic adjustment cost, as in Rotemberg (1982). The terms $\gamma_w$ and $\gamma_p$ are the gross growth rates of the real wage and inflation along the balanced growth path. The adjustment cost is a quadratic function of the difference between the balanced growth path growth rate of nominal wages and the actual growth rate of nominal wages, governed by the parameter $\phi_w$, which captures the size of the adjustment cost, and proportional to the wage bill of labor type $l$. The wage adjustment cost is motivated by the fact that agencies have to negotiate wages each period and this activity consumes real resources. Each agency member incurs in an equal share of the wage adjustment cost, which here takes the form of the fee $F_t$.

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83 Since the agency does not generate profits and is not owned by anyone—but just collects the wage adjustment cost and sets a common wage that is optimal for both types given their relative weight—it is not obvious what is the right discount factor. An alternative is to use savers’ discount factor, which produces very similar results. A more natural approach would be to introduce each household type’s discount factor within the brackets, and inside each household type’s piece of the objective function, but the FOC gets more complicated and non-recursive. In any case, all of these choices are expected to generate virtually similar results.

84 In the model, the existence of wage adjustment costs makes the dynamics of employment, investment, and productivity much closer to the data.
J Model appendix

This section includes the proofs for the model equilibrium results.

Let \( \Omega := \arg\min_x \frac{R_t(x) + W_t}{A_t x} \).

**Proposition 1.** For all \( x \notin \Omega \): \( R_t(x) = 0 \). **Proof:** Suppose that there is an efficiency level \( \tilde{x} \notin \Omega \) such that \( R_t(\tilde{x}) > 0 \). From Proposition 1, \( h^{(i)}(\tilde{x}) = 0 \) \( \forall i \in [0, 1] \). However, given that \( R_t(\tilde{x}) > 0 \), capital owners wish to rent all of their machines of efficiency level \( \tilde{x} \). The capital rental market does not clear, so this cannot be part of an equilibrium.

**Proposition 2.** For all \( x_1, x_2 \in \Omega \): \( \frac{R_t(x_1) + W_t}{A_t x_1} = \frac{R_t(x_2) + W_t}{A_t x_2} = c \). **Proof:** This follows directly from the definition of \( \Omega \). Suppose that \( \exists \tilde{x}_1, \tilde{x}_2 \in \Omega \) such that \( \frac{R_t(\tilde{x}_1) + W_t}{A_t \tilde{x}_1} > \frac{R_t(\tilde{x}_2) + W_t}{A_t \tilde{x}_2} \). Then, it must follow that \( \tilde{x}_1 \notin \Omega \).

**Corollary 1.** From propositions 1-2, \( R_t(\cdot) \) must take the following form:

- For all \( x \notin \Omega \): \( R_t(x) = 0 \). This implies that \( \frac{R_t(x) + W_t}{A_t x} = \frac{W_t}{A_t x} \).
- For all \( x \in \Omega \): \( R_t(x) = cA_t x - W_t \). This comes from the fact that \( \frac{R_t(x) + W_t}{A_t x} = c \).

**Proposition 3.** \( \Omega = [T_t, \infty) \), where \( T_t \) is defined by \( \int_{T_t}^\infty xH_t(x)dx = Y_t \). **Proof:** First, note that the Rotemberg assumptions imply that, in equilibrium, \( P_t(i) = P_t \) and \( Y_t(i) = Y_t \) for all \( i \in [0, 1] \).

For a contradiction, suppose \( \Omega \neq [T_t, \infty) \).^85

From corollary 1 and the definition of \( \Omega \), we have that:

\[
\frac{R_t(x_1) + W_t}{A_t x_1} = c < \frac{W_t}{A_t x_2}, \ \forall x_1 \in \Omega \text{ and } \forall x_2 \notin \Omega
\]  

---

^85I will focus on the case where \( [T_t, \infty) \notin \Omega \). A case where \( [T_t, \infty) \subset \Omega \) may be part of an equilibrium, but capital rental and goods market clearing would imply that \( R_t(x) \leq 0 \) for all \( x < T_t \), and no machine with \( x < T_t \) would be rented, so the equilibrium outcome would be exactly the same as in the \( \Omega = [T_t, \infty) \) case. Let me ignore such a case without loss of generality.
Now, note that, in order to have $h^{(i)}_t(\tilde{x}) > 0$ for some $i \in [0, 1] \text{ and }\tilde{x} \in [0, \infty)$, it must be the case that $R_t(\tilde{x}) \geq 0$ (as otherwise the capital owners would not rent any machine with efficiency level $\tilde{x}$).

This implies that there must be a $\tilde{x} \in \Omega$ such that $R_t(\tilde{x}) \geq 0$(otherwise no machine would be rented and the goods market would not clear).

Moreover, note that, from corollary 1, for any arbitrary $x_1, x_2 \in \Omega$ such that $x_1 < x_2$, it must be the case that $R_t(x_1) < R_t(x_2)$.

Therefore, if $[T_t, \infty) \not\subset \Omega$, it must be the case that there exist a $\hat{x} \in \Omega$ such that $R_t(\hat{x}) \geq 0$ (otherwise no machine would be rented and the goods market would not clear).

Take an arbitrary $\bar{x} \notin \Omega$ such that $\bar{x} > T_t$ (this exists since we are in the $[T_t, \infty) \not\subset \Omega$ case).

From equation (3) we know that $c < \frac{W_t}{A_t \bar{x}}$. Also, as $R_t(\hat{x}) \geq 0$ and $\hat{x} \in \Omega$, we know that $R_t(\hat{x}) = c A_t \hat{x} - W_t \geq 0$, so $c \geq \frac{W_t}{A_t \hat{x}}$. These implies that:

$$\frac{W_t}{A_t \hat{x}} \leq c < \frac{W_t}{A_t \bar{x}}$$

Which implies that $\hat{x} > \bar{x}$. But this is a contradiction, since $\hat{x} < T_t < \bar{x}$.

**Proposition 4.** $c = \frac{W_t}{A_t T_t}$

**Proof:** For a contradiction, suppose not. Suppose that $c = D < \frac{W_t}{A_t T_t}$.

- (i) $R_t(x) \geq 0 \iff R_t(x) = DA_t x - W_t \geq 0 \iff x \geq \frac{W_t}{DA_t}$. Given that $D < \frac{W_t}{A_t T_t}$, we have that $x \geq \frac{W_t}{DA_t} > \frac{W_t}{\frac{W_t}{A_t T_t} A_t}$.
- (ii) $R_t(T_t) = DA_t T_t - W_t < \frac{W_t}{A T_t} A_t T_t - W_t = 0$.
- (iii) $DA_t x - W_t = 0 \iff x = \frac{W_t}{DA_t}$.
- (iv) $\frac{\partial [DA_t x - W_t]}{\partial x} = DA_t$; $\frac{\partial^2 [DA_t x - W_t]}{\partial x^2} = 0$.

Then, we can define the set $\Phi := [T_t, \frac{W_t}{DA_t}]$. By (i)-(iv), $R_t(x) < 0$ for any $x \in \Psi$, so no

---

86 There could not be an equilibrium where $c > \frac{W_t}{A_t T_t}$, since $\frac{W_t}{A_t T_t} = \inf_{x \not\in \Omega} \frac{R_t(x) + W_t}{A_t x}$, so that would contradict the definition of $\Omega$. 

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machine with \( x \in \Psi \) would be rented by the capital owners. Hence, from proposition 3 and (i)-(iv), only machines with \( x \geq \frac{W_t}{D_A} > T_t \) could be rented. Since \( \Psi \subset \Omega \), and given that \( \Omega = [T_t, \infty) \) and the definition of \( T_t \), goods market could not clear.

Figure J.1: Illustration.
Appendix References


