The COVID-19 pandemic that began in the winter of 2020 has shaken the entire world, and the world trading system has certainly not been spared. Governments everywhere are now worried about the possibility of future pandemics that might be even more disrupting to normal patterns of life. A possible “worst-case” scenario is a new virus that arises in some country and that is so dangerous that it causes countries to shut their borders to all international trade (i.e., revert to autarky) in an attempt to prevent the virus outbreak in one country from turning into a global pandemic. You have been hired by the Biden Administration to evaluate such a scenario and settle a within-Administration dispute.

Specifically, a disagreement has erupted within the Biden economic team as to the short run and long run impacts on US relative prices of closing US borders to international trade. Biden Team 1 claims that the relative price of the US import good to its export good will rise by more in the short run than in the long run if the US closes its borders to international trade, while Biden Team 2 claims that this relative price will rise by less in the short run than in the long run. Contributing to the dispute is whether, in the short run it is producers who are constrained in their responses relative to the long run, or rather consumers who are constrained in their responses relative to the long run, or both.

Using the Basic Trade Model with the US initially trading freely with the world and exporting good y in exchange for imports of good x, and assuming that US preferences are homothetic (i.e., along any ray from the origin, all US indifference curves have the same slope), please answer the following 3 questions to resolve the dispute within the Biden Administration:

a) If in the short run producers cannot respond to relative price changes but consumers can respond fully, while in the long run both producers and consumers can respond fully to relative price changes, will the relative price of good x to good y in the US rise by more or by less in the short run as compared to the long run if the US closes its borders to trade (i.e., moves from free trade to autarky)?

b) If in the short run producers can respond fully to relative price changes but consumers cannot substitute one good for another in their consumption choices (i.e., they cannot change the ratio in which they consume the two goods in the short run), while in the long run both producers and consumers can respond fully to relative price changes, will the relative price of good x to good y in the US rise by more or by less in the short run as compared to the long run if the US closes its borders to trade (i.e., moves from free trade to autarky)?

c) If in the short run producers cannot respond to relative price changes and consumers cannot substitute one good for another in their consumption choices (i.e., they cannot change the ratio in which they consume the two goods in the short run), while in the long run both producers and consumers can respond fully to relative price changes, will the relative price of good x to good y in the US rise by more or by less in the short run as compared to the long run if the US closes its borders to trade (i.e., moves from free trade to autarky)?
Part II. Answer either question 1 or question 2 below.

1. (20 points) Demonstrate graphically using the Basic Trade Model (you can focus on a small country that is trading freely and takes world prices as given) that a country that finds itself trading in the direction contrary to its comparative advantage will lose from trade, by showing that:

   a) If the country has a comparative advantage in good y, and the government of the country is adamant that the country should nevertheless export good x and employs a production subsidy on good x sufficient to induce the country to export x, then the country will achieve a level of utility which is below its autarky utility level (i.e., it would lose from such trade); and

   b) If the country has a comparative advantage in good y, and the government of the country is adamant that the country should nevertheless export good x and employs a consumption tax on good x sufficient to induce the country to export x, then the country will achieve a level of utility which is below its autarky utility level (i.e., it would lose from such trade).

2. (20 points) Consider the Basic Trade Model with countries A and B trading freely and country A exporting good x in exchange for imports of good y from country B. Suppose that countries A and B have identical and homothetic preferences (i.e., the indifference curves for countries A and B are identical, and along any ray from the origin, all indifference curves have the same slope). Please answer the following two questions (note: you may appeal to your answer from part (a) to make answering part (b) easier).

   a) Show that if country B makes a transfer of purchasing power to country A, there is no “secondary burden or blessing” (i.e., the world relative price of good x to good y will not change as a result of the transfer, and the Ohlin case holds).

   b) Then show that if the transfer of purchasing power that country B makes to country A is in exchange for A providing B with some of the secrets to its superior technologies that have the effect of shifting out B’s PPF uniformly (i.e., radially – implying that B’s new PPF has the same slope along any ray from the origin as did its old PPF, but is just shifted out), then the world relative price of good x to good y will rise as a result of the transfer of purchasing power from B to A combined with the transfer of technology from A to B.
Part 1

Sketch of Answers
Midterm Exam Econ 39F 2021-22

We are told that the US is initially freely trading with the world and exporting good Y in exchange for imports of good X, and that US preferences are homothetic (meaning that along any ray from the origin, the slopes of all US indifference curves are the same).

If the US is forced into autarky by a future pandemic threat, we are asked to determine whether the relative price of good X to good Y in the US will rise by more in the short run or in the long run.

We are given 3 distinct definitions of "short run" and asked to answer this question for each definition.

A) In the short run producers cannot respond to relative price changes but consumers can respond fully, while in the long run both producers and consumers can respond fully. The graph on the next page shows that
The price of good X relative to good Y \( P_x/P_y \) will rise in the US by more in the short run than in the long run.

As the figure above reflects, in the long run the autarky price is as usual determined by the tangency between an indifference curve and the PPF. The relative price is given by the slope labeled \(-\frac{P_x}{P_y}\) in the long run.

In the short run, output is fixed at \( q_x^{SR} \) and \( q_y^{SR} \), the free trade levels. This means that the short run autarky price will be the slope of the indifference curve passing through the short run
production point \((q^s, q^s)\). [In the short run, this is just like an exchange economy with endowments \(e_x = q^s\) and \(e_y = 2q^s\).] That slope is labeled as \(-\frac{(px)}{(py)}\) in the figure. We know that

\[
\left(\frac{px}{py}\right)_{US} > \left(\frac{px}{py}\right)_{LR}
\]

because we are told that preferences are homothetic, hence the slope of each indifference curve as it passes through the ray from the origin drawn through the long run autarky point is the same (and equal to \(-\frac{(px)}{(py)}\) LR).

But then the curvature of the indifference curves guarantees that \(\frac{(px)}{(py)}_{US} > \left(\frac{px}{py}\right)_{LR}\), as claimed.

Hence, we can conclude that \(\frac{px}{py}\) will rise more in the short run than in the long run if "short run" is defined in this way.
b) In the short run, producers can respond fully to relative price changes but consumers cannot change the ratio in which they consume $x$ and $y$, while in the long run, both producers and consumers can respond fully. The graph below shows that, again, $p_x/p_y$ will rise in the US by more in the short run than in the long run.

As the figure above reflects, in the short run, consumers must consume $x$ and $y$ in the same ratio they do under...
free trade, which is given by \( \frac{C^F}{C^T} \)

the slope of the ray through the origin drawn in the figure. This implies that the short run autarky prices will be given by the slope of the PPF where a short run (right angle) indifference curve along the ray just touches the PPF, as depicted by the dashed right angle indifference curve. The slope of the PPF at that point is labeled \(-\frac{P_x}{P_y}\) in the figure. The long run autarky price is as usual, determined by the tangency between a long run indifference curve and the PPF. The relative price is given by the slope labeled \(-\frac{P_x}{P_y}\). We know that

\[
\frac{(P_x)_{aur}}{(P_y)_{aur}} > \frac{(P_x)_{LR}}{(P_y)_{LR}}
\]

because with homothetic preferences, the slope of the long run indifference curve that passes through the short run autarky point must be \(-\frac{P_x}{P_y}\), which is the slope of the PPF at the free trade production point.
And therefore flatter than the slope of the PPF at the short run austark point. But then given the tangency condition that defines \( \frac{(P_x)_{US}}{(P_y)_{LR}} \) as the curvature of the PPF guarantees that \( \frac{(P_x)_{US}}{(P_y)_{LR}} = \frac{(P_x)_{US}}{(P_y)_{LR}} \) as claimed.

Hence we can conclude that \( P_y \) will rise more in the short run than in the long run if "short run" is defined in this way.

c) If in the short run producers cannot respond to relative price changes and consumers cannot change the ratio in which they consume x and y, while in the long run both producers and consumers can respond fully, the graph on the next page shows that again \( P_y \) will rise in the US by more in the short run than in the long run. As the figure reflects, in the long run the overall price is, as usual, determined by the tangency between a (long run) indifference curve and the PPF. The relative price is given by the slope labeled \( \frac{(P_x)_{US}}{(P_y)_{LR}} \).
In the short run, output is fixed at $q^y_x$ and $q^y_y$, the free trade levels. But consumption must occur at the free trade ratio given by $C^F_{y,us}, C^F_{y}$, the slope of the ray through $q^F_{y,us}$, $C^F_{x,us}$. The origin drawn in the figure above. That means that there will be excess supply of good $y$ and $P_y = 0$, while $P_x > 0$, and supply will equal demand for $x$. In other words, we have $(P_x)^{Us}_{g,y} = \infty$, as depicted in the figure.
Therefore \((P_x)_{sr}^u \geq \infty > (P_x)_{LR}^u\) as claimed.

Hence we can conclude that \(P_x\) will rise more in the short run than in the long run if "short run" is defined in this way. [And for this case we did not have to use the homotheticity of preferences.]

**Part II**

1. We are asked to demonstrate that a country trading in the direction contrary to its comparative advantage will lose from trade, by considering two cases:

   a) The country has a comparative advantage in \(y\), but the government of the country exploits a production subsidy on \(x\) sufficient to induce the country to export \(x\) and import \(y\). We are told to assume that the country is small in world markets and trades freely taking world prices as given. The figure on the next page depicts the situation.
Absent the production subsidy to good $X$, the country would exploit its comparative advantage in $y$, $\frac{(P_y)}{(P_x)} \text{ exceeds } \frac{(P_y)}{(P_x)}^{W*}$

and export $y$ and import $X$ (trade triangle in dashed lines). But a production subsidy will raise the price received by producers of $X$ in the country (including payment) by $P_x^{*} = (1 + s)P_x$, and we have $P_y = P_x^{*}$, implying $\frac{P_x^{*}}{P_y} = (1 + s)\frac{P_x}{P_y}$.

Consumers, however, continue to face the
world prices $p_{dx} = p_{wx}$, $p_{dy} = p_{wy}$ as a result of free trade with the world. And their budget constraint is a line with slope $-\left(\frac{p_{wy}}{p_{wy}}\right)$ through the production point since we can write the budget constraint accounting for the funds to pay for the production subsidy as

$$p_{dx} C_x + p_{dy} C_y = p_{dx} x_x + p_{dy} y_y - 5p_{wx} x_x$$

which, using the pricing relationships, simplifies to

$$p_{dy} [y_y - C_y] = p_{dx} [x_x - C_x]$$

balanced trade at world prices. In the above figure, a production subsidy to $x$ has raised $\frac{p_{dx}}{p_{dy}}$ sufficiently high to $p_{dy}$ induce a production point on the MVP (tangent to the price line with slope $-\left(\frac{p_{dx}}{p_{dy}}\right)$) and a consumption point on the budget line passing through that production point (where an indifference curve is tangent to the budget line) that imply that the county is exporting $x$ ($x$) and importing $y$ ($y$). As depicted, the utility level $U$ is below the autarky.
utility level \( u^a \). And it is clear from the figure that for any product or subsidy to \( x \) that achieves trade in this direction (exports of \( x \) and imports of \( y \)), the consumption point will have to lie inside the PPF and hence on a lower indifference curve than the actually indifference curve labeled \( u^a \).

b) The country has a comparative advantage in \( y \), but the government of the country employs a consumption tax on good \( x \) sufficient to induce the country to export \( x \) and import \( y \). Again we are told to assume that the country is small in world markets and trades freely taking world prices as given. The figure on the next page depicts the situation.

Absent the consumption tax on good \( x \), the country would exploit its comparative advantage in \( y \) \( (\frac{p_y}{p_x} > \frac{w_x}{w_y}) \) and export \( y \) and import \( x \) (trade triangle in dashed lines). But a consumption tax \( t \) will raise the (tax-inclusive) price that
Consumers in the country pay for good $X$,

$$p_{dc} = (1 + t) p_x$$

and we also have $p_{dc} = p_y$,

implying

$$\frac{p_x}{p_{dc}} = \left(1 + t\right) \frac{p_x}{p_y}$$

Producers continue to face the world prices $p_{dx}^w = p_x$ and $p_{dy}^w = p_y$ as a result of free trade with the world. And the budget constraint for consumers is a line with slope $-\frac{(p_{dx})}{(p_{dy})}$ through the production point, since we can write the budget constraint accounting for the tax

\[ \frac{-p_{dx}}{p_{dy}} \]
revenue collected as

\[ p_x C_x + p_y C_y = p_x 2x + p_y 2y + t x C_x \]

Which, using the pricing relationships, simplifies to

\[ p_y [q_y - 2y] = p_x [C_x - 2x] \]

or balanced trade at world prices. In the above figure, a consumption tax on \( x \) has raised \( \frac{p_{xc}}{x} \) sufficiently high to induce a consumption point on the budget line. (where an indifference curve is tangent to the price line with slope \( \frac{p_{xc}}{p_y} \)) which implies less consumption of \( x \) than production \( a \) and hence exports of \( a x \) and imports of \( b y \). As depicted, the utility level \( U \) is below autarky utility level \( U^a \). And it is clear that any consumption point on the budget line to the northeast of the production point (which is required to generate exports of \( a x \)) must correspond to an indifference curve that lies below \( U^a \).
2. We are asked to consider two transfer scenarios in a setting where countries A and B trade freely. Country A exports good $x$ in exchange for imports of good $y$ from B, and where countries A and B have identical and homothetic preferences.

a) In the first scenario, B makes a transfer of purchasing power to A. We are asked to show that the world relative price of $x$ to $y$ will not change as a result of the transfer (i.e., there will be no secondary burden or blessing associated with the transfer). The figures on the following page depict the answer. Prior to the transfer, the two countries trade at the equilibrium relative world price $\left(\frac{P_x}{P_y}\right)$ depicted in the bottom figure.

In the top left figure, A is shown to be exporting $x$ in the amount $E^{x,0}_A$ and importing $y$ in the amount $E^{y,0}_B$ prior to the transfer, as reflected in its trade triangle. Similarly, prior to the transfer, B is shown to be importing $x$ in the amount $E^{x,0}_B$ a-d
Exporting y in the amount $E_y^0$, as reflected in its trade triangle. And as reflected in the bottom figure, the world relative price at which they initially trade, $(P_x^0)^w$, ensures that $E_x^0 = M_x^0$ (and through Walras' Law, that $E_y^0 = M_y^0$ too).

We analyze the transfer by asking at fixed world prices $(P_x^0)^w$ how the transfer shifts the export of x by A and the import of y by B. Due to the assumption of identical and homothetic preferences across the two countries, we know that A and B were both consuming on the same indifference curve from the origin and will continue to do so after the transfer at the fixed original world price ratio $(P_x^0)^0$. This is reflected in the top two figures, and the resulting transfer triangle (in dashed lines) is therefore identical across A and B with A increasing its consumption of x at the fixed $(P_x^0)^0$ by exactly the same amount as B decreases its consumption of x. And at the fixed $(P_x^0)^w$, production of x is unchanged in each country. Therefore, at fixed world prices $(P_x^0)^w$, A's export supply $F_x^A = 2_x - C_{x}$ shifts in by exactly the same amount as B's import demand $M_x^B = C_{x} - 2_x$. 

$E_x^0$ = $M_x^0$
Shifts in. This is reflected in the bottom figure, which implies that the transfer does not lead to any change in the equilibrium relative world price \( \left( \frac{P_A}{P_B} \right)_w = \left( \frac{P_A}{P_B} \right)_o \) as labeled in the figure.

b) In the second scenario, the transfer of purchasing power that B makes to A is in exchange for a transfer of technology from A to B that shifts B's PPF out uniformly (radially). We are told that this means B's new PPF will have the same slope along any ray from the origin as did its old PPF but is just shifted out. We are asked to show in this scenario that the world relative price of \( x \) to \( y \) rises as a result of this transfer of purchasing power in exchange for technology.

To answer this, we can exploit our answer to part (a) by breaking this second scenario into its two component parts: first, a transfer of technology from A to B that shifts B's PPF out radially; and second, a transfer of purchasing power from B to A. For the fixed (new) PPF of B and of A, we have already answered the second step in part (a):
due to identical and homothetic preferences, we know from our answer to part (a) that the transfer of purchasing power will not alter relative world prices. So all we need to now show is that the transfer of technology will lead to a rise in the relative world price of X to Y. This is depicted in the figures on the following page. Initially, before the transfer of technology (and before the transfer of purchasing power), the two countries are trading according to the trade triangles with labels $E_x^0$ and $M_x^0$ for country A (top left figure) and $M_y^0$ and $E_y^0$ for country B (top right figure). And the bottom figure depicts the original export supply curve of A and import demand curve of B whose intersection determines the original equilibrium relative world price $(p_x^0)$. The transfer of technology shifts B's PPF radially outward. At fixed relative world prices $(p_x^0)$, this has no impact on A's export supply of X, so in the bottom figure A's export supply curve does not shift when it transfers technology to B. But at fixed world prices $(p^0)$, B's import demands will now change. To determine
how they will change, we use the fact that the PPF shifts radially outward and the fact that B's preferences are homothetic. The first fact implies that, at fixed world prices $(\ell^*_w, p^*_w)$, B's output will shift out along the ray through the origin in the top right figure that passes through B's original (i.e., with the old PPF) production point. The second fact implies that, at fixed world prices $(\ell^*_w, p^*_w)$, B's consumption will shift out along the ray through the origin in the top right figure that passes through B's original consumption point. As a consequence, B's new trade triangle (dotted lines) with its original trade triangle (solid lines) confirms this means that B's import demand for X will increase at the original world prices $(\ell^*_w, p^*_w)$, which in turn implies a shifting out of B's import demand curve for X, as depicted in the bottom figure and a consequent rise in the Equilibrium $(\ell^*_t, p^*_t)$, as we were asked to show.

Recalling that from how the transfer of purchasing power from B to A has no impact on $(\ell^*_w, p^*_w)$, as we showed in part (c), we are done.