The strong lensing convergence power spectrum as a dark matter probe

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LCDM on large scales

A 6(+) parameter model that is extremely successful on cosmological scales.
LCDM on small scales

Much harder to gauge LCDM on small scales (galactic/sub-galactic):

➔ Deep in the nonlinear regime at low redshifts; require N-body sims for predictions.
➔ Cannot ignore baryonic physics/astrophysics.
➔ Stellar formation becomes increasingly inefficient with decreasing halo mass.
➔ Dark matter models that behave like CDM on large scales can have very different effects on sub-galactic scales.
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Let’s use the smallest scales to falsify/corroborate the CDM paradigm.
Strong gravitational lensing

$y(x) = x - \alpha(x) = x - \nabla \psi(x)$

Source plane  Image plane

SDSS J1038+4849
Mao & Schneider (1998): perturbations caused by substructures near lensed quasars can explain anomalous fluxes.

Baryon-independent measurement.

In this talk I focus on galaxy-galaxy lenses.
Strong gravitational lensing as a small-scale probe

Dark structures lying close to an image (in projection) can distort it.

Reconstructed surface mass density

Vegetti et al. (2012)
Many studies originally assumed that perturbations were caused by **substructure** within the main halo doing the lensing, but it is now clear that the entire line of sight volume has **interloper** halos that can act as **perturbers**.
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Understanding whether the subhalos or interlopers are the dominant contribution is crucial when translating detections into DM constraints!
Strong gravitational lensing as a small-scale probe

Direct detection: resolve individual, pretty massive perturbers and infer properties (mass, position). Requires postprocessing and combining many images to convert detections into DM constraints.

Indirect/statistical detection: exploit CDM expectation of a large number of unresolved low-mass structures to statistically detect their collective perturbations on images (marginalizing over individual subhalo properties).
Strong gravitational lensing as a small-scale probe

**Direct detection**: resolve individual, pretty massive perturbers and infer properties (mass, position). Requires postprocessing and combining many images to convert detections into DM constraints.

**Indirect/statistical detection**: exploit CDM expectation of a large number of unresolved low-mass structures to statistically detect their collective perturbations on images (marginalizing over individual subhalo properties).
Convergence = surface mass density in units of critical density for lensing

What we want

Dark matter theories

(Sub)halo MF

Convergence power spectrum

Strong lens images

Deflection vectors on the lens plane
Let us start off considering the limit where all perturbers are substructure.

\[
\kappa \equiv \frac{\Sigma(r)}{\Sigma_{\text{crit}}}
= \nabla^2 \psi(r)
\]

\[
\Sigma_{\text{crit}} = \frac{c^2 D_{\text{os}}}{4\pi G D_{\text{ol}} D_{\text{ls}}}
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\kappa_{\text{tot}}(r) = \kappa_0(r) + \kappa_{\text{sub}}(r)
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\textit{Diaz Rivero+ (2018, 1)}
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\[ \kappa_{\text{tot}}(\mathbf{r}) = \kappa_0(\mathbf{r}) + \kappa_{\text{sub}}(\mathbf{r}) \]

\[ \kappa_{\text{sub}}(\mathbf{r}) = \sum_{i=1}^{N_{\text{sub}}} \kappa_i(\mathbf{r} - \mathbf{r}_i, m_i, q_i) \]
Let us start off considering the limit where all perturbers are substructure.

\[ P_{\text{sub}}(k) = P_{1\text{sh}}(k) + P_{2\text{sh}}(k) \]

\[
P_{1\text{sh}}(k) = \frac{(2\pi)^2 \bar{\kappa}_{\text{sub}}}{\langle m \rangle \Sigma_{\text{crit}}} \int dm \, dq \, m^2 \, P_m(m) \, P_q(q|m) \times \left[ \int dr \, r \, J_0(k \, r) \hat{\kappa}(r, q) \right]^2
\]
Substructure convergence power spectrum

\[ \approx \frac{\bar{c}_{\text{sub}} m_{\text{eff}}}{\Sigma_{\text{crit}}} \]

Largest subhalos
Concentration, subhalo mass function
Inner profile

High-k slope can distinguish e.g. cusp vs. core

Diaz Rivero+ (2018, 1)
Substructure convergence power spectrum

Diaz Rivero+ (2018, 2)

Cyr-Racine+ (2018)
Substructure convergence power spectrum

Exotic DM scenario:
low-mass cutoff
+ self-interactions

Diaz Rivero+ (2018)
What if we include line-of-sight (LOS) structure?

Recursive multi-lens plane equation

\[
\vec{y} = \vec{x}_1 - \sum_{i=1}^{N} \vec{\alpha}_i(\vec{x}_i)
\]

\[
\vec{x}_j = \vec{x}_1 - \sum_{i=1}^{j-1} \beta_{ij} \vec{\alpha}_i(\vec{x}_i), \quad \text{where} \quad \beta_{ij} \equiv \frac{D_{ij}D_s}{D_j D_{is}}
\]

\[
\kappa_i(\vec{x}) \equiv \frac{\sum_i (D_i \vec{x})}{\sum_{cr,i}}
\]

\[
\Sigma_{cr,i} \equiv \frac{c^2 D_s}{4\pi G D_i D_{is}}
\]

Sengul+ (2020)
LOS convergence power spectrum

Have to define an **effective convergence**, treating interlopers as effective subhalos.

\[
\kappa_{i,\text{eff}}(s) = \frac{\Sigma(s; m_{\text{eff},i}, r_{s,\text{eff},i}, \tau_i)}{\Sigma_{cr,l}}
\]

\[
P_1(k) = \left(\frac{4\pi G}{c^2}\right)^2 D_i^2 \int_0^{\chi_s} d\chi \frac{W_I^2(\chi)}{g^2(\chi)\chi^2} \\
\times \int dm n(m, \chi) m^2 \\
\times \int d^2 \tilde{q} \mathcal{P}(\tilde{q} | m, \chi) \left| \tilde{\phi} \left( \frac{D_i r_s}{g(\chi) D_\chi} k; \tau \right) \right|^2
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Convergence profile (e.g. projected NFW)

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Mass distribution
Convergence profile (e.g. projected NFW)

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\[ W_1 \equiv W_{fg} + W_{cp} + W_{bg} = \frac{f(\chi)D_{xs}\chi^2}{D_\chi D_s} \]

\[ f(\chi) = \begin{cases} 1 - \beta_{xl} & \chi \leq \chi_l \\ 1 - \beta_{lx} & \chi > \chi_l \end{cases} \]

**Interloper selection function**

**Mass distribution**

**Convergence profile** (e.g. projected NFW)

**Other profile properties**

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Interloper selection function

Lensing kernel

\[ \tau_{\text{s,eff}}(\chi) = \frac{D_l}{g(\chi)D \chi} \tau_s \]

\[ m_{\text{eff}}(\chi) = f(\chi) \frac{\Sigma_{cr, l}}{\Sigma_{cr, \chi}} \left( \frac{D_l}{g(\chi)D \chi} \right)^2 m \]

Sengul+ (2020)
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LOS convergence power spectrum

Ratio of substructure to interloper power spectrum amplitude

- $f_{\text{sub}} = 0.4\%$
- $f_{\text{sub}} = 2\%$
- $f_{\text{sub}} = 4\%$

Fraction of dark matter halo mass in substructure $f_{\text{sub}}$

Sengul+ (2020)
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Error due to projection?

\[ \kappa_{\text{div}} = \kappa_{\text{eff}} \equiv \frac{1}{2} \nabla \cdot \vec{\alpha} \]

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Sengul+ (2020)
GL is the best baryon-independent way we have of probing the low-mass end of the HMF (and only way outside the LG), and consequently probing an untested regime in CDM.

The convergence power spectrum relates length scales to mass scales, bridging the gap between strong lens images and dark matter theories. It is sensitive to lower masses than direct detection methods.

The interloper contribution cannot be ignored: it likely dominates the signal for the SLACS and BELLS galaxy-galaxy lenses.

This is good news! The HMF is a cleaner probe of dark matter than the SMF, which is subject to messy astrophysics.
Questions?

Talk based on:

arXiv: 1707.04590 (ADR, F.Y.-Cyr-Racine, C. Dvorkin)
arXiv: 1809.00004 (ADR, C. Dvorkin, F.-Y. Cyr-Racine, J. Zavala, M. Vogelsberger)