The Insurance Value of Financial Aid

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Abstract

Financial aid programs exist to enable students with fewer financial resources to pay less to attend college than other students with greater financial resources. When income is uncertain, a means-tested financial aid formula that requires more of an Expected Family Contribution (EFC) when income and assets are high and less of an EFC when income and assets are low provides insurance against that uncertainty. Using a stochastic, life-cycle model of consumption and labor supply, we show that the insurance value of financial aid is substantial. Across a range of parameterizations, we calculate that financial aid would have to increase by enough to reduce the net cost of attendance by about a third to compensate households for the loss of the income- and asset-contingent elements of the current formula. This compensating variation is net of the negative welfare consequences of the disincentives to work and save inherent in the means-testing of financial aid.

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1 Introduction

Attending college is an important pathway to higher earnings. Unconditional estimates of the gap in median earnings between year-round, full-time workers with bachelor’s degrees and those with only high school diplomas are about 67 percent, for both men and women. Carneiro, Heckman, and Vytlacil (2011) estimate the marginal returns to a year of college and find an earnings premium of 8 percent per year of college.

The high returns to college have made the financing of college an important topic for both academic research and public policy over the last several decades. High returns to college have been accompanied by high and rising costs of college attendance. Launched as part of the Great Society programs of the 1960s, the federal financial aid system has grown in scale and complexity to help ever more students from low- and middle-income families afford college and the access to higher earnings it can provide. Most states and institutions of higher education also operate financial aid programs. For all families, and particularly for those who have higher incomes or who aspire to more expensive institutions, how to pay for college is often a savings decision that begins when the child is born.

Financial aid programs exist to enable students from families with fewer financial resources to pay less to attend college than other students from families with greater financial resources. As implemented through both federal and institutional formulas, the amount of financial aid a student receives declines with both the income and assets of his or her family at the time of enrollment. The inclusion of family assets in the formulas to determine a student’s “Expected Family Contribution” (EFC) toward college expenses has attracted considerable attention from economists, who have highlighted the resulting disincentive for families to save for college expenses and who have produced varying estimates of its impact on household saving. The concept of the “financial aid tax” dates back to Case and McPherson (1986). Edlin (1993) provided an early, readable discussion of the financial aid tax, and Feldstein (1995) estimated a large crowding out of saving due to this tax, spawning a small literature testing the robustness of those initial results.

Omitted from this literature is the recognition that the saving disincentives due to the financial aid tax comprise only the incentives side of a standard incentives-insurance tradeoff. In general, providing insurance against risks beyond a household’s control will distort incentives along margins that the household can control. When income is uncertain, a financial aid formula that re-

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2 See Oreopoulos and Petronijevic (2013) for a review of estimates to the returns to college and a discussion of how and why the college earnings premium has changed over time.
3 See Dynarski and Scott-Clayton (2013) for an evaluation of financial aid policy today.
4 The implicit tax on assets also figures prominently into practitioner guidance on saving for college. See, for example, http://www.forbes.com/sites/troyonink/2014/02/14/how-assets-hurt-college-aid-eligibility-on-fafsa-and-css-profile/.
quires more of an EFC when income and assets are high and less of an EFC when income and assets are low provides insurance against that uncertainty. The incentives-insurance tradeoff is well understood in the literature on optimal redistributive taxation. What has not yet been recognized is that, as redistributive mechanisms based on assets and income, an analogous tradeoff is present in financial aid formulas.

The contribution of our paper is to estimate the insurance value of financial aid using a stochastic, life-cycle model of consumption and labor supply in which households save in anticipation of a planned retirement, uncertain income, and the college education of their children. The main results show that the insurance value of financial aid is substantial. We calculate the insurance value of financial aid by comparing lifetime expected utility under two financial aid systems—one in which there is a stylized version of the current financial aid formula and one in which colleges give aid by simply discounting their tuition, regardless of a household’s income or assets. This comparison is analogous to substituting a lump sum tax for a distortionary tax, which under certainty would be expected to make the household better off. However, given sufficient income uncertainty and risk aversion, the substitution may lower welfare by removing the insurance value of financial aid. Across a range of parameterizations, we calculate that financial aid would have to increase by enough to reduce the net cost of attendance by about a third to compensate households for the loss of the income- and asset-contingent elements of the current formula. For our preferred parameterization, a dollar of financial aid delivered through the current formula is worth $1.30 in lump sum tuition discounts. Further, this compensating variation is net of the negative welfare consequences of the disincentives to work and save inherent in the means-testing of financial aid.

The remainder of the paper is organized as follows. Section 2 describes the key features of the financial aid system and briefly reviews the literature on the relationship between financial aid and household saving. Section 3 develops the stochastic life-cycle model of consumption and labor supply that will be used to simulate work and saving decisions and thus measure the insurance value of financial aid. The main results on the insurance value of financial aid are presented in Section 4, along with sensitivity analyses. Section 5 discusses directions for further research, and Section 6 concludes.

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5See Eaton and Rosen (1980) and Varian (1980) for early analyses of this tradeoff in optimal income tax systems. Using variation in tax and transfer systems across U.S. states, Grant, Koulovatianos, Michaelides, and Padula (2010) show that state-level measures of redistributive taxation correlate negatively with the standard deviation of the within-state consumption distribution, consistent with an insurance effect of redistributive taxation. Using the Panel Study of Income Dynamics, Hoynes and Luttmer (2011) show that while the redistributive value of state tax-and-transfer programs declines sharply with income, the insurance value of these programs is increasing in income.

6In this respect, the analysis is similar to prior papers that have examined the insurance aspects of other tax and expenditure policies. See Hubbard, Skinner, and Zeldes (1995) for precautionary saving and social insurance, Engen and Gruber (2001) regarding precautionary saving and the unemployment insurance system, and recent papers by Rostam-Afschar and Yao (2014) on precautionary saving and progressive taxation and Athreya, Reilly, and Simpson (2014) on the insurance value of the Earned Income Tax Credit.
2 The Financial Aid Tax

Most need-based financial aid is governed by either of two formulas: a “Federal Methodology” set by Congress that determines eligibility for federal financial aid and an “Institutional Methodology” set by the College Scholarship Service that is used by many selective colleges and universities to determine eligibility for institutionally provided aid. While both require information on income and assets, they differ principally in that the Institutional Methodology considers more sources of income and assets. The key omission from both formulas is assets held in retirement accounts like 401(k) plans and IRAs.

Aid awarded under the Federal Methodology is based on information reported on the Free Application for Federal Student Aid (FAFSA). The FAFSA combines information on family structure, income, and assets to generate the Expected Family Contribution (EFC), and financial need is calculated by subtracting the EFC from the student’s cost of attendance at a given school. The components of the formula are presented in the EFC Guide published each year and form the basis of the algorithm used in this paper to calculate financial aid. Students who are unmarried and sufficiently young apply as dependents of their parents. Those who are older, married, veterans, or have dependents of their own apply under the more favorable status of independent students.

As our focus is the parents’ saving decisions, we consider students as dependents and simplify the calculations by zeroing out the student’s contributions. We further simplify the modeling of financial aid by using the formula in the Federal Methodology for combining assets and income to calculate the EFC but a fully general measure of assets and income that is more consistent with the Institutional Methodology.

Following the Federal Methodology, the EFC is obtained by considering a family’s “Adjusted Available Income,” (AAI) which is the sum of “Available Income” (AI) and the “Contribution from Assets” (CA), defined as follows:

\[
AI = \text{Adjusted Gross Income} - \text{Federal Income Tax Paid} + \text{State and Other Tax Allowance} + \text{Social Security Tax Allowance} + \text{Income Protection Allowance} + \text{Employment Expense Allowance}
\]

\[\text{(1)}\]

See https://professionals.collegeboard.com/profdownload/FM%20%20IM%20Diﬀerences.pdf for a summary of the key differences between the two methodologies.

The methodology for determining the EFC is found in Part F of Title IV of the Higher Education Act of 1965, as amended, and governs awards for federal Pell grants, subsidized Stafford loans, Perkins loans, federal work-study programs, and other opportunities. For the latest and archived “EFC Guide” publications, see: http://ifap.ed.gov/ifap/byAwardYear.jsp?type=efcformulaguide.

In the absence of this simplification, each different type of asset in the formula would necessitate both a state variable and a choice variable in the model below. Other alternatives are possible. For example, all assets could be treated as 529 plan assets that accumulate tax free.
Available Income begins with the parents’ adjusted gross income (AGI) from their tax return and subtracts allowances based on other payments that a household would make in order to earn that income. As implemented below, AGI is just the sum of labor income and asset income, and federal income taxes paid are approximated by a simplified version of the federal tax schedule based on that income. Marginal tax rates under this schedule range from 10 percent at very low levels of income to 39.6 percent at the highest income levels. Other allowances are made for State and Other Taxes, Social Security Taxes, Employment Expenses, and Income Protection. Each of these other allowances is as specified in the EFC Guide, with a state tax allowance of 4.5 percent chosen to reflect the middle of the distribution of state tax rates.10

The Contribution from Assets is zero if assets do not exceed the Asset Protection Allowance specified in the EFC Guide and 12 percent of any excess of assets over that allowance otherwise. These two components are added together in Equation (2) to obtain AAI. In Equation (2), the scalars $k$ and $j$ are both equal to 1 but could be altered in simulations of alternative financial aid formulas. Given AAI, the EFC is calculated as:

$$EFC = 0.22 \cdot \min(14,600, \max(AAI, -3,409)) + 0.25 \cdot \max(0, \min(AAI, 18,400) - 14,600) + 0.29 \cdot \max(0, \min(AAI, 22,100) - 18,400) + 0.34 \cdot \max(0, \min(AAI, 25,900) - 22,100) + 0.40 \cdot \max(0, \min(AAI, 29,600) - 25,900) + 0.47 \cdot \max(0, AAI - 29,600)$$

The EFC is a piecewise-linear spline in AAI with marginal conversion rates that increase progressively from 22 to 47 percent. Two aspects of these formulas are noteworthy. First, the top marginal conversion rate of 0.47 is reached at a fairly low level of AAI, or $29,600 in the 2012-2013 formula used below. Second, while the marginal conversion rates of 12 percent in Equation (2) and 22 – 47 percent in Equation (4) have stayed the same over the years, the various nominal amounts in Equation (4) and the dollar values in the allowances in Equations (1) and (2) have increased over time. The modeling framework below fixes these dollar values in real terms based on the 2012-2013 formula.

Table 1 shows the EFCs that result from these formulas for a range of income and assets. For illustrative purposes, these calculations use the allowances for a married couple with one parent working and one child in college in which the

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10The income and payroll taxes are based on the formulas in place in 2013. A payroll tax of 6.2 percent for the employee’s share of Social Security is levied on pre-tax labor income up to a maximum taxable earnings limit of $113,700, and an analogous Medicare payroll tax of 1.45 percent is levied on all such income.
older parent is 45 years old. The table shows that the EFC is monotonically increasing in both assets and income. The EFC remains at zero for combinations of income and assets that do not exceed the various allowances. At any college using the Federal Methodology to allocate aid, any EFC that fell below the costs of attendance would make the student eligible for financial aid, potentially up to the difference between those costs and the EFC if the institution committed to meet full demonstrated need. For the highest values of income and assets, the EFCs exceed the costs of attendance even at the most expensive colleges, resulting in no financial aid.

Table 2 presents the implied marginal tax rates on income inherent in the EFC amounts in Table 1. Each cell holds the asset level constant (at the value specified in the row heading) and then calculates the incremental change in the EFC for a $15,000 increase in labor income. At low levels of income and assets, at which the EFC is zero, the implied marginal tax rates are also zero. Once they become positive, marginal tax rates can be as high as 40 percent. The intuition from Equations (1) and (4) is that the implied marginal tax rate is the conversion rate, up to 0.47, multiplied by one minus the marginal income tax rate on labor income. Recognizing that the marginal tax rate includes Social Security and Medicare payroll tax rates, the numbers in the table might be 0.47 · (1 − 0.35) = 0.3055, or about 30 percent. Lower rates obtain at higher income levels for which the combined state and federal taxes are higher, despite the decline in the payroll tax at the maximum taxable earnings limit. Higher or lower rates may occur at lower income levels for which the marginal labor income tax rates may be lower but the marginal conversion rates are also lower.

These marginal tax rates are in addition to the marginal income tax rates from the payroll tax, federal income taxes, and state income taxes, implying potentially high combined tax rates on labor income during the years in which parents have children in college. This possibility is noted but not explored in Feldstein (1995). Using cross-sectional regressions of actual financial aid awards, Dick and Edlin (1997) estimate lower income-sensitivity of financial aid than these theoretical predictions, noting that actual awards are typically not as progressive as the formulas imply.

Table 3 shows the implied marginal tax rates on assets inherent in the EFC formula. Analogous to Table 2, each cell of Table 3 holds labor income constant (at the value specified in the column heading) and then calculates the incremental change in the EFC for a given increase in assets. Over most of the table, the implied marginal tax rate on assets is about 7 percent. This “asset tax” comes from two sources. The first is in Equations (2) and (4), in which 12 percent of assets are available and are converted at rates up to 47

11 The formula used to generate the table ignores the Simplified Needs Analysis in the Federal Methodology that excludes assets from the EFC calculation for low-income households. This likely understates the insurance value of financial aid for households with low permanent income.

12 The EFC is divided by the number of children in college, so with more children in college (and adjusting for the impact of more children on the income allowance) there would be the possibility of financial aid even at these income and asset levels.
percent: \(0.47 \cdot 0.12 = 0.0564\). The second is in Equations (1) and (4), in which the assets generate income, net of taxes on that income at the federal and state levels, and then are converted at rates up to 47 percent. With a 3 percent rate of return and a 30 percent combined marginal income tax rate on asset income, this would yield an additional \(0.47 \cdot 0.03 \cdot (1 - 0.3) = 0.0099\). Combining these two components gives approximately the 7 percent figure found in much of the table. Lower rates obtain at higher income levels for which the combined state and federal taxes are higher and at lower income levels for which the marginal tax rates may be lower but the marginal conversion rates are also lower.

An implied marginal tax rate of 7 percent may not seem too high, but it is important to note that it applies in each successive year of college attendance to the remaining assets. Thus, a dollar of assets at the start of college is reduced by \(0.07 \cdot [1 + (1 - 0.07) + (1 - 0.07)^2 + (1 - 0.07)^3] = 0.252\), or about 25 percent over four years in college. Thus, the financial aid formula levies a substantial tax on assets over a broad range of income and asset combinations.\(^{13}\)

These implied marginal tax rates on assets are the impetus for the empirical literature that has estimated whether households respond to the asset tax by saving less. The literature starts with Feldstein (1995), who estimated a reduction of about 50 percent in asset accumulation due to the financial aid tax. His estimation sample was a cross-section of 161 households in the Survey of Consumer Finances 1986. Long (2003) argues that a household’s estimate of the implicit tax on assets that would discourage saving is more complicated than suggested in Feldstein (1995) and in Table 3, noting that it depends on factors such as the likelihood of children going to college, the expected cost of college (since the marginal tax rate is zero if the EFC exceeds the cost of attendance), and the possibility that the college does not meet all need, in which case an additional dollar of assets will reduce unmet need rather than financial aid. His methodology generates smaller taxes at the margin and no correlation between those marginal tax rates and asset accumulation. Later studies by Monks (2004) using the National Longitudinal Study of Youth and Reyes (2008) using the Panel Study of Income Dynamics find weak evidence consistent with lower asset accumulation, but at magnitudes much less than Feldstein (1995).\(^{14}\)

The implied marginal tax rates on both income and assets in Tables 2 and 3 are also the source of the insurance value of financial aid. As was noted by Eaton and Rosen (1980), even a simple proportional income tax in which the proceeds are redistributed as a lump sum will raise welfare when income is uncertain. As with the prior literature on the disincentive effects of the asset tax, the degree of insurance in the financial aid formula depends on whether the college commits to provide financial aid equal to the difference between the costs

\(^{13}\)These estimates for the asset tax are broadly consistent with those of Dick and Edlin (1997), who estimated marginal asset tax rates of up to 30 percent using cross-sectional data from the 1987 National Postsecondary Student Aid Survey.

\(^{14}\)The implicit tax on income in the financial aid formula has received no consideration to date as a source of economic distortions. Handwerker (2011) uses the Health and Retirement Study to show that parents delay retirement while paying for a child’s college education. She finds little evidence that paying for a child’s education has any impact on work intensity for those who are working.
of attendance and the EFC. This is assumed in the analysis below and is true of
the most well funded colleges and universities, for which the analysis in general
is most applicable. However, this issue is not as critical for the insurance value
of financial aid as it is for the disincentives of the asset tax. Even if there may be
some income or asset ranges over which an institution may not boost financial
aid dollar-for-dollar with demonstrated need, generating lower marginal tax
rates than in Tables 2 and 3, the insurance provided on the inframarginal need
is still present.

3 Stochastic Life-Cycle Model of Consumption
and Labor Supply

This section presents a stochastic, life-cycle model of consumption and labor
supply in which the traditional retirement motive for saving is augmented by a
precautionary motive to save against income uncertainty and a potential need to
pre-fund a child’s college education. We begin by specifying the model fully, fol-
lowed by a discussion of the solution method, and concluded with a justification
for the parameters chosen.

3.1 Model Specification

The basic structure of the model is that in each period of life, \( s \), the household
chooses values of consumption, \( C_s \), and labor, \( L_s \), as functions of the two state
variables in the model, current assets, \( A_s \), and labor income from fulltime work,
\( Y_s \). The individual’s value function in period \( t \), \( V_t(A_t, Y_t) \), is defined as:

\[
V_t(A_t, Y_t) = \max_{\{C_s, L_s\}} \sum_{s=t}^T \beta^{s-t} (u(C_s) + \theta v(L_s))
\] (5a)

\[
u(C) = \frac{C^{1-\gamma}}{1-\gamma}
\] (5b)

\[
v(L) = \frac{(\bar{L} - L)^{1-\mu}}{1-\mu}
\] (5c)

\[\hat{Y}_s = Y_s \left( \frac{L_s}{L^F} \right)\]

\[X_s = A_s + \hat{Y}_s - h(\hat{Y}_s) - z_s(A_s, \hat{Y}_s)\]

\[A_{s+1} = (1 + r)(X_s - C_s) - g(\hat{Y}_s + r(X_s - C_s))\]

\[A_s \geq 0, \forall s\]

\[L_s^\text{min} \leq L_s \leq L_s^\text{max}, \forall s\]

The value function is equal to the sum of the expected utility of consumption
and leisure in each period from the current period \( t \) to the final period \( T \),
discounted by a factor of $\beta$ each period.\textsuperscript{15} The discount factor governs the utility tradeoff across periods – values closer to 1 reflect greater patience. The utility of consumption each period shown in Equation (5b) is assumed to take the Constant Relative Risk Aversion (CRRA) form, where $\gamma$ is the coefficient of relative risk aversion. With a utility function such as CRRA that has a convex marginal utility function (i.e. $u''(C) > 0$), there is a precautionary motive for saving, and greater uncertainty in the income process will induce greater saving.\textsuperscript{16} In Equation (5c), leisure is defined as the difference between a time endowment, $L$, and the amount of labor supplied, $L_s$. The parameter $\theta$ governs the relative weight placed on the utilities of consumption and leisure each period. The functional form for the utility of leisure is the same as for consumption, with curvature parameter, $\mu$.

Equation (5d) defines labor income, $Y_s$, as a function of fulltime income, $Y_s$, and the labor choice, $L_s$, scaled by an amount, $L^F$, such that if the household worked exactly, $L^F$, its labor income would be $Y_s$. We can think of the ratio $\left(\frac{L_s}{L^F}\right)$ as the fraction of a fulltime year worked or, alternatively, of the ratio $\left(\frac{Y_s}{L^F}\right)$ as an annual wage at which the household is compensated for each unit of labor, $L_s$. Equation (5e) defines the concept of “cash on hand” that is available to finance consumption and income taxes each period. To obtain cash on hand, $X_s$, assets are augmented by labor income but reduced by payroll taxes, $h \left(\hat{Y}_s\right)$, and costs of college attendance, $z_s \left(A_s, \hat{Y}_s\right)$, which may depend on assets and labor income through the financial aid formula described in Equations (1) – (4).\textsuperscript{17} The term, $z_s \left(A_s, \hat{Y}_s\right)$, can also incorporate the impact on cash on hand of loans taken out to fund educational expenses.\textsuperscript{18}

Equation (5f) shows how assets accumulate from one period to the next. Cash on hand is used to finance both consumption and the income taxes, $g \left(\hat{Y}_s + r \left(X_s - C_s\right)\right)$, that are due based on capital income and labor income.

\textsuperscript{15}In addition to the additive separability of consumption and leisure, the specification for the value function and the within-period utility makes two simplifying assumptions. The first is that there is no adjustment to the argument of the utility function for the size of the household, even after the child has left for college. The second is that there is no mortality risk and thus no accidental bequests. Further, there is no planned bequest motive. See Samwick (2010) for a similar model that includes mortality risk and bequest motives.

\textsuperscript{16}The use of the CRRA utility function is standard in both the empirical and theoretical literature on precautionary saving. CRRA utility means that a consumer remains equally willing to engage in gambles over a constant proportion of current wealth as wealth increases. An alternative, and perhaps more realistic assumption, might be that the consumer will accept larger proportional risks as wealth increases. See Kimball (1990) for a discussion and derivation of the key results for precautionary saving.

\textsuperscript{17}The payroll tax includes coverage for disability insurance, but the impact of disability is not modeled in this paper. See Chandra and Samwick (2008) for a similar model that includes mortality risk and bequest motives.

\textsuperscript{18}We do not include the income tax deduction for tuition and fees, for which households can claim a deduction of the lesser of tuition and fees or $4,000 ($2,000) if their MAGI is less than $130,000 ($160,000) for those married filing jointly (or half those thresholds for single filers). Hoxby and Bulman (2016) find no evidence that the post-secondary tax deduction affects college-going behavior or other aspects of college financing.
To avoid the complexity of an additional state and choice variable, the portfolio decision is restricted to a single riskless asset paying a return, \( r \), each period. Thus, the amount of saving is \( X_s - C_s \), and capital income is just \( r \times (X_s - C_s) \). The household’s taxes are calculated based on the 2013 tax schedule for a married couple with one child who does not itemize deductions and receives all capital income as interest or dividends rather than capital gains. Payroll taxes are assumed to be paid as the labor income is earned, prior to the consumption decision each period. Since income taxes depend on capital income and thus the outcome of the consumption decision during the period, they are assumed to be paid at the end of the period.

The last two elements of Equation (5) are the constraints on the optimal choices. Equation (5g) is the liquidity constraint, which requires assets, \( A_s \), to be positive in each period – the individual cannot borrow against future income to finance current consumption. This is a simplification that nonetheless acknowledges the credit constraints that prevent individuals from borrowing too heavily against future income outside of a secured or collateralized relationship.\(^{19}\) Equation (5h) imposes minimum, \( L_{s \min} \), and maximum, \( L_{s \max} \), constraints on labor supply. In retirement, \( L_s = L^F = L_{s \min} = L_{s \max} \), by assumption and without loss of generality.

The processes that describe the uncertainty in and evolution of fulltime income are as follows.

Before retirement:

\[
\begin{align*}
\ln(Y_s) &= \ln(P_s) + u_s \quad (6a) \\
\ln(P_{s+1}) &= \nu_s + \ln(P_s) \quad (6b) \\
u_{s+1} &= \rho u_s + \varepsilon_{s+1} \quad (6c) \\
\varepsilon_{s+1} &\sim i.i.d. N(0, \sigma^2) \quad (6d)
\end{align*}
\]

At retirement:

\[Y_{s+1} = RR \cdot Y_s\]  

After retirement:

\[Y_{s+1} = Y_s\]

Prior to retirement, the natural log of fulltime income is equal to the natural log of permanent income, \( P_s \), plus a shock to income, \( u_s \), that follows an AR(1) process. Permanent income is assumed to grow at a deterministic annual rate of \( \nu_s \), which may vary over time. The innovations to that AR(1) process are assumed to be independently and identically drawn from a normal distribution with mean zero and variance \( \sigma^2 \).\(^{20}\) In this model, the individual retires at a

\(^{19}\) The outcomes of the model are not greatly affected by allowing a fixed amount of unsecured borrowing. It also imposes the liquidity constraint directly, rather than including a much higher rate for borrowing that would discourage but not prohibit large amounts of unsecured borrowing. See Hurst and Willen (2007) for an analysis of consumption with a richer modeling of credit constraints.

\(^{20}\) In the simulations, the mean of the shock to the level (not log) of income is normalized to be one in all periods.
planned date that is known from the beginning of the working life. By assumption, there is also no impact of labor supply, $L_s$, on any current or future value of fulltime income. At retirement, fulltime income falls by a factor $(1 - RR)$, where $RR$ is the replacement rate. This replacement rate is meant to capture the income from Social Security, as it is assumed that if the household has an employer-provided pension, it is not a defined benefit plan and would those be captured in $A_t$.\footnote{These modeling choices for income are designed to avoid additional state variables and choices unrelated to the college savings decision. A richer model would include the risk of involuntary retirement due to health or other reasons and a choice over the retirement age based on economic factors. It would also be possible to include a better approximation of the Social Security benefit formula, at the cost of additional complexity in the model.}

### 3.2 Solution Method

The solution method for stochastic optimization problems with multiple state and control variables is discussed in detail in Carroll (2001). As a dynamic programming problem, Equation (5a) can be written recursively for any period $t$ as:

$$V_t(A_t, Y_t) = \max_{\{C_t, L_t\}} \left[ u(C_t) + \theta v(L_t) + \beta E_t [V_{t+1}(A_{t+1}, Y_{t+1})] \right]$$

The optimization proceeds backwards through time, from period $T$ to the first period, generating a series of rules for consumption and leisure that determine optimal consumption and leisure as a function of the state variables in that period. In periods before retirement that have both a leisure choice and a consumption choice, the period-by-period solution to Equation (9) can be found by solving a system of two first order conditions (one for $C_t$ and one for $L_t$) and three constraints in Equations (5g) – (5h). Thus, it is a system of 5 equations in 5 variables (the two choice variables plus the three Lagrange multipliers). To simplify the solution, we break each within-period problem into two sub-period problems, with the leisure choice occurring in the first sub-period and the consumption choice occurring in the second sub-period.

More formally, we define the two sub-period problems in period $t$ as follows. In the first sub-period, the household chooses labor supply, $L_t$, according to:

$$V L_t(A_t, Y_t) = \max_{L_t} \theta v(L_t) + V C_t(X_t, Y_t)$$

\begin{align}
\hat{Y}_t &= Y_t \left( \frac{L_t}{L^F} \right) \\
X_t &= A_t + \hat{Y}_t - h(\hat{Y}_t) - z_t(A_t, \hat{Y}_t) - g(\hat{Y}_t) \\
L_t^{\min} &\leq L_t \leq L_t^{\max}
\end{align}
In the second sub-period, the household chooses consumption, $C_t$, according to:

$$VC_t(X_t, Y_t) = \max_{C_t} u(C_t) + \beta E_t [VL_{t+1}(A_{t+1}, Y_{t+1})]$$  \hspace{1cm} (11a)$$

$$A_{t+1} = (1 + r (1 - g'(Y_t))) (X_t - C_t)$$  \hspace{1cm} (11b)$$

$$A_{t+1} \geq 0$$  \hspace{1cm} (11c)$$

In Equations (10) and (11), $VL_t(A_t, Y_t)$ and $VC_t(X_t, Y_t)$ are the value functions for the labor and consumption sub-period problems, respectively. The main change from the original formulation of the problem in Equation (5) is in the way income taxes are calculated, which must be approximated when there are two sub-periods. In the first sub-period, income taxes are collected on labor income assuming that capital income, which is determined in the second sub-period, is zero. This is the term, $g(Y_t)$, in Equation (10c). The income tax function is progressive in labor income, i.e. both $g' \geq 0$ and $g'' \geq 0$. This approximation means that, with capital income set to zero in the first sub-period, the marginal tax rate on labor income may be understated. In the second sub-period, income taxes are collected on capital income, $r(X_t - C_t)$, at a rate of $g'(Y_t)$, as shown in Equation (11b). The income tax on capital income is set equal to the marginal income tax rate based on fulltime income, $Y_t$, multiplied by the amount of capital income. The approximations mean that the marginal tax rate is constant at $g(Y_t)$ (rather than progressive) and uses the state variable, $Y_t$, rather than the prior sub-period’s choice variable, $Y^t$, as the base. This latter simplification is required in order to avoid adding $Y^t$ as an additional state variable in the second sub-period.

In this new formulation of the household’s problem, the first-order condition for the labor supply choice in the first sub-period is:

$$\theta v' (L_t) + \frac{\partial VC_t(X_t, Y_t)}{\partial X_t} \left( \frac{Y_t}{L_t^F} \right) \left( 1 - h' \left( \hat{Y}_t \right) - z_t Y_t A_t, \hat{Y}_t - g' \left( \hat{Y}_t \right) \right) = \mu^{\text{max}} - \mu^{\text{min}}$$  \hspace{1cm} (12)$$

The first term is the marginal utility of an additional unit of labor supplied, with $v'(L_t) < 0$. At an interior optimum, this disutility must be equal in magnitude to the gain in utility that occurs in the second sub-period due to the higher consumption made possible by this additional unit of labor supplied. This utility gain has three components – the marginal utility of another dollar of cash on hand to start the second sub-period, $\frac{\partial VC_t(X_t, Y_t)}{\partial X_t}$; the pre-tax "wage" from fulltime work, $\frac{Y_t}{L_t^F}$; and one minus the marginal tax rates on labor income due to the payroll tax, financial aid formula, and income tax, $\left( 1 - h' \left( \hat{Y}_t \right) - z_t Y_t A_t, \hat{Y}_t - g' \left( \hat{Y}_t \right) \right)$. 

11
The first-order condition for the consumption choice in the second sub-period is:

\[
u'(C_t) - \beta (1 + r (1 - g'(Y_t))) \left( E_t \left[ \frac{\partial V L_{t+1} (A_{t+1}, Y_{t+1})}{\partial A_{t+1}} \right] \right) + \lambda = 0 \quad (13)\]

The first term in the first-order condition is the marginal utility of an additional dollar of consumption in period \( t \). The second term is the discounted value of saving that dollar to be used in period \( t + 1 \). The dollar grows by the after-tax interest rate and has a marginal value of \( \frac{\partial V L_{t+1} (A_{t+1}, Y_{t+1})}{\partial A_{t+1}} \) at that time. This marginal value is uncertain because of the shock to income received in period \( t + 1 \). In this expression, \( r \cdot g'(Y_t) \) is the marginal tax on another dollar of saving. The marginal utility of a dollar of assets at time \( t + 1 \) is discounted back to period \( t \) utility by a factor of \( \lambda \). The difference between the marginal utility of consumption and the marginal utility of assets in the next period is zero at the optimal level of consumption.

We can use the Envelope Theorem to obtain analytical expressions for the terms that appear in these first-order conditions. Applying the Envelope Theorem to Equation (11a) yields an expression for \( \frac{\partial V C_t (X_t, Y_t)}{\partial X_t} \):

\[
\frac{\partial V C_t (X_t, Y_t)}{\partial X_t} = \beta (1 + r (1 - g'(Y_t))) \left( E_t \left[ \frac{\partial V L_{t+1} (A_{t+1}, Y_{t+1})}{\partial A_{t+1}} \right] \right) + \lambda \quad (14)
\]

which is equal to \( u'(C_t) \) by Equation (13). This substitution can be made in Equation (12) to get a new first-order condition for the labor supply choice:

\[
\theta \nu' (L_t) + u' (C^*_t) \left( \frac{Y_t}{L^F} \right) \left( 1 - h' \left( \frac{Y_t}{L^F} \right) - z^Y_t \left( A_t, \frac{Y_t}{L^F} \right) - g' \left( \frac{Y_t}{L^F} \right) \right) = \mu_{\text{max}} - \mu_{\text{min}} \quad (15)
\]

Note that the term, \( z^Y_t \left( A_t, \frac{Y_t}{L^F} \right) \), indicates that there is a financial aid tax on earning income while the child is in college. The higher is this financial aid tax, the lower the value of \( L_t \). Applying the Envelope Theorem to Equation (10a) yields an expression for \( \frac{\partial V L_t (A_t, Y_t)}{\partial A_t} \):

\[
\frac{\partial V L_t (A_t, Y_t)}{\partial A_t} = \frac{\partial V C_t (X_t, Y_t)}{\partial X_t} \frac{\partial X_t}{\partial A_t} = u' (C^*_t) \left( 1 - z^A_t \left( A_t, \frac{Y_t}{L^F} \right) \right) \quad (16)
\]

Advancing this equation to period \( t + 1 \) and substituting it into Equation (13) yields a new first-order condition for the consumption choice:
\( u'(C_t) = \beta (1 + r (1 - g'(Y_t))) \times \left( E_t \left[ u'(C'_{t+1}) \left( 1 - z^A_{t+1} \left( A_{t+1}, L^*_{t+1} \left( \frac{Y_{t+1}}{L} \right) \right) \right) \right] + \lambda \) \) 

At an interior optimum (i.e. one in which the liquidity constraint in Equation (11c) does not bind and thus \( \lambda = 0 \)), the marginal utility of consumption in period \( t \) is equal to the discounted expected marginal utility of consumption in period \( t + 1 \), accounting for the effects of both the after-tax interest rate in period \( t \) and the financial aid tax on assets in period \( t + 1 \), \( \left( 1 - z^A_{t+1} \left( A_{t+1}, L^*_{t+1} \left( \frac{Y_{t+1}}{L} \right) \right) \right) \). The higher is the financial aid tax, the lower is this term, and thus the higher is the value of \( C_t \) at which the first-order condition will hold.

When the household is in retirement, the only choice each period is for optimal consumption, and there is no remaining uncertainty in the income process. The solution begins in the last period of life, \( T \), when the problem is trivial because the household simply consumes all of its assets and after-tax income, yielding an optimal value for \( C_T \) as a function of the state variables, \( A_T \) and \( Y_T \). For all retirement periods prior to the last period of life, optimal consumption (when the liquidity constraint does not hold with equality) is given by a first-order condition analogous to Equation (17):

\[ u'(C_t) = \beta (1 + r (1 - g'(Y_t) + r (X_t - C_t))) \times \left( 1 - z^A_{t+1} \left( A_{t+1}, Y_{t+1} \right) \right) u'(C'_{t+1} (A_{t+1}, Y_{t+1})) \]

Note that the absence of income uncertainty means that there is no expectations operator around the marginal utility of consumption next period. Further, there is no need to approximate the income tax function in retirement periods. Finally, in the application of the model considered below, college expenses are assumed to occur before retirement in the model, so the \( \left( 1 - z^A_{t+1} \left( A_{t+1}, Y_{t+1} \right) \right) \) term is always 1 during retirement years.\(^{22}\)

Once the optimal consumption and leisure rules have been obtained, the model can be simulated forward by specifying initial values of the state variables, drawing random shocks to income, and applying the leisure and consumption rules to generate distributions of asset balances in each period. In the simulations below, the model is evaluated using the distributions generated based on 1,000 independent random draws of the income profile.\(^{23}\) The key outcome of

\(^{22}\)When the liquidity constraint that \( A_{t+1} \) cannot be negative is binding, then consumption in period \( t \) is given by \( X_t - g'(Y_t) r X_t \).\(^{23}\)The closest antecedent in the literature is the model of Dick, Edlin, and Emch (2003), who estimate preference parameters for education and saving to determine the asset reductions due to the financial aid system and simulate the asset and welfare changes that would result from changes to that system. The saving framework in that paper is based on a non-stochastic life-cycle model and thus cannot measure the insurance value of financial aid.
the model is a value for the expected value of $VL_1(A_1, Y_1)$, computed as the average value of this term across the 1,000 income profiles and a starting asset value of zero at the beginning of the work life. This expected value is a metric by which different financial aid systems can be compared, as those with higher values of $E[VL_1(A_1, Y_1)]$ are the ones in which the household is better off.

### 3.3 Model Parameterization

Our baseline parameters in Equations (5) - (8) are as follows. Economic life lasts 60 periods, with retirement in the 40th period, corresponding roughly to an adult life of ages 25 - 85. The child is born in the fourth period of economic life, with college starting in the 22nd period. The parameters of the income process are typical of those found in the literature, with deterministic growth of 1.5 percent per year ($\nu_s = 0.015$), a retirement replacement rate of 50 percent ($RR = 0.5$). The stochastic elements of the income process are an AR(1) coefficient of $\rho = 0.95$ and a standard deviation of the annual income shock of $\sigma = 0.15$. The interest rate, $r$, is 0.03, and the coefficient of relative risk aversion, $\gamma$, is set to 3. The curvature of the utility function for leisure, $\mu$, is also set to 3.

Tuition, which serves as the maximum value of the EFC, is $60,000, which corresponds to the total costs of attendance at highly selective institutions in

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24 The replacement rate of 50 percent is consistent with the calculations in Biggs and Springstead (2008), who use the Social Security Administration’s Modeling Income in the Next Term (MINT) model to compute replacement rates from Social Security relative to multiple pre-retirement earning concepts. For our analysis, the “CPI average” method is the natural comparison, as average earnings are measured in real terms. Median benefit replacement rates under this concept are 56 and 53 percent for individuals and couples, respectively, who were age 64 – 66 in 2005. There are two reasons why these calculations are higher than our estimate. First, their model indicates that these replacement rates will fall over time (by about a percentage point per decade), and with our household sending kids to college under the 2013 EFC formula, they are about 25 years younger. Second, MINT model used by Biggs and Springstead (2008) considers only earnings up to the Social Security Maximum Taxable Earnings limit, inflating the replacement rate (particularly for households with incomes as high as in our simulations) relative to the replacement rate on total earnings that we use here.

25 These parameters are based on the estimates in Hubbard, Skinner, and Zeldes (1994), which are standard in the consumption-savings literature. They find $\rho = 0.95$ for all 3 education groups and roughly $\sigma = 0.15$ for the middle group, with variance decreasing with education. They also include a MA(1) term, which we ignore here under the notion that it would capture primarily measurement error. Note that Guvenen (2009) shows that estimates of Equation (6c) on panel data yield values of have $\rho$ that are upward biased when the parameter $\nu_s$ in Equation (6b) is constrained to have zero cross-sectional variance. We consider alternative specifications that modify these parameters below.

26 The curvature parameter, $\mu$, is related to the Frisch elasticity of labor supply. In the macroeconomic literature on consumption and labor supply, the more typical formulation of the disutility of labor is $\nu(L) = -\frac{L^{1+\frac{1}{\mu}}}{1+\frac{1}{\mu}}$, where $\eta = \frac{\nu'}{L^{1+\frac{1}{\mu}}}$ is the Frisch elasticity of labor supply. In our formulation, $\eta = \left(\frac{2 + \nu}{L}\right) \left(\frac{1}{\mu}\right)$. With values of $\underline{T} = 168$ total hours in a week and setting $L = L^F = 40$ hours for fulltime work, a value of $\mu = 3$ corresponds to a value of $\eta$ of about 1, which is intermediate between the micro- and macro- estimated elasticities found in the literature. See Reichling and Whalen (2012) for a review.
2013, the year we use for our EFC and income tax schedules. In Equation (3), \( j = k = 1 \), in the baseline, to implement the EFC formula as specified. In addition to financial aid offered through the EFC formula, the household is assumed to be able to take out a loan of \( \$10,000 \) per year of college at the riskfree rate of \( r = 0.03 \) to be repaid over a period of 20 years. For a patient household, the discount factor, \( \beta \), is 0.97. Initial assets \( A_1 \), are set to 0. In the simulations of the model, results are presented for a range of \( Y_1 \) from \( \$50,000 \) to \( \$200,000 \). The sensitivity of the main results to these parameter choices is documented below.

For each set of values of the other parameters in the model, we choose \( \theta \) so that the average value of labor supply during the working years is equal to \( L^F \). In presenting our results, we set \( L^F = 40 \), corresponding to the number of hours in a fulltime workweek. But the model does not distinguish between different methods of increasing our decreasing work – it could equally be a secondary earner entering or dropping out of the labor force as it is a primary earner increasing or decreasing hours per week or weeks per year. When the model is solved with variable labor supply, we set \( L_{s_{min}} = 35 \) and \( L_{s_{max}} = 45 \) in Equation (5h), corresponding to a range of ±12.5% in labor supply.

Figure 1 summarizes the age profiles of average income, consumption, assets, and college costs for our baseline parameters for a patient household, with \( \beta = 0.97 \). The top panel holds labor supply fixed at \( L^F \), and the bottom panel allows labor supply to be chosen optimally between \( L_{s_{min}} \) and \( L_{s_{max}} \). In Figure 1, \( Y_1 = \$100,000 \). Looking first at the top panel, income starts at this amount, and average income grows by 1.5 percent per year before retirement, at which time it falls by 50 percent. The figure shows income net of payroll and income taxes on labor income. Consumption is smoothed from working years into retirement – consumption is below income before retirement and then above income in retirement. Asset accumulation makes this possible, as assets rise to a peak of roughly 4 times pre-retirement income on the eve of retirement. Assets are depleted over the years in which the child is in college and then again, to zero, in the retirement period. Average consumption decreases only slightly during the college years.\(^{27}\) Over the life cycle, consumption rises as retirement approaches – with \( \beta \cdot (1 + r) \) approximately 1, it is the need for precautionary saving that generates the upward-sloping consumption profile.

In the bottom panel, all of the profiles are affected by the ability of the household to vary its labor supply. The flexibility to increase labor supply later in the working life in response to adverse income draws early in the working life allows the household to have higher consumption in those early years. With higher early consumption, the household accumulates fewer assets prior to retirement and prior to the college years. By itself, lower asset holding will lower the EFC the family pays for the child to attend college. The EFC is also lowered by the

\(^{27}\)The smoothness of consumption around the years of college attendance is consistent with the evidence in Souleles (2000), who shows in the Consumer Expenditure Survey that households’ non-education consumption does not decrease over the academic year in proportion to college expenditures in the fall. That is, at least over short horizons, the household is able to smooth consumption.
ability of the household to reduce its labor supply during the years when labor income will be included in the EFC calculation. This reduction in labor supply is evident in the income profile in the bottom panel of the figure and is presented in more detail in Table 4. The table shows average labor supply, relative to $L^F = 40$, in the periods before, during, and after the college years. Before college, labor supply averages 40.25 but falls to 36.30 during the college years before rising back to 40.67 after college. During college, the likelihood that the household is supplying the minimum amount of labor increases to 69.60 percent. After college, the likelihoods of being at the labor supply minimum or maximum are higher than before college, as continued shocks to income cause the variability of all quantities around the average profiles shown in Figure 1 to increase.

Figure 2 shows the analogous figure for the impatient household, with $\beta = 0.92$. In the top panel, with labor supply fixed, the average income profile is the same as in Figure 1. Due to impatience, asset accumulation is less rapid early in the life cycle. This generates a lower EFC but a decline in assets during the college years nonetheless. Assets peak at less than 2 times pre-retirement income and are more rapidly spent down in retirement, even as the consumption profile in retirement slopes downward. The comparison between the fixed and variable labor graphs in the two panels of the figure are as they were for the patient household – early consumption is higher, pre-retirement asset accumulation is lower, and labor supply and consumption both fall during the college years. As shown in Table 4, labor supply for the impatient household shows less variation over the working life than for the patient household, in part because asset accumulation has been lower than for the patient household and affords less of a buffer against income uncertainty.

Table 5 shows the differences in average EFCs and pre-college asset accumulation for the baseline parameters, holding labor supply fixed to better highlight the impact of saving on college costs. Four different parameterizations are shown, with and without income uncertainty for patient and impatient households. In the row for initial income of $100,000, the first two entries for EFC show that the difference in college costs for the households depicted in the top panels of Figures 1 and 2 is about $7,600 per year. Impatient households save less and thus receive more financial aid. The difference is comparable in magnitude for income levels up to $150,000, after which it begins to taper off. This is the "financial aid tax" that has been the focus of the prior literature, as a greater desire to save for identical income paths yields a higher cost of college.

The right panel of the table repeats the comparisons when there is no income uncertainty. Without uncertainty, the patient household has a lower EFC.

28 Carroll (1992) argues for discount factors as low as 0.9 to motivate a Buffer-Stock model of saving in which households maintain a target saving rate rather than accumulate resources for retirement early in their life cycle. Cronqvist and Siegel (2015) use data on identical and fraternal twins to suggest that genetic differences account for about a third of observed differences in savings behavior across individuals.

29 The EFC may differ for each year in college, as income fluctuates and assets are spent or accumulated. The EFC value shown in the table is the equivalent annuity value of the four individual EFCs, using the assumed baseline interest rate of 3 percent.
at low income levels and a higher EFC at high income levels. With no uncertainty, there is no chance that low initial incomes become unusually high in mid-career and result in higher EFCs. Similarly, there is no chance that high initial incomes become unusually low in mid-career and result in lower EFCs. Looking across columns for a given initial income, without large differences in asset accumulation across patient and impatient households, the "financial aid tax" is negligible.

4 Model Results

This section calculates the insurance value of financial aid by solving the model described in Section 3 under the current financial aid formula and an alternative in which financial aid does not depend on income or assets. Instead, the college changes the cost of attendance by raising or lowering tuition but giving no other aid. This change effectively converts the potentially distortionary taxes on income and assets in the financial aid formula into revenue-equivalent lump sum taxes. In the absence of income uncertainty, such a change would make the household worse off. To quantify this welfare loss, we could solve for the amount, $\delta$, such that by adjusting the average college cost, $E[z_s(A_s,Y_s)]$, by $\delta$, the household achieves the same ex ante utility, $E[VL_1(A_1,Y_1)]$, that it obtained under the current formula. In this case, $\delta$ is a compensating variation, and in the absence of income uncertainty, it will be negative. That is, the college could give less aid in the form of a lump sum than it gives on average through the current formula and leave the household as well off while saving on its aid budget.

However, when the household faces income uncertainty, the welfare gains due to the insurance value of financial aid will counteract and may even outweigh the welfare losses due to the disincentives to supply labor and save under the current formula. For each of our parameterizations, we solve the model three times – under the current formula, under a revenue-equivalent system in which the average amount of financial aid from the current formula is replaced by a lump sum, and under a utility-equivalent system in which that lump sum is adjusted to restore the household to same level of expected utility as under the current formula. Postive (negative) adjustments to the lump sum indicate that the household is better (worse) off under the current formula.

Our main results are presented in Table 6 for the patient household. The left panel shows the EFC, financial aid, and additional aid required to achieve the same utility as the current formula when there is no income uncertainty. As the level of initial income rises, financial aid falls and the EFC rises. For all levels of initial income in which financial aid is given, the compensating variation is negative, rising in magnitude from almost nothing at initial income of $50,000 to $1,864 at initial income of $100,000 to a peak of $3,589 at initial income of $150,000. That is, a household with initial income of $100,000 facing no income uncertainty would be willing to receive $1,864 less in aid, raising its EFC by 7.4% from $25,314 to $27,178, in order to avoid the distortionary taxes on labor.
supply and saving in the current financial aid formula.

The right panel shows the same information when the household faces income uncertainty with a standard deviation of the income shock equal to 15%. For all initial income levels, the compensating variation is now positive, indicating that the household would need to be compensated for losing the insurance value of financial aid. Note that these compensating amounts are net of the welfare costs of the disincentives illustrated in the left panel. The magnitude of this compensation rises from $5,105 for an initial income of $50,000 to over $11,000 for all initial income values of $125,000. These additional aid amounts represent further discounts in the cost of college of over 20%, even at the highest income levels. For an initial income of $100,000, the compensating variation of $9,783 represents a reduction in the EFC of about a third and an increase in financial aid of 30%. Put differently, a dollar of financial aid is worth $1.30 in lump sum discounts to tuition, because the financial aid is targeted to scenarios of low income and assets when its marginal value is higher.

Figure 3 shows the impact of this compensation on the average consumption profile of the patient household under the baseline parameters. The solid curve shows the same average consumption profile from the bottom panel of Figure 1. As shown in Table 6, financial aid is $31,581 on average. The long-dashed curve pertains to the alternative in which financial aid is $31,581 regardless of income and assets. The present value of lifetime resources, and therefore consumption, is the same in this “Revenue Equivalent” alternative. That the latter starts out lower and ends higher is due to the need for additional precautionary saving in the absence of the insurance provided by the financial aid formula. With that insurance under the current system, the household can spend more early in life when consumption is relatively low. The effect on asset accumulation is noticeable: households accumulate about 33 percent less under the current system on the eve of college-going compared to the “Revenue Equivalent” alternative. The lower asset accumulation results in lower consumption later in life. Thus, the current formula better allows the household to smooth consumption over time. The other dashed curve in Figure 3 is the average consumption profile that obtains when additional $9,783 of financial aid is provided to allow the household to achieve the same lifetime expected utility as under the current financial aid formula. The promise of this additional aid allows the household to raise consumption early in life under this "Utility Equivalent" alternative relative to the "Revenue Equivalent" alternative, but not to the extent as under the current formula.

Table 7 presents the analogous calculations for the impatient household under the baseline parameters. The welfare consequences of the income- and asset-contingent aspects of the current formula are comparable to those for the patient household. The negative welfare consequences of the distortionary taxes in the left panel with no income uncertainty are more pronounced, despite similar amounts of financial aid. When income is uncertain, compensating variations

Note that this reduction is due to both incentives, in the form of the implicit tax on assets and income, and insurance, with a lessened need to save for precautionary reasons.
are smaller at lower initial income levels and larger at higher initial income levels compared to the patient household. In all cases, because impatient households save less and thus receive more aid, the compensating variations are somewhat higher as a share of the EFCs. The impatient household with initial income of $100,000, for example, has a compensating variation of $9,033, which is 38.5% of the EFC and about a quarter of the existing financial aid award.

Table 8 presents a sensitivity analysis of the compensating variation as the key baseline parameters are changed in isolation. The first row of the table repeats the baseline results from Table 6 for a patient household with initial income of $100,000 facing income uncertainty with $\sigma = 0.15$. The next several rows consider changes in parameters that affect the amount of risk in the income profile or the household’s risk aversion. Changing these parameters should have a noticeable impact on the insurance value of financial aid. The compensating variation falls to $3,471 when the magnitude of the income shock falls to $\sigma = 0.10$ and to $-599$ with a shock of $\sigma = 0.05$. From these results, it is clear that a standard deviation of income shocks of slightly more than 0.05 is required to fully offset the negative welfare consequences of the distortionary taxes on income and assets in the financial aid formula, conditional on the other parameters. The next two rows change the persistence of the income shock, raising it with an AR(1) parameter of $\rho = 0.99$ and lowering it with $\rho = 0.90$. With higher persistence, the compensating variation doubles, and with lower persistence, its magnitude is halved. The next two rows lower the curvature of the utility functions, setting $\gamma = \mu = 2$ and then $\gamma = \mu = 1$. Less curvature makes the household less risk averse and more willing to substitute consumption and leisure intertemporally in response to changes in the budget constraint like an alternative financial aid formula. Lowering the curvature reduces the compensating variation to $6,439$ and $4,262$ for parameters of 2 and 1, respectively.

The final four rows change aspects of the budget constraint other than the risk profile. Lowering the retirement replacement rate from $RR = 0.5$ to $RR = 0.25$ generates more saving and less financial aid. The compensating variation rises in dollar terms, to $10,605$, but falls as a share of the EFC, to 31.8%. Increasing the size of the loan from $10,000$ to $30,000$ per year generates a slight decrease in aid and the compensating variation. Starting out life with initial assets of $100,000$ or $200,000$ instead of zero decreases financial aid, as some of the initial assets are saved for later periods. The compensating variations increase in dollar terms and decrease as a share of the EFC, by small amounts in both cases. Overall, Table 8 shows that the compensating variation is appropriately sensitive to assumptions about the household’s risk aversion and the degree of risk faced but generally robust to other changes in the budget constraint.
5 Discussion

The source of the welfare gain in our model is that in scenarios in which income realizations have been low, the current financial aid formula reduces the cost of attending college whereas the alternatives do not. Such estimates of the insurance value of financial aid will be sensitive to how we model other choices that might alleviate the burden of a high tuition payment in the face of low assets and income. Two choices are already included – increasing labor supply and taking out loans. Another would be to attend a college that costs less but (as must be the case in equilibrium) delivers lower benefits. To the extent that those lower benefits are lower earnings in the future, the timing of the cash flows mimics that of borrowing. Consumption falls less today but income to support future consumption (here thought of as the collective income of all members of the household) is lower.\footnote{In the model (as in reality), the loan opportunity exists in both the current formula and the alternatives and, perhaps as a result, the sensitivity analysis in Table 8 indicates that it has only a small impact on the compensating variation. If the borrowing opportunity is to better resemble sacrificing future earnings by going to a lower-cost school, it would be available only in the alternative formulas.}

There are several possible directions for further research, most of which would expand the complexity of the model beyond the framework of two choice variables and two state variables used here. First, we do not consider the real growth in the cost of attending college or the uncertainty surrounding it, even though this cost growth and uncertainty are prominent in policy discussions regarding access to higher education. Incorporating this growth and uncertainty would likely increase the insurance value of financial aid, since the EFC that comes from the Federal Methodology does not depend explicitly on college costs except as a maximum. Indirectly, the growth in the cost of attending college influences the indexing of the dollar values in the financial aid formulas.

Second, as noted above, not all assets are included in the measure of assets used in the financial aid formula. Retirement accounts are excluded from both the Federal and Institutional Methodologies, and home equity is also excluded from the Federal Methodology. A more general model of saving decisions in the presence of financial aid would include a state variable, say $W$, to represent excluded assets and a choice variable, say $m$, to reflect net saving in these excluded assets. The household's problem would then be to maximize the same objective function as in Equation (5) by choosing all three of $C(A_t, Y_t, W_t)$, $L(A_t, Y_t, W_t)$, and $m(A_t, Y_t, W_t)$ each period. This is a considerably more complicated problem. Similarly, we do not consider the growing industry of tax-advantaged college saving vehicles, like 529 plans and Coverdell accounts, that make saving for college relatively cheaper than in our model.

Third, we do not consider parents' payments for college in a more general context of intergenerational transfers to children. In such a framework, payments for college could be replaced by direct payments of cash if the value proposition in college becomes less favorable. They could also be replaced by
larger bequests, accumulated over a longer period and thus less of a drag on consumption during the working life.

Finally, as suggested by Dick and Edlin (1997), assets are reflecting lifetime income – information beyond what is available in current income. For a given level of current income, a low level of assets indicates that prior income shocks were sufficiently low that the household found it optimal to consume most of its income. Including assets in the financial aid formula allows the formula to partially insure against those prior shocks as well. Future work can consider how household welfare might be improved by introducing a measure of lifetime average earnings into the financial aid formula, allowing the sensitivities to assets and current income to be lessened.

6 Conclusion

Prior literature has conjectured, and provided mixed empirical evidence, that the implicit tax on assets in the financial aid formula creates a distortion in saving behavior. The literature has not considered as extensively that there is also an implicit tax on labor earnings in the formula. Our analysis is the first to recognize that these implicit taxes are merely one component of a standard incentives-insurance tradeoff. Using a stochastic, life-cycle model of consumption and labor supply in which households have precautionary, retirement, and college motives for saving, we show that in a model without income uncertainty, the implicit taxes have modest negative consequences for household welfare. When households face income uncertainty, the insurance value of a financial aid formula based on the current Federal and Institutional Methodologies is substantial. Across a range of parameterizations, we calculate that financial aid would have to increase by enough to reduce the net cost of attendance by about a third to compensate households for the loss of the income- and asset-contingent elements of the current formula. For our preferred parameterization, a dollar of financial aid delivered through the current formula is worth $1.30 in lump sum tuition discounts, due precisely to the targeting of the financial aid to scenarios in which the household has low income or assets and thus a greater marginal value of additional resources.

Without considering income uncertainty, the welfare losses due to the disincentives in the financial aid tax appear to be the economic costs of the explicitly redistributive financial aid formula. The formula transfers resources from those with (predictably) higher assets and income to those with (predictably) lower assets and income. However, when reasonable amounts of income uncertainty are added to the model, we see that across all levels of initial income, the compensating variation switches sign, and the progressive nature of the implicit taxes confers the benefits of insurance against that income uncertainty. That is, a justification for the implicit taxes on assets and income based on a desire to redistribute across \textit{ex ante} different groups is not needed. A justification based on a desire to redistribute within an \textit{ex ante} identical group suffices. Because of the insurance, every group distinguished by initial income can be
made better off *ex ante*. For reasonable parameterizations, the insurance value of means-tested financial aid more than offsets the disincentive costs of means-tested financial aid. Put differently, governments and institutions that provide financial aid according to this formula are able to give less aid than they would have to otherwise in order to keep the family of the college student equally well off.

The cost to the providers of financial aid of offering this insurance has not been modeled in the analysis, but such costs are likely to be small. Providers like governments and colleges offer financial aid based on this formula to a large population of students – those from families who have been lucky and those who have not. To the extent that the income uncertainty these families face is idiosyncratic in nature, the aggregation of aid awards across this population diversifies away the risk. To the extent that there are more systemic shocks to the families’ income, the long time horizons for governments and colleges allow them some opportunity to smooth these fluctuations over time.
References


Figure 1: Baseline Model Results, Patient Household

Fixed Labor

Variable Labor
Figure 2: Baseline Model Results, Impatient Household

### Fixed Labor

![Graph showing Fixed Labor results](image)

### Variable Labor

![Graph showing Variable Labor results](image)
Figure 3: Average Age-Consumption Profile by Financial Aid Formula, Baseline Parameters, Patient Household