Two weeks ago, the New York Times ran an article with the headline “6 Months Before Brexit, Many in U.K. Fear ‘It’s Looking Very Grisly’.” As the Times article noted:

“When Theresa May appears on stage at the Conservative Party’s annual meeting this week, it will take all her determination to drown out the ticking of an invisible clock. One hundred and eighty days stand between Britain and an uncontrolled exit from the European Union. Then it will be 179, 178 ... After two years of negotiation, Britain has reached a moment of consequence for the process known as Brexit. The insulating layer of time that had protected the country from a potentially failed divorce from the bloc is thinning. Soon, it will be gone, with the threat of major new trade restrictions closing in.”

Theresa May is getting nervous and she wants to know how bad this could get for the UK if there is indeed a “hard Brexit” (i.e., an uncontrolled exit from the European Union). Specifically, as she gears up for the final 6 months of negotiations with the European Union, May is interested in an estimate of what she should be willing to pay on behalf of the UK to avoid being pushed back to autarky (she knows that a hard Brexit is not really the equivalent of autarky, but she wants to be prepared for autarky just in case).

The economists at the UK Department for International Trade have been tasked with coming up with such an estimate, and you have been hired to assist them using the 2 good (x and y) 2 country (UK importing x and EU importing y) Basic Trade Model. They have data on the current UK import penetration ratio (the value of UK imports divided by the value of UK GNP, all measured in units of x at current prices), but that is all the data they have at the moment. They are trying to figure out what additional data they will need in order to provide Theresa May with a meaningful estimate of the compensating variation for the UK in moving from the pre-Brexit status quo to autarky, expressed as a fraction of UK GNP in the pre-Brexit status quo. Two issues have arisen, and they are seeking answers from you. Please advise:

(i) One group of economists within the UK Department for International Trade claims that the only additional information needed is the elasticity of the UK import demand curve, and that with this additional piece of information they can then use the UK import penetration ratio to provide either an upper bound or a lower bound on the desired compensating variation measure, depending on what that elasticity turns out to be. Use the Basic Trade Model diagrams to evaluate the validity of this claim.

(ii) A second group of economists within the UK Department for International Trade claims that, when coming up with bounds on the desired compensating variation measure, in addition to the elasticity of the UK import demand curve it would be helpful to also know the elasticity of the foreign (European Union) export supply curve that the UK faces. Use the Basic Trade Model diagrams to evaluate the validity of this claim.
Part II. Answer either question 1 or question 2 below.

1. (20 points) Consider a small x-importing country in the 2-good (x and y) Basic Trade Model. Suppose that this country wants to achieve a production target for good x, and in particular wants to produce a specific level of x that is higher than the level it would produce under free trade.

Rank the following policy options from best (lowest cost to the economy) to worst (highest cost to the economy): (a) a tariff on imports of good x set at the ad valorem level that achieves this production target; (b) a subsidy to the production of good x set at the ad valorem level that achieves this production target combined with a tax on consumption of good x set at the same ad valorem level as the production subsidy; and (c) a subsidy to the production of good x set at the ad valorem level that achieves this production target combined with a tax on consumption of good x set at the ad valorem level that raises just enough tax revenue to fund the production subsidy (you may assume that the consumption tax is set to be on the “right” side of the Laffer Curve, i.e., it is the lowest consumption tax that generate the desired revenue).

2. (20 points) Using the 2-good 2-country Basic Trade Model, (a) prove algebraically that Walras’ Law continues to hold in the presence of trade imbalances; and (b) show graphically that Walras’ Law continues to hold in the presence of trade imbalances (for this part, you can assume the Ohlin case to simplify your graphs).
(i) We are asked to use the Basic Trade Model diagrams to evaluate the validity of the claim that, with the UK import penetration ratio and information on the elasticity of the UK import demand curve, we can provide a meaningful estimate of the compensating variation for the UK in moving from the pre-Brexit status quo to autarky, expressed as a fraction of UK GNP in the pre-Brexit status quo. Specifically, the question before us is whether, with this information, we can say that the import penetration ratio is either an upper bound or else a lower bound on the desired compensating variation measure.

To assess the validity of this claim, let's first draw the Basic Trade Model diagrams for the UK illustrating the two cases: case (a) where the import penetration ratio is an upper bound and case (b) where the import penetration ratio is a lower bound. Then we can see...
While each case implies about the elasticity of UK import demand,

Case (a): Upper Bound

Here we illustrate an arbitrary indifference curve $U_{a,k}$ and arbitrary prices $(p_{a,k})^T$ for the UK where

$$\frac{P_x M_{x}}{G N P_{X}} \geq \frac{C V}{G N P_{T}}$$
Case (b): Lower Bound

Here we illustrate an arbitrary indifference curve $U^{UK}_T$ and arbitrary prices $(\frac{p^x_{t}}{p^T_{t}})_a$ for the UK where

$$\frac{CV}{GNP^UK_T} > \frac{P^T_{x}M^UK_{x}}{GNP^UK_T}$$
In both cases (a) and (b), the UK imports the volume $M_x$ at trading prices $(\frac{P_x}{P_o})_T$. The difference between the two cases is where we have positioned the autarky indifference curve $U_a$, and hence the autarky prices for the UK, $(\frac{P_x}{P_o})a$.

In case (a), $(\frac{P_x}{P_o})a$ is not that much greater than $(\frac{P_x}{P_o})_T$. In case (b), $(\frac{P_x}{P_o})a$ is much greater than $(\frac{P_x}{P_o})_T$.

Hence, in case (a), the price change from autarky to trade --- $(\frac{P_x}{P_o})a$ to $(\frac{P_x}{P_o})_T$ --- that induced the import volume change from zero to $M_x$, was a relatively small price change. And in case (b), the price change from autarky to trade --- $(\frac{P_x}{P_o})a$ to $(\frac{P_x}{P_o})_T$ --- that induced the import volume change from zero to $M_x$, was a relatively large price change.

This means that in case (a) the UK import demand is relatively elastic, while in case (b) it is relatively inelastic.
We can conclude that this group of economists is correct in their claim. If we know that the UK import demand is sufficiently elastic, then we can say that the UK import penetration ratio is an upper bound on the compensating variation measure desired by Theresa May. And if we know that the UK import demand is sufficiently inelastic, then we can say that the UK import penetration ratio is a lower bound on the compensating variation measure desired by Theresa May.

(iii) We are asked to use the Basic Trade Model diagrams to evaluate the validity of the claim of a second group of economists that the foreign (EU) export supply elasticity is also relevant— in addition to the UK import demand elasticity and the UK import penetration ratio—for putting bounds on the desired compensating variation measure. Using the diagrams of the Basic Trade Model, we can show that this claim is false.

To see this, let's suppose, for example, that we have measured the UK import demand
elasticity and we know we are in case (a) -- that is, the UK import demand is very elastic so the import penetration ratio is an upper bound on our desired compensating variation measure. Then, using our equilibrium determination diagram from the Basic Trade Model we can see that the elasticity of the EU export supply curve that generated the equilibrium $M_x^U$ that we observe is irrelevant to our calculations because it is irrelevant to $M_x^U$ and irrelevant to $(\frac{P_x}{P_M})_a$ as depicted below:
As depicted, the three EU export supply curves will all generate the same observed data -- a trading price \( (P_x) \) and a UK import volume \( M_x^u \) -- but the export supply curve labeled \( \text{EU}_x \) is very elastic while the export supply curve labeled \( \text{EU}_u \) is very inelastic and the export supply curve labeled \( \text{EU}_v \) is in the middle. But as long as \( M_x^u \) and \( (P_x) \) are given and as long as the elasticity of the UK import demand curve is given, the UK autoly price \( (P_x)^u_k \) will be given and the desired compensating variation measure will be the same, regardless of which of these three EU export supply curves happens to be generating the data.
Part II

10. We are asked to rank three policy packages for a small x-importing county to achieve a production goal of a higher $\bar{z}_x$, i.e., a $\bar{z}_x$ that is higher than the $\bar{z}_x$ under free trade. Let's use the Basic Trade Model to depict the impact of each policy package, and then we can compare.

a) an import tax on $x$

This is our basic tariff diagram.

\[ \frac{p_x}{p_y} = \frac{\frac{c_x}{p_x}}{\frac{c_y}{p_y}} \]

So that $p_x$ rises above $\frac{p_x}{p_y}$ to the level, as depicted, where production of $x$ is equal to the target level $\bar{z}_x$.

The cost to the economy is given by $\frac{EV}{p_x}$ as depicted.
(15) A subsidy to the production of $X$ combined with a tax on the consumption of $X$ set at the same ad valorem level as the production subsidy. This is a combination of the production subsidy and the consumption tax that we studied in class.

Let's first write down the price relationships. There are no tariffs or export taxes so producers can always sell at world prices $P^W_x$ and consumers can always buy at world prices $P^W_y$. And thus producers receive the subsidy and consumers must pay the tax. So we have:

$$p^d_x = (1 + s)P^W_x \quad p^d_y = P^W_y$$

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so

$$\frac{p^d_x}{p^d_y} = (1 + s)\frac{P^W_x}{P^W_y} \quad \frac{p^d_x}{p^d_y} = (1 + s)\frac{P^W_x}{P^W_y}$$

But we are also told that $S = \text{tax}$, so let's just use the notation

$$t = S = \text{tax}.$$ 

Then we have

$$\frac{p^d_x}{p^d_y} = (1 + t)\frac{P^W_x}{P^W_y} = \frac{p^d_x}{p^d_y}.$$ 

So just as with a tariff, domestic producer and consumer relative prices are equalized. Some can use $(\frac{p^d}{p^d})$. 
Therefore, the diagram looks just like the diagram for the task of part (a):

With $t = s = t(x)$ and

\[ \frac{\partial x}{\partial y} = \frac{\text{pdc}}{\text{pd}} \equiv \frac{p_x}{p_f}, \]

the diagram is exactly the same as for part (a).
(c) Same as part (b), but with the consumption tax set at a level that raises just enough tax revenue to fund the production subsidy.

We know from part (b) that when the consumption tax is set at the same ad valorem level as the production subsidy, the resulting policy package is equivalent to a tax which raises positive revenue. So we can conclude that we must reduce the consumption tax (the problem tells us to ignore the possibility of satisfying this revenue constraint by being on the wrong side of the Laffer curve, i.e., with a very high consumption tax that could raise the needed revenue with a high-tax-low-consumption outcome).

So we can conclude that for this policy package, $S > \text{tax}$.

So without pricing relationships from part (b), we now have

\[
\frac{\frac{\partial q}{\partial x}}{\frac{\partial c}{\partial y}} = (1 + s) \frac{p_w}{p_y} > (1 + t) \frac{p_w}{p_y} = \frac{\frac{p_d}{p_c}}{\frac{p_d}{p_c}}.
\]
Finally, before drawing the Basic Trade Model diagram, let's check the budget constraint:

\[
\text{Subsidy} = \frac{\text{Value of products}}{\text{Subsidy payments}} = \frac{\text{Value of consumption}}{\text{Subsidy payments}}.
\]

Substitute price relationships:

\[
(1 + s)P^w_x 2_x + P^w_y 2_y = 2P^w_x 2_x + t \cdot x P^w_x C_x = (1 + t \cdot x)P^w_x C_x + P^w_y C_y
\]

\[
\Rightarrow P^w_x [C_x - 2_x] = P^w_y [2_y - C_y]
\]

\[
\text{BT at work}.
\]
As the figure from part (c) depicts, with
\[ \frac{p_{dx}}{p_x} > \frac{p_{dc}}{p_y} \]
the consumption distortion is reduced relative to cases (a) + (b) where
\[ \frac{p_{dx}}{p_x} = \frac{p_{dc}}{p_y} \Rightarrow \]
and hence the \( \frac{EV}{p_x} \) depicted in the figure of case (c) is smaller than the \( \frac{EV}{p_x} \) depicted in cases (a) and (b).

So with
\[ \frac{EV}{p_x}(a) = \frac{EV}{p_x}(b) > \frac{EV}{p_x}(c) \]
we can conclude that the policy package in case (c) is the best while the policy packages in cases (a) and (b) are equivalent to each other and provide a more costly way to achieve \( q_x^* \) than case (c).
2. (a) We are asked to prove algebraically that Walras' Law holds in the presence of trade imbalances. We are told to use the 2-good 2-county Ricardian Trade Model.

Let $TB^A$ denote the trade balance of County A with $TB^A > 0$ implying that A runs a trade surplus (transfers purchasing power to B) and $TB^A < 0$ implying that A runs a trade deficit (B transfers purchasing power to A).

Walras' Law says that if we find prices $P_x$ and $P_y$ that clear the $x$ market, so that

$$[C^A_x - q^A_x] = [q^B_x - C^B_x]$$

And if both A and B are satisfying their budget constraints--taking account of any transfers they make to each other--then the $y$ market must also clear at these prices, or

$$[q^A_y - C^A_y] = [C^B_y - q^B_y].$$
So suppose \( p_x \) and \( p_y \) clear the \( x \) market, so that
\[
(1) \quad [x^A_x - q_x^A] = [x^B_y - c_y^B].
\]

Let's write down the --- inclusive of trade imbalance --- budget constraints for \( x \) and \( y \), which must hold at any prices, including the \( p_x \) and \( p_y \) that ensure \((1)\) is
\[
(2) \quad p_x q_x^A + p_y q_y^A = p_x c_x^{A} + p_y c_y^{A} + TB^A
\]
\[
(3) \quad p_x q_x^B + p_y q_y^B + TB^A = p_x c_x^{B} + p_y c_y^{B}
\]

From \((2)\) and \((3)\), we get
\[
(2') \quad \frac{p_x}{p_y} [x^A_x - q_x^A] + \frac{TB^A}{p_y} = [x^B_y - c_y^B]
\]
\[
(3') \quad \frac{p_x}{p_y} [x^B_x - c_y^B] + \frac{TB^A}{p_y} = [c_y^{B} - q_y^{B}]
\]

But \((1)\), \((2')\) and \((3')\) then imply that
\[
[x^A_x - q_x^A] = [x^B_y - q_y^B]
\]
which is the market clearing condition for \( y \).
(b) We are asked to show graphically that Walras' Law holds in the presence of trade imbalances. And we are told that we can assume the Ohlin case to simplify our graphs, which is the case where the receiver of the transfer (i.e., the country running the trade deficit) spends the money just as the grantor of the transfer (i.e., the country running the trade surplus) would have spent the money. For the Ohlin case, the trade imbalance does not change the world prices, so our graph is nice and simple, as the following page shows, with country A running a trade deficit (receiving a transfer) and country B running the corresponding trade surplus (making the transfer).
Tr is A's trade deficit and B's trade surplus.
For the Ohlin case, the graphical confirmation of Walras' Law is easy, because with the base of the trade triangle and the base of the transfer triangle adding to the same length across County A's diagram and County B's diagram by the market clearing condition for $x$ as depicted, the height of the trade triangle minus the height of the transfer triangle is then also the same across County A's diagram and County B's diagram, which ensures that they market clears too.