Cost Allocation and Net Load Variability

Jacob Mays

***Abstract*—A basic principle of electricity market design is that cost allocation should follow cost causation as closely as possible. As renewable resources play a larger role in electricity generation, system operators have become increasingly concerned about costs related to variability in net load. While variable generation resources have attracted a great deal of attention along these lines, load is a larger source of variability in most systems. This paper introduces a Shapley value framework to assess the total system cost attributable to electricity market participants that either exacerbate or alleviate net load variability. Further, this paper investigates the extent to which these costs are adequately reflected in wholesale market prices. Numerical tests indicate that the cost of variability on the hourly time scale is in most cases internalized by market-integrated generators and buyers, mitigating concerns that variability imposes a socialized cost from ramping or cycling of thermal generators.**

***Index Terms*—Cost allocation, electricity markets, energy policy, net load variability, renewables integration**

I. Introduction

DUE to a combination of falling costs and policy goals, recent years have seen rapid growth in wind and solar electricity generation. In 2015, for instance, wind represented 41 percent of capacity additions in the U.S., while solar accounted for 26 percent [1]. Accordingly, many researchers have begun to investigate the implications of asking these resources to supply increasingly high percentages of total electricity load, with some studies even proposing that 100 percent renewable generation is possible [2]. Transitioning to these resources can create a dilemma for grid operators. While the marginal cost of energy from these sources is zero, the variability and uncertainty that accompany them may lead to increased costs elsewhere in the system. All other things equal, these costs may limit the share of energy these resources can economically provide [3]. These concerns, embodied in the "duck chart" published by the California Independent System Operator (CAISO) [4], have led to a wealth of work investigating the integration cost of wind and solar resources (see, e.g., [5]–[7]).

Less examined in the context of variability and uncertainty, however, is the role of the demand side. While load has always been the main source of variability in electricity systems, recent developments in advanced metering infrastructure and smart devices have created new opportunities for electricity users to manage their consumption. The existing potential of end users to shift load is large in applications such as water heating, space heating and cooling, agricultural pumping, and wastewater treatment [8], [9]. Further growth in the ability to shift demand may come with installation of storage [10], the increasing popularity of electric vehicles [11], and extensions to demand-side bidding formats [12]. Taken together, these strategies can have a substantial impact on the overall shape of load in the system [13]. Moreover, in addition to individual benefits for buyers of electricity, demand-side participation can help the overall health of the market by ensuring reliability in times of peak usage and preventing the exercise of market power [14].

For system operators focused on fairness and efficiency, the magnitude of the "cost of variability" is less important than ensuring that this cost is allocated to the generator or load that caused it. As part of this effort, many have called for greater market integration of variable resources [15] as well as increased utilization of time-varying retail rates [16]. In a market context, the most straightforward way to accomplish such an allocation is by charging prices based on the marginal cost to produce electricity at any given time. However, much of the concern with variability is its interaction with the operating constraints of thermal generators. Since marginal costs do not reflect, e.g., constraints associated with generator start-up decisions and minimum operating levels, it is not guaranteed that costs related to variability are incorporated in prices [17]. Thus, while others have made the case that greater market integration is a necessary response to the cost of variability [6], [18], it is also worth asking if it is a sufficient one.

These observations motivate the two contributions of this study. The first contribution is a methodology to fairly partition the cost of variability between various classes of generation and load based on principles of cost causation. By considering the contributions of loads, this methodology expands the scope of previous studies and offers a more complete picture of the cost of variability. A simple example demonstrates the value of such an approach. For transmission system operators that have limited visibility below the transmission-distribution interface, distributed energy resources have the appearance of reduced load; accordingly, treating the now-decreased load as given would constitute unequal treatment of generation resources connected above and below the interface. Costs related to variability and uncertainty appear on various timescales [5]. While the methodology could be adapted to smaller or longer timescales or to uncertainty, this paper focuses on the intraday variability of primary concern for generator ramping and cycling.

The second contribution is a series of experiments intended to test the degree to which the cost of variability is internalized by energy market participants. The key distinction for this analysis is between the *marginal* costs used for pricing versus the *incremental* costs used to assess causation. Prices serve a twofold purpose: to encourage efficient behavior in the short term, and promote efficient investment in the long term. While marginal cost pricing is efficient for the short run, incremental

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costs are arguably a more effective signal for the long run. Underpinning much market analysis is the assumption that the two track together. Because of the interaction between variability and generator cycling, however, it is possible for incremental costs to be greater than marginal costs. While some of these costs are not visible to market operators [19], a clear case arises when an additional generator must be brought online. When this occurs, market participants can be in a sense insufficiently penalized for contributing to variability and insufficiently rewarded for alleviating it. To test the robustness of marginal cost pricing to additional variability in the system, I apply the cost causation methodology to a series of counterfactual scenarios. While the experiments confirm that incomplete internalization of cost can arise in a realistic market setting, there is little evidence to support the claim that increased variability will lead to consistent socialization of cost on the considered timescale.

The cost causation methodology builds on the idea of a "proxy resource" commonly used in integration cost studies [20]. Conceptually, the least expensive way to supply a fixed amount of energy over the course of the day occurs when the net load (i.e., total load minus generation from solar and wind) is unchanging.[1] In this setting, system operators can simply dispatch the least expensive thermal generating units for the entire day, avoiding much of the cost associated with start-ups, ramping, and congestion. Accordingly, a consumer that brings the overall system closer to this ideal (e.g., by matching their consumption to the output of wind) should impart less cost on the system. By contrast, a consumer that exacerbates variation in net load (e.g., by using more in peak hours in the early evening) is likely to impart more cost on the system. The total cost of variability is calculated as the difference between the actual dispatch and this idealized setting.

The contribution of each load and variable generator to this cost is dependent on the size and direction of its departure from this ideal. Assume that we are given a set of loads and variable generators, each with a 24-hour profile. It is straightforward to calculation a "deviation profile" equaling the difference between the actual profile and a uniform reference profile. Further, assume we are given an order in which we should consider these deviation profiles. Starting from the idealized reference, these deviation profiles are added one at a time, recomputing the optimal dispatch at each step. The incremental cost attributable to a deviation can then be calculated as the total dispatch cost *after* it is included minus the total dispatch cost *before* it is included. This change in cost gives a more complete estimate of the cost to serve the load: if an additional generator is turned on in order to support the new dispatch, the start-up cost will be directly attributed to that load. This is in contrast to allocating based on marginal costs, which contain no information as to which load "caused" the generator to be committed. A second contrast is that, whereas prices typically give the marginal cost in a single time period, the procedure assesses the total cost for the load profile over the course of the day. Due to generator minimum run times and ramping constraints, the total cost to serve a load profile is not necessarily equal to the sum of its hourly parts.

One clear feature of the above procedure is that the incremental cost will depend on the order in which the deviations enter the dispatch. A natural response is to simply repeat the calculation for every possible entrance order, in the end attributing to each load the average of the resultant incremental costs. This approach can be formalized using the Shapley value solution concept from cooperative game theory [21]. In addition to its intuitive appeal, the Shapley value has been shown to have several features that make it attractive as a cost causation metric. First, two identical participants will be assessed the same cost. Second, a participant who causes the same incremental cost regardless of the other participants in the market will be assessed exactly that cost. Third, the cost measurement for several different services combined (e.g., energy and reserves) will be the same as the sum of the measurements for each of them calculated individually. As the unique partition that satisfies all three of these criteria, the Shapley value has long been seen as a fair way of dividing total cost [22], [23].

A simple example as well as general models for measuring cost causation and pricing are developed in Section II. Data and results of the numerical tests are described in Section III. Conclusions and possible implications are discussed in Section IV.

II. Cost Causation Methodology

A. Motivating Example

Consider a system with an inflexible load of 320 MW in hour t_1 and 420 MW in hour t_2. Meanwhile, wind farms in the system will produce 80 MW in t_1 and 100 MW in t_2 at a marginal cost of \$0 and without any uncertainty. Accordingly, while the load increases by 100 MW between the two periods, the output of wind alleviates the variability and reduces the ramp requirement by 20 MW. The remaining load is served by two thermal generators with characteristics described in Table I, both of which are offline at the beginning of the period.

TABLE I
Generator Parameters

Unit	Min Output (MW)	Max Output (MW)	Start-up Cost (\$)	Energy Cost (\$/MWh)
Coal	50	200	4,000	15
Gas	40	300	0	20

Since the net load of 320 MW in t_2 is greater than can be supplied by either thermal generator individually, it is clear that both must started. The efficient dispatch for this system, with LMPs calculated by fixing the binary start-up variables at the optimal solution, is summarized in Table II.

By selling a total of 400 MWh at \$20/MWh, the coal generator earns \$8,000. This revenue is insufficient to cover its operating cost of \$6,000 plus start-up cost of \$4,000. Wholesale markets typically have a mechanism for make-whole payments in these situations to ensure generator incentive compatibility. In this case, the make-whole payment amounts to \$2,000. Meanwhile, the gas generator sets the LMP

[1]This is not strictly true in practice since generator offers can change throughout the day, most explicitly in the case of unplanned outages.

TABLE II
OPTIMAL DISPATCH RESULTS

Period	Load (MWh)	Wind (MWh)	Coal (MWh)	Gas (MWh)	LMP ($/MWh)
t_1	320	80	200	40	20
t_2	420	100	200	120	20
Total	740	180	400	160	-

in both periods and has no start-up cost. Hence, its revenue and cost are both equal to $160 \text{ MWh} \cdot \$20/\text{MWh} = \$3{,}200$. The total system cost is therefore $\$13{,}200$.

Because the LMP is the same in both periods, there is no direct incentive to shift load from the second into the first period. Such an incentive could arise via the allocation of the make-whole payment. A number of pricing schemes have been proposed to limit side payments [24]. Several potential strategies are available, e.g., allocating to loads based on consumption 1) in the period during which the start-up cost was incurred, 2) over the entire planning horizon, or 3) during the peak period. Although allocating based on strategy 1 may make sense a priori, the result in this case is counterintuitive: adding cost to t_1 would merely encourage *more* variability. However, lacking a standard by which to judge these or other pricing strategies, it is impossible to say which is "correct."

B. Cost Causation and Allocation

A basic principle of electricity market design is that cost allocation should follow cost causation as closely as possible. This principle suggests a two-part problem for market operators: first, defining a metric for cost causation, and second, designing tariffs that follow that metric. In the electricity context, a useful framing for the first problem is provided by [25]. The two most natural ways to judge cost causation are the marginal cost of additional consumption and the total cost of all consumption. This study focuses on the total cost for two reasons. First, as indicated by the example, variability may not be appropriately captured by marginal costs. Second, defining cost causation in terms of marginal cost begs the question of fair cost allocation: with causation defined marginally, setting prices equal to the marginal cost trivially provides a perfect match between causation and allocation.

An axiomatic approach to the second problem, with a specific application to allocating the cost of uncertainty among variable generators, is developed in [26]. The authors propose that cost allocation rules based on cost causation should satisfy six axioms. Two of these axioms are especially relevant for the present study: market participants causing cost should pay for it, and market participants mitigating cost should be rewarded for it. In this context, the contributions of this paper are to 1) develop a new cost causation metric based on total cost and 2) test whether existing pricing schemes conform with the axioms of [26] by comparing to this metric.

Crucially, this paper does not propose a new cost allocation scheme. As discussed in [25], there are important reasons for market operators to set prices based on marginal costs regardless of the underlying cost causation metric. The intent of the present study is to ensure that doing so does not lead to a violation of equally viable notions of fairness that arise from total cost.

C. Shapley Value

It can be seen from the motivating example that marginal cost gives only a partial assessment of cost causation; at issue is not only the cost of an additional unit of electricity in each time period, but also the interaction between the two periods. Accordingly, a holistic measure of cost causation should consider the cost of the entire load profile. As described in Section I, the idea employed in this paper is to allow deviations from the uniform ideal to enter the dispatch one at a time, calculate the incremental cost to serve each deviation, and then take an average over every possible order in which they could enter.

This strategy is equivalent to the Shapley value solution concept in cooperative game theory. While the theory behind this concept is well developed, its practical application has been limited due to the complexity of its calculation. Nevertheless, the Shapley value has been widely applied in areas such as transportation, natural resources, and electricity transmission [27], [28]. More recent uses include evaluating deposit insurance premia in the financial sector [29], assessing input uncertainty in simulation modeling [30], and establishing prices for cloud computing services [31]. The related Aumann-Shapley value has also been widely utilized in applications including cost allocation for electricity transmission service and losses [32], [33]. Because the Aumann-Shapley value relies on integration, it is not well-suited to the discontinuous unit commitment problem considered in this paper.

Three studies have explored the use of the Shapley value in the context of power generation. The analysis of [34] discusses the Shapley value alongside two other concepts from cooperative game theory, the core and the nucleolus, for the purpose of allocating start-up costs and no-load costs. Due to its straightforward interpretation as the average incremental cost over all possible orders in which loads can enter the dispatch, as well as the fairness attributes mentioned in Section I, I limit the discussion to the Shapley value. Central to the concepts of the core and the nucleolus are the ideas of bargaining power and willingness to leave the market if unhappy with a given cost allocation. While these considerations are important to ensure a stable cost allocation, they have no bearing on cost causation.[2] Turning the attention to fair compensation of large numbers of distributed energy resources, [35] proposes a method to efficiently estimate Shapley values, testing on coalitions of up to 1200 members. In a renewables-only context, [26] develop and compare several game theoretic approaches to allocating the cost of uncertainty among variable generators. In this setting, the focus is on comparing allocation strategies given a contractually defined causation metric. Since the underlying cost causation metric is a continuous, time-separable function, the Aumann-Shapley value is preferred as a cost allocation strategy. By contrast, in the present study the Shapley value serves as the cost causation metric itself.

[2]It should also be noted that, in the non-convex setting of the unit commitment problem, the core may be empty.

A precise calculation requires the solution of 2^n unit commitment problems for each day under study. As discussed in Section II-B, the calculation is intended as a cost causation metric for offline analysis rather than a cost allocation strategy necessitating real-time price formation. Accordingly, implementation challenges are not examined here. The numerical study in this paper instead computes exact values for a smaller number of generator and customer classes. Nevertheless, estimation through sampling has been utilized in both the electricity context [35] and more generally [36], indicating that extension to a larger system would be possible. That said, the exact calculation employed here may in fact be sufficient for many applications: for example, many utilities are interested in potential cross-subsidization between a small number of customer classes (e.g., solar and non-solar residential consumers).

A more general way to consider an electricity market as a cooperative game would include all the generators and loads as individual participants in a value game. I have chosen to focus on sources of variability both for both computational efficiency and conceptual clarity. The computational advantage arises due to the nature of the Shapley value, which requires a number of calculations that is exponential in the number of market participants. The conceptual advantage stems from the limited participation of the demand side in current markets: any surplus calculation would rely on the assumed value of electricity to each consumer, potentially clouding any insight from the numerical results. Further, for the sake of discussion I assume that all loads are fixed; however, the model could be easily extended to allow price-responsive demand by first computing the optimal dispatch, then fixing the cleared load for all subsequent calculations.

D. Unit Commitment

Given a set G of generators and a set L of loads participating in the day-ahead market, let each $l \in L$ be characterized by a vector $\mathbf{d}^l$ consisting of the cleared demand for each time period $t \in T$ of the day. Further, let $\mathbf{d}$ be a vector for the total cleared demand, such that $d_t = \sum_{l \in L} d_t^l$. With additional notation in the Appendix, the total cost of the day-ahead market can be expressed as a function of demand:

$$c(\mathbf{d}) = \min_{\mathbf{u},\mathbf{v},\mathbf{p}} \sum_{g \in G} \sum_{t \in T} (k_g u_{gt} + s_g v_{gt} + \sum_{s \in S} c_{gs} p_{gst}) \tag{1}$$

$$\text{s.t.} \sum_{g \in G} \sum_{s \in S} p_{gst} = d_t \qquad \forall t \in T \tag{2}$$

$$\sum_{g \in G} p_{gt}^a \geq r_t^a \qquad \forall a \in A, t \in T \tag{3}$$

$$(\mathbf{u}, \mathbf{v}, \mathbf{p}) \in \mathcal{G}^C. \tag{4}$$

The objective function in (1) equals the sum of generator start-up, no-load, and energy costs over the planning horizon. The power balance constraint is reflected by (2), while (3) guarantees a minimum level of ancillary services. Lastly, (4) requires that each generator follow an operating schedule that is technically feasible, following, e.g., minimum run times and ramping constraints. Since generator start-up decisions and on-off status in each time period are represented by binary variables, the problem is non-convex.

E. Load Profiles

The first goal of this paper is to isolate and attribute the "cost of variability." This can be defined by comparing the actual dispatch cost against an ideal profile that aligns with the total output of wind and solar in the system. For each contributor to variability in the system, it is possible to construct a *deviation* profile reflecting what changes they would need to make to achieve this ideal. In the case of loads these changes can be interpreted as shifts in demand, while in the case of generators they can be interpreted as contracts with storage or demand response providers. For each $l \in L$, define the deviations from a uniform reference profile $\Delta\mathbf{d}^l$, with

$$\Delta d_t^l = d_t^l - \frac{1}{|T|} \cdot \sum_{t' \in T} d_{t'}^l. \tag{5}$$

Let the set $G_V \subseteq G$ represent variable generators, and let each $g \in G_V$ be dispatched at p_{gt} in time period $t \in T$. A deviation profile for each variable generator can be specified in an analogous manner with

$$\Delta d_t^g = -p_{gt} + \frac{1}{|T|} \cdot \sum_{t' \in T} p_{gt'}. \tag{6}$$

Then, defining $V = L \cup G_V$ as the set of all sources of variability, it is clear that

$$\sum_{t \in T} \Delta d_t^v = 0 \qquad \forall v \in V. \tag{7}$$

The idealized profile that leaves a flat load to be met with thermal generators can then constructed as

$$\bar{\mathbf{d}} = \mathbf{d} - \sum_{v \in V} \Delta\mathbf{d}^v. \tag{8}$$

F. Cost of Variability Calculation

With this setup, establishing a cost causation metric amounts to partitioning the cost $c(\mathbf{d}) - c(\bar{\mathbf{d}})$ across the sources of variability in the system. Let the set R represent all possible permutations of the set V, and let P^r represent all the elements of V that precede v in order $r \in R$. The Shapley value, measuring the average incremental cost of the deviation, can be expressed as

$$\begin{aligned}\phi_v(c) = \frac{1}{|V|!} \sum_{r \in R} [c(\bar{\mathbf{d}} + \sum_{i \in P^r} \Delta\mathbf{d}^i + \Delta\mathbf{d}^v) \\ - c(\bar{\mathbf{d}} + \sum_{i \in P^r} \Delta\mathbf{d}^i)].\end{aligned} \tag{9}$$

In each permutation, the incremental cost is found by taking the difference in dispatch cost after including the deviation of v with all of its predecessors. Since the last entrant in any permutation restores the original profile $\mathbf{d}$, it can be seen that $\sum_{v \in V} \phi_v(c) = c(\mathbf{d}) - c(\bar{\mathbf{d}})$, i.e., the total cost of variability.

G. Pricing Schemes

For a given $\mathbf{d}$, solving the unit commitment problem gives optimal dispatch decisions for the day-ahead market. However, in the presence of nonconvexities, market clearing prices do not in general exist. Several pricing schemes have been proposed to support the efficient dispatch while limiting side payments. Here I focus on marginal cost pricing with make-whole payments, described in [37]. Tests on alternate schemes, including the dispatchable and convex hull pricing schemes in [17], do not appreciably change the results. Under marginal cost pricing, binary variables are fixed at their optimal values and this restricted model is re-solved. The dual variables π_t associated with the power balance constraints in (2) and π_t^a associated with the ancillary services requirements in (3) then become the prices for energy and ancillary services in each time period. Lastly, a make-whole payment δ_g for each generator can be calculated as a function of the dispatch instructions to ensure at least zero profit within each commitment interval.

In line with typical practice, assume that loads are allocated energy charges based on their consumption in each period, ancillary services charges based on their load share in each period, and make-whole payments based on their load share for the entire day. Assume that variable generators are paid based only their energy production and neither provide nor pay for ancillary services. Then, with $\Delta d_t^l \ll d_t$ the implicit price $\phi_l(\pi)$ that a load l pays (or is paid) for its variability can be approximated as

$$\phi_l(\pi) = \sum_{t\in T} \pi_t \cdot \Delta d_t^l + \sum_{t\in T}\sum_{a\in A} \pi_t^a \cdot r_t^a \cdot \frac{\Delta d_t^l}{d_t}. \tag{10}$$

For variable generator g, the corresponding payment is

$$\phi_g(\pi) = -\sum_{t\in T} \pi_t \cdot \Delta d_t^g. \tag{11}$$

H. Internalization of Costs and Benefits

The second goal of this paper is to assess how well the cost of variability is internalized by standard pricing schemes. The cost causation metric described in Section II-F accounts for the entire incremental cost of adding variability to the system. In general, the incremental cost $\phi_v(c)$ should have the same sign as $\phi_g(\pi)$, but be smaller in magnitude. Having the same sign indicates that the prices are producing a cost allocation that is at least directionally correct; market participants who exacerbate variability pay for it, while those who alleviate volatility are paid for it. Being smaller in magnitude is a result of the fact that as variability increases, the marginal cost of adding still more variability tends to increase. As long as these conditions hold, it can be said that the cost of variability is internalized by the market participant: they pay for the cost they impose on the system through higher prices. If, on the other hand, the price falls below the cost in magnitude, the market participant has (incrementally) imposed a cost on the system that is not recovered in prices. For these situations, a normalized measure of socialized cost can be calculated as

$$\text{SOC}_l = \frac{\phi_l(c) - \phi_l(\pi)}{\sum_{t\in T} d_t^l} \tag{12}$$

for load l and

$$\text{SOC}_g = \frac{\phi_g(c) - \phi_g(\pi)}{\sum_{t\in T} p_{gt}} \tag{13}$$

for generator g. A positive value can be interpreted as a socialized cost per MWh of total consumption or production, while a negative value indicates that the participant is insufficiently rewarded for alleviating variability in the system.

I. Example Calculation

With the procedure in place, it is possible to return to the example system proposed in Section II-A. Starting with Section II-E, a deviation profile is calculated for both of the sources of variability in the system. Since the total demand grows from 320 MW in t_1 to 420 MW in t_2, the deviation profile for the load $\Delta\mathbf{d}^l$ consists of -50 MW in t_1 and +50 MW in t_2. Since the output from the wind farm increases from 80 MW to 100 MW, its deviation profile $\Delta\mathbf{d}^w$ is +10 MW in t_1 and -10 MW in t_2. Subtracting these two deviation profiles from the original demand results in an idealized demand profile $\bar{\mathbf{d}}$ of 360 MW in t_1 and 380 MW in t_2.

The procedure now turns to assessing the cost of the deviations. Since there are only two participants, there are two possible orders in which they can enter the dispatch. In either order, it is first required to compute the cost $c(\bar{\mathbf{d}})$. The optimal dispatch in this scenario is summarized in Table III. In this scenario, with zero variability in net load, it is no longer required to start the coal generator. Despite the higher energy cost, the avoided start-up cost means that total system cost falls to $560 \text{ MWh} \cdot \$20/\text{MWh} = \$11{,}200$. Comparing the system cost for the actual dispatch in Section II-A against this idealized reference, the total cost of variability amounts to \$2,000.

TABLE III
OPTIMAL DISPATCH WITH IDEAL PROFILE

Period	Load (MWh)	Wind (MWh)	Coal (MWh)	Gas (MWh)	LMP ($/MWh)
t_1	360	80	0	280	20
t_2	380	100	0	280	20
Total	740	180	0	560	-

To continue the procedure, first consider if the deviation of the load enters the dispatch first. The dispatch resulting from to meet the demand $\bar{\mathbf{d}} + \Delta\mathbf{d}^l$ is shown in Table IV. The deviation is sufficient to bring net demand in t_2 above 300, compelling the dispatch of the coal generator. Furthermore, the minimum operating level constraint of the gas generator is binding in this case, resulting in the coal unit becoming marginal in t_1. Total system cost in this case comprises the \$4,000 start-up

TABLE IV
OPTIMAL DISPATCH WITH LOAD DEVIATION

Period	Load (MWh)	Wind (MWh)	Coal (MWh)	Gas (MWh)	LMP ($/MWh)
t_1	310	80	190	40	15
t_2	430	100	200	130	20
Total	740	180	390	170	-

cost and energy costs of 390 MWh $\cdot$ \$15/MWh + 170 MWh $\cdot$ \$20/MWh = \$9,250, for a total of \$13,250. In this permutation, the incremental cost attributed to the deviation profile of the load is \$13,250 − \$11,200 = \$2,050. As the last entrant, adding $\Delta\mathbf{d}^w$ restores the total demand to its original value. It follows that the incremental cost caused by the deviation of wind in this permutation is \$13,200 − \$13,250 = −\$50.

Now consider the opposite order, in which the deviation of wind is added first. After including $\Delta\mathbf{d}^w$, the total system demand is 370 MW in both periods. The optimal dispatch for this scenario is summarized in Table V. While the wind introduces some variability into net load, it is not large enough to require the commitment of the coal generator. The total system cost in this case is equal to the energy cost of the gas generator, \$11,200. Since this is equal to the system cost in the ideal scenario, the incremental cost attributed to wind in this permutation is \$0. It follows that the incremental cost attributable to load in this ordering is \$2,000. Having exhausted the potential orders, it is possible to compute the values $\phi_l(c) = (\$2{,}050 + \$2{,}000)/2 = \$2{,}025$ and $\phi_w(c) = -\$25$. In essence, the load is assigned 101% responsible for the cost of variability, while wind is given 1% credit for partially alleviating the volatility. However, since the LMP remains constant at \$20/MWh through both periods, it is clear that the price of variability $\phi_l(\pi) = \phi_w(\pi) = 0$ for both variable market participants.

TABLE V
OPTIMAL DISPATCH WITH WIND DEVIATION

Period	Load (MWh)	Wind (MWh)	Coal (MWh)	Gas (MWh)	LMP (\$/MWh)
t_1	370	80	0	290	20
t_2	370	100	0	270	20
Total	740	180	0	560	-

The Shapley value methodology rigorously confirms the intuition that the load has a greater contribution to the cost of variability in the example system. However, in this small test system, marginal cost pricing is in fact wholly inadequate to the task of reflecting the cost of variability. The socialized cost amounts to $\text{SOC}_l = \$2{,}025/740$ MWh $= \$2.74/\text{MWh}$. Meanwhile, the wind farm is not compensated for its beneficial variability, and $\text{SOC}_w = -\$0.14/\text{MWh}$.

III. Numerical Study

While the example system can provide an illustration of the problem and the methodology, it can also lead to misleading conclusions. In particular, uplift represents a much smaller portion of total payments in typical real-world systems, averaging from \$0.04 to \$0.16/MWh in U.S. day-ahead markets [38]. In order to better characterize the interaction between the shape of the load curve and pricing, I constructed a test system for a set of loads participating in a single-bus system with demand and generation characteristics modeled on ERCOT. Several system characteristics are reported directly by ERCOT; the test uses actual wind generation, solar generation, and required levels of each ancillary service in ERCOT for Tuesday, July 19, 2016 [39]. More complicated tasks are the development of a set of generators with realistic cost structures and technical attributes and a set of loads that accurately reflects the diversity of customers in the system.

Because the baseline scenario is constructed from a single day in a single system, it is worth emphasizing that the intent of this study is not to produce a general assessment of the cost of variability of any particular technology or load class. Instead, the goal is to test the ability of marginal cost pricing (or competing pricing schemes) to allocate the cost of variability in a fair way.

A. Generator Attributes

A complete set of offers and technical attributes for thermal generators was taken from the FERC Unit Commitment Test System [40]. Since this test system is somewhat larger than ERCOT, a subset of 456 generators approximating the total installed capacity of each fuel source in ERCOT was selected [41]. Since the generator data comes from outside ERCOT and from a different year, note that the priority is not to precisely match the actual cost seen in the ERCOT system, but to appropriately reflect the balance between start-up, no-load, energy, and ancillary services costs in the system. Separate bids for the provision of ancillary services were not available; the price of each ancillary service is assumed to be entirely determined by the lost opportunity cost from energy sales.

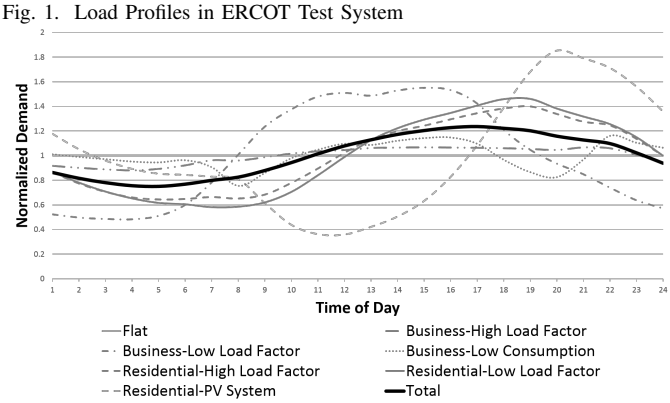
This set of thermal generators is supplemented by the actual hourly output of wind and solar on the modeled day. This maximum output is treated as completely curtailable. Both technologies followed typical output patterns on the day in question. Wind generation amounted to 8133 MW in the hour ending at 1AM, fell steadily to a nadir of 1666 MW in the hour ending at 1PM, and climbed back to 8459 MW for the hour ending at midnight, with total output equaling 10.2 percent of energy consumption. Solar, meanwhile, peaked at 292 MW in the hour ending at 1PM and provided a total of 0.2 percent of energy consumed over the course of the day.

B. Load Profiles

ERCOT divides end users in the region into 28 different segments and 8 geographical zones based on historical consumption patterns, for a total of $28 \cdot 8 = 224$ potential load profiles. In addition to providing the number of meters assigned to each segment and geography on a regular basis, ERCOT publishes a backcasted daily load profile for each customer segment and geography [42]. Since precise interval data are not available for every customer, a sum of all these backcasted profiles does not precisely match the total usage in the system; nevertheless, they provide a good indication as to the division of total consumption between customer classes. In order to find a subset of these 224 profiles that represents the diversity of customers in the system without adding undue computational time, three steps were taken. First, I chose to focus on diversity between customer classes rather than diversity between geographies. By taking a weighted average of each segment profile based on their prevalence in the 8 zones, I produced a composite profile for each customer class. Second, I manually reduced this set of 28 profiles down to 10 by

removing several with very low representation and combining similar profiles. Third, I sought a linear combination of the remaining 10 that matched the actual hourly load seen in ERCOT as closely possible, with each profile constrained to represent between 0 and 20% of total demand. This process resulted in a set of 7 loads that provide a nearly precise match to actual load ($R^2 = 0.999$). Figure 1 depicts the shapes of the seven load profiles used in the study, as well as the total demand in the system.

Fig. 1. Load Profiles in ERCOT Test System



While ERCOT names are retained for the load profiles, some license is taken in the following descriptions of the less straightforward labels. The Flat profile corresponds to certain industrial users (e.g., Oil and Gas) that operate at consistent levels, while Business-High Load Factor profiles correspond to other large industrial users. The Business-Low Load Factor depicts a more typical commercial profile, while the Business-Low Consumption profile is an amalgam of many classes of users with low demand. The inclusion of street lighting in this class likely explains the atypical decrease in consumption from 7-8 AM and increase from 8-10 PM exhibited in its load profile.

C. Baseline Results

The methodology in Section II was then applied to this more realistic test system. The total cost of the dispatch in the system amounted to \$34.4M, while an ideal demand profile would have cost \$32.5M. As a result, the total cost of variability in the system is assessed at \$1.9M. The total percentage consumption or production of each source of variability, its contribution to the cost of variability $\phi_v(c)$ on both a relative and absolute basis, and the price it pays for its variability $\phi_v(\pi)$ are summarized in Table VI.

It can be seen that the load profiles with relatively higher usage in the early morning and late evening—Flat, Business-High Load Factor, Business-Low Consumption, and Residential-PV System—contribute little to the total cost of variability relative to their total share of load. The opposite holds for the other three load profiles, which have relatively higher usage near the system peak at 4-5 PM. Meanwhile, with its higher generation during times of low usage at night, wind contributes to the cost of variability, while solar's favorable timing means it actually alleviates variability in the system.

TABLE VI
COST AND PRICE OF VARIABILITY IN TEST SYSTEM

	Load (%)	$\phi_v(c)$ (%)	$\phi_v(c)$ (\$K)	$\phi_v(\pi)$ (\$K)
Flat	20.0	0.0	0	0
Bus-High Load Factor	16.3	5.4	103	231
Bus-Low Load Factor	15.9	27.1	516	1324
Bus-Low Consumption	4.9	1.6	30	78
Res-High Load Factor	20.0	25.4	483	1144
Res-Low Load Factor	20.0	28.4	541	1299
Res-PV System	2.8	-0.1	-2	-16
Solar	0.2	-1.1	-20	-47
Wind	10.2	13.2	250	597

Notes: Load percentages represent coefficients found in regression rather than actual level in ERCOT. $\phi_v(c)$ refers to the cost of variability, while $\phi_v(\pi)$ refers to the price of variability.

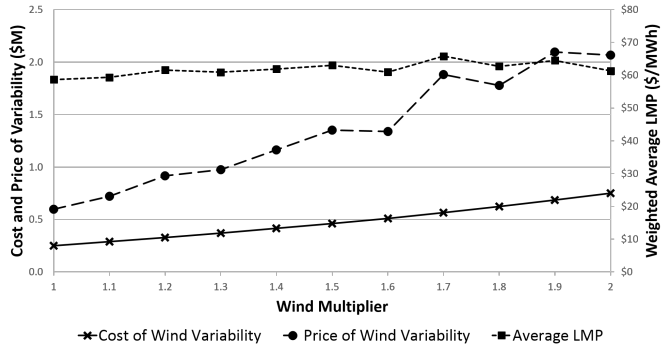
LMPs in the test system ranged from \$29/MWh at 4 AM to \$153/MWh at 4 PM. Accordingly, market participants whose consumption was higher in the afternoon pay more for their variability. For all nine sources of variability, the price $\phi_v(\pi)$ is both directionally correct and greater than the cost $\phi_v(c)$. Accordingly, we can conclude that no cost or benefit has been socialized.

D. Counterfactual Scenarios

While the chosen summer day does have some degree of net load variability, concern about socialized cost is likely to grow with higher penetration of variable resources. Accordingly, I constructed a total of 35 counterfactual scenarios by increasing the quantity of solar and wind in the system. The first ten of these were constructed by adding increments of 10 percent of the current production of wind (up to double its present value), balancing this by adding the same amount of total energy to the demand of the Flat load profile. Key results from the addition of wind are shown in Figure 2. As wind is added to the system, the cost of its variability increases. Its contribution to the total cost of variability (not shown) also increases, from 13 percent in the baseline to 27 percent when doubled. However, the price of its variability retains a comfortable margin above the cost, indicating that the cost of wind's variability is completely internalized in these scenarios. Also shown in Figure 2 is the weighted average LMP for the scenarios. Since the added wind is complemented by additional demand, this value remains steady over the considered scenarios. A further ten scenarios were constructed by increasing the total solar production by multiples of 10 up to 100, also balancing the total energy production with added Flat profile consumption. Figure 3 shows the key results for this set of scenarios. Unlike in the case of wind, the addition of up to 50 times the current level of solar has a negative cost of variability.[3] Until a multiplier of 40, solar is rewarded for its mitigation of variability, as the price of its variability is a larger negative number than its cost. However, the scenario with solar multiplied by 50 provides an example of socialized benefits: despite having a negative cost of variability, solar has a positive price of variability. Subsequent additions of solar

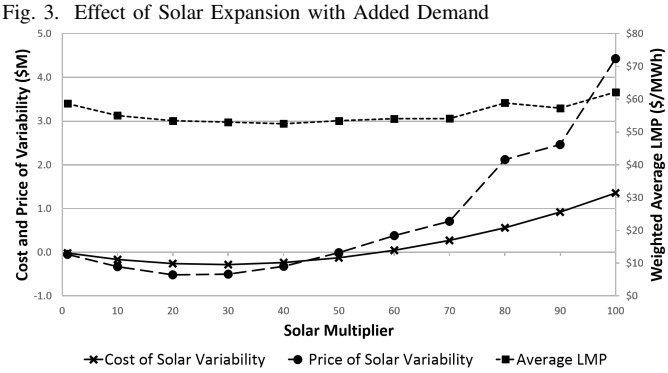
[3]This result is contingent on the selection of a summer weekday for the numerical tests, and should not be interpreted as a general finding.

Fig. 2. Effect of Wind Expansion with Added Demand



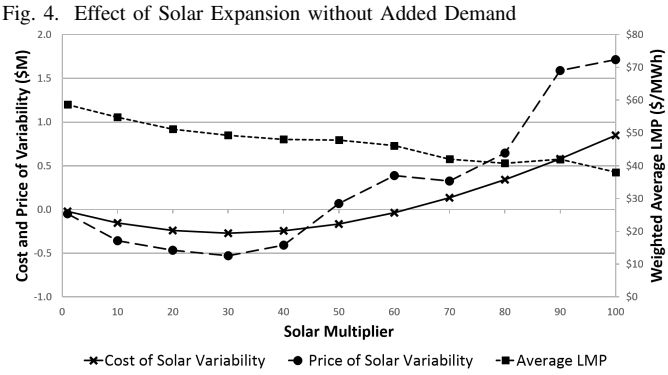
quickly increase the cost of variability, up to 76 percent of the total cost of variability in the 100x scenario. As in the case of wind, addition of solar with complementary demand does not have a consistent impact on weighted average LMP. A third group of ten scenarios added this same quantity of

Fig. 3. Effect of Solar Expansion with Added Demand



solar generation, but without adding any extra demand. Results from this set of scenarios are shown in Figure 4. Curves for the cost and price of variability mimic those in the previous set of scenarios in relative terms. However, both cost and price are lower in an absolute sense, as additional solar displaces the most expensive thermal generators. Correspondingly, the weighted average LMP decreases steadily as supply is added to the system.

Fig. 4. Effect of Solar Expansion without Added Demand



Because no large-scale socialization of costs or benefits was identified in these settings, an additional five scenarios were constructed to ensure that the same trends are likely to continue with more extreme variability. These scenarios resulted from taking two and three times the current quantity of wind without adding demand, 150 and 200 times the current quantity of solar without adding demand, and doubling wind plus taking 50 times the current quantity of solar while adding demand. In realistic systems, expansion of these resources would result in geographic smoothing and accompanying changes in the generation mix and usage patterns. For the purpose of assessing prices under conditions of high variability, however, an overestimate of variability does not pose a problem.

With 9 sources of variability and 35 scenarios, there are 315 opportunities to compare the cost of variability with its price. In 16 of the 315 cases, cost imposed on the system was partly socialized. In the largest such instance, the Bus-Low Consumption underpaid for its cost of variability by $\mathrm{SOC}_l = \$0.56/\mathrm{MWh}$. In 3 of the 315 cases, benefits provided to the system were incompletely reflected in lower prices. In the largest such instance, solar generators were under-compensated by $\mathrm{SOC}_g = \$3.09/\mathrm{MWh}$. The existence of these cases shows that incomplete internalization of costs due to variability can arise in electricity markets. However, the relative rarity and small magnitudes involved indicate that such socialization is unlikely to occur on a consistent basis.

IV. Conclusion

The expansion of variable resources in many power systems has led many to consider how market design must adapt [43]. A question of particular interest is whether markets provide adequate incentives for flexibility. This paper addresses the inverse of that question, assessing the degree to which the cost of variability is internalized under current pricing schemes. Tests across an array of high-variability scenarios indicate that, in most cases, the cost of variability is adequately conveyed in prices generated by wholesale markets. This by itself does not guarantee adequate incentives for flexibility: suppression of volatility in prices at both the wholesale and retail level means that this productive signal may be lost. These results should bolster the case, however, that with appropriate market integration, market participants are unlikely to create a socialized cost due to variability.

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Appendix A
Notation for Unit Commitment Problem

Indices and sets:

- $g \in G$: generators
- $l \in L$: loads
- $t \in T$: time periods
- $a \in A$: ancillary services
- $s \in S$: bid steps, allowing generator cost functions to be represented with up to ten piecewise-constant segments
- $\mathcal{G}^C$: feasible region for generator output schedule given capacity constraints, ramping constraints, and minimum up and down times

Variables:

- u_{gt}: (binary) commitment status of generator g at time t
- v_{gt}: (binary) startup decision for generator g at time t
- p_{gst}: offer cleared for generator g at bid step s at time t
- p^a_{gt}: amount of ancillary service a provided by generator g at time t

Parameters:

- k_g: no load cost for generator g
- s_g: startup cost for generator g
- c_{gs}: marginal cost for generator g in bid step s
- r^a_t: amount of ancillary service a required at time t

Appendix B
Additional Notation

Demands:

- d^l_t: cleared demand for load l at time t
- $\mathbf{d}^l$: vector of cleared demands for load l
- $\mathbf{d}$: vector of total demand cleared for all loads
- Δd^l_t: deviation of load l from a flat reference profile at time t
- $\Delta\mathbf{d}^l$: vector of deviations of load l
- $\bar{\mathbf{d}}$: vector of idealized demand profile for all loads

Indices and sets:

- $g \in G_V$: set of variable generators
- $l, g, v \in V$: set of all sources of variability (loads and variable generators)

Prices and payments:

- π_t: clearing price for energy at time t
- π^a_t: clearing price for ancillary service a at time t
- δ_g: make-whole payment due to generator g

Functions:

- $c(\mathbf{d})$: total cost of demand cleared in day-ahead market
- $\phi_v(c)$: contribution to cost of variability for market participant v
- $\phi_v(\pi)$: price of variability paid by market participant v
- SOC_v: socialized cost or benefit arising from the variability of market participant v

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