

CAPTURING THE STRUCTURE OF JAPANESE MATHEMATICS LESSONS: SOME FINDINGS OF THE INTERNATIONAL COMPARATIVE STUDIES

Yoshinori Shimizu
Faculty of Education, Tokyo Gakugei University
shimizu@u-gakugei.ac.jp

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Abstract: This paper examines the findings of two international comparative studies on classroom practices to explore the characteristics of Japanese mathematics lessons focusing on how Japanese teachers approach to teaching mathematics in the structured problem solving mode and how their students perceive such lesson structure. First, some findings of the TIMSS Videotape Classroom Study are examined. A special attention is given to those findings that relates to the emphasis in the classrooms on discussing alternative solutions to a problem in the recurring lesson pattern, as the features that appear to make Japanese lessons different from the other two countries, Germany and the United States. Second, a preliminary analysis of the data from the Learner's Perspective Study is presented. The analysis of videotaped sequences of ten consecutive mathematics lessons in a public school in Tokyo suggests the need to identify the "lesson structure in the sequence" as compared with the lesson pattern identified by the analysis of a set of single lessons. The pattern relates to the connections constructed by the teacher between lessons and the structure of the actual lesson itself. Finally, based on the findings of two studies, the key characteristics of Japanese approach to teaching mathematics are discussed.

Introduction

The video component of the Third International Mathematics and Science Study (TIMSS) was the first attempt ever made to collect and analyze videotapes from the classrooms of national probability samples of teacher at work (Stigler & Hiebert, 1999; Stigler et al., 1999). Focusing on the actions of teachers, it has provided a rich source of information regarding what goes on inside eighth-grade mathematics classes in Germany, Japan and the United States with certain contrasts among three countries. The findings of the study include aspects of mathematics lessons as identified with a strong resemblance between Germany and the United States with Japan seemingly unique. One of the sharp differences between the lessons in Japan and those in the other two countries relates to how lessons were structured and delivered by the teacher. The structure of Japanese lessons was characterized as "structured problem solving".

The Learner's Perspective Study (LPS), on the other hand, is a nine-country study of the practices and associated meanings in "well-taught" eighth-grade mathematics classrooms with a focus on learner practice (David, 2001a). In part, the study is motivated by the postulated cultural specificity of teacher and by a strongly felt belief that the characterization of the practices of the mathematics classroom must attend to learner practice with at least the same priority as that

accorded to teacher practice. The data of this study includes videotaped classroom data for ten consecutive mathematics lessons and post-lesson video-stimulated interviews with the teacher and students in each of three participating eighth grade classes. The data set in the LPS has the potential to look more closely into the students' perception of lesson structure.

In this paper, based on the findings of the two international comparative studies, the author examines some characteristics of Japanese mathematics lessons, with a particular attention to how Japanese teachers approach to teaching mathematics in the structured problem solving mode and how their students perceive such lesson structures. Focusing on the features where Japanese lessons appear differently from the other two countries in the TIMSS Videotape Classroom Study, we shall examine how Japanese teachers intend to promote mathematical thinking in the classroom with the emphasis of discussing alternative solutions to a problem. Then, a preliminary analysis of the videotaped sequences of ten consecutive mathematics lessons in a public school in Tokyo, a subset of the data in the Learner's Perspective Study, is presented. The analysis is intended to explore the lesson structure in the sequence of lessons as compared with the lesson pattern identified by the analysis of a set of single lessons. Also, based on the analysis of post-lesson interviews, discrepancies in perceptions of lesson structures between the teacher and the students are described. Finally, reflecting on the findings of two studies, the key characteristics of Japanese approach to teaching mathematics are discussed.

Emphasis on Students' Thinking in the Classrooms: Findings of the TIMSS Video Study Goals of Japanese Lessons Reported by Teachers

In the TIMSS Videotape Classroom Study teachers' responses on the questionnaire were analyzed, "What was the main thing you wanted students to learn from today's lesson?" There was a significant difference between the reported goals of teachers in Japan and teachers in the other two countries. A majority of Japanese teachers reported that fostering mathematical thinking was the main goal for their lessons, while 55 percent of German teachers and 61 percent of U.S. teachers reported that development of skills was the main thing to be learned (See Table 1.). Teachers' responses were coded as "Mathematical Thinking" when they emphasized students' exploration, development, and comprehension of mathematical concepts, or the discovery of multiple solutions to a problem.

Table 1
Teachers' Responses on the Questionnaire about the Goal of Lesson (%)
(Stiglar et al., 1999, p.46)

	Germany	Unite States	Japan
Skills	55	61	25
Thinking	31	21	73

The result of teachers' questionnaire shows that the Japanese teachers described their goals of lessons in consistent with the goals of teaching mathematics described in the Course of Study in which fostering students' mathematical thinking is emphasized (Ministry of Education, Sports, Science, and Culture, 1989). Also, goal statements which are similar to what teachers described in the questionnaires as goals of their lessons can be found in the teacher's edition of textbooks with concrete descriptions of them in terms of mathematical content to be dealt in the lesson.

One of the Japanese mathematics lessons in the sample coded as "JP-012" in the study, for

example, was on the equal areas of triangles within parallel lines. The problem on which students worked in the lesson was the “land problem”, which asks students to change the boundary between two lands without changing the area of them. Students were expected to apply what they had learned in the previous lesson; the area of triangles obtained by using the same base but translating the vertex opposite the base along a path parallel to the base, thus keeping the height constant. In the teacher’s edition of a eighth grade textbook, three goals for the lesson on this topic are described from the learner’s perspective with an emphasis on understanding and thinking (Tokyo Shoseki, 1996, p.219).

- *Students can understand that the distance between two parallel lines is a constant.
- *Students can understand the area of two triangles that share the base and have the same height are equal and use this property to solve the problem
- *Students can change the shape of a polygon without changing the area of it.

There is a follow-up problem in the textbook that asks students to change a quadrilateral into a triangle without changing its area. The “land problem” is included in the teachers’ edition as a “related problem” which is expected to appear at certain point in the sequence of lessons. Also, software for the specific purpose is available, to demonstrate the dynamic movement of vertex of triangle without changing the base and height, for example, as was used in the videotaped lesson. Teachers can easily access to those materials designed as a whole for specific goals of lessons, that is, fostering mathematical thinking through students’ exploration or the discovery of multiple solutions to a problem. Thus, Japanese teachers could naturally respond to the question of the main thing they wanted students to learn from today’s lesson by reporting “mathematical thinking” as the main goal.

Thinking and Discussing Alternative Solution Methods for a Problem

In the TIMSS Videotape Classroom Study, tasks on which students were working during seat work were coded into three mutually exclusive categories; Practice Routine Procedures, Invent New Solutions/Think, and Apply Concepts in New Situations. As to the average percentage of seat work time spent in three kinds of tasks, Japan differed markedly from the other two countries, spending less time on practice of routine procedures during seat work and more time inventing new solutions or thinking about mathematical problems (See Table2.).

Table 2
Average Percentage of Seat Work Time Spent in Three Kinds of Tasks
(Stiglar et al., 1999, p.102)

	Germany	Unite States	Japan
Practice Routine Procedures	89.4	95.8	40.8
Invent New Solutions /Think	4.3	0.7	44.1
Apply Concepts in New Situations	6.3	3.5	15.1

Invent New Solutions/Think, in particular, was coded to describe tasks in which students had to create or invent solution methods, proofs, or procedures on their own, or in which the main task was to think or reason. The result shown in Table 2 suggests that Japanese teachers intended to organize a mathematics lesson with the inclusion of solution methods produced by their students to the problem posed. The expectation is that different students will come up with different solution methods and that they can learn from each other in classroom.

When we consider the difference of time spent on Invent New Solutions/Think between

Japan and the other two countries, therefore, it seems interesting to explore whether the lesson contains alternative solution methods and who provides the alternatives. In the TIMSS Videotape Classroom Study, whether an alternative solution method was presented by the teacher, or by students, during the course of each lesson was coded.

Table 3 shows the percentage of lessons that included alternative solution methods of each type. Table 4 shows the average number of alternative solution methods of each type presented in the lessons of the three countries (Stiglar et al., 1999, p.54). The U.S. lessons included significantly more teacher-presented alternative solution methods than the Japanese. Japanese lessons included significantly more student-presented alternative solution methods than either German or U.S. lessons. As shown in Table 3 and Table 4, the study identified the sharp contrasts between Japan and the other two countries with respect to the presentation of alternative solution methods.

Table 3
Percentage of Lessons That Included Teacher-presented and Student-presented Alternative Solution Methods (Stiglar et al., 1999, p.54)

	Germany	Unite States	Japan
Teacher-presented	14	19	7
Student-presented	12	8	42

Table 4
Average Number of Teacher-presented and Student-presented Alternative Solution Methods Presented per Lesson (Stiglar et al., 1999, p.54)

	Germany	Unite States	Japan
Teacher-presented	0.4	0.5	0.1
Student-presented	0.2	0.2	1.7

As was suggested by the results of the study, one of the striking characteristic of mathematics lessons in Japanese elementary and lower secondary schools relates to the frequent exposure of students to alternative solution methods to a problem (Shimizu, 2000). Discussing multiple solutions to a problem in a whole-class mode is a common style for teaching mathematics in Japanese schools.

For the inclusion of a discussion on the solution methods presented by the students in the class, the teacher needs to plan their lesson with a clear idea about both the topics to be taught and expected students' responses. "Kyozaikenkyu", in Japanese, that means analyzing the topic carefully in accordance with the objective(s) of a lesson, is a necessary part in the preparation of a lesson with a clear idea about the topic. It includes an analysis of the mathematical connections between the current topic and previous topics (and forthcoming ones in many cases), an analysis of mathematical content within the topic, anticipation for students' approaches to the problem to be presented, and planning of instructional activities based on them.

A mathematics lesson in Japan, which lasts fifty minutes in secondary schools, can typically be divided into several segments (Becker et al., 1990; Shimizu, 1999). A common organization of lessons is comprised of the following segments which often serve as the "steps" or "stages" both in teachers' planning and in teaching-learning processes (Shimizu, 1996).

Presentation of a problem; problem solving by students; a whole-class discussion about the methods for solving the problem; summing up by the teacher; (exercises/extensions). A simpler organization of lessons may also be used; an introduction, developments, and summary. A further analysis is needed to describe how those alternative solution methods are treated in such processes in order to attain the main goal of the lesson.

Lesson Structure Reconsidered: A Preliminary Analysis of the Learner's Perspective Study

Study Design of the Learner's Perspective Study

The Learner's Perspective Study (LPS) is a nine-country study of the practices and associated meanings in "well-taught" eighth-grade mathematics classrooms (Clarke, 2001a). In part, the study is motivated by the postulated cultural specificity of teacher practice (Stigler & Hiebert, 1999), and by a strongly felt belief that the characterization of the practices of the mathematics classroom must attend to learner practice with at least the same priority as that accorded to teacher practice. The nine participating research teams bring to the project the perspectives of their countries of origin: Australia, Germany, Hong Kong/China, Israel, Japan, The Philippines, South Africa, Sweden, and USA.

The technique for undertaking this research involved the development of complex "integrated data sets" that combined split-screen video records of teacher and students with transcripts of post-lesson interviews and copies of relevant printed or written material. Building upon the methodological precedent (Clarke, 2001b), this project integrates complementary analyses of the substantial international data set generated through the combined efforts of the participating researchers. Each participating country in the Learner's Perspective Study used the same research design to collect videotaped classroom data for ten consecutive math lessons and post-lesson video-stimulated interviews with at least twenty students in each of three participating 8th grade classrooms. Data collection involved videotaping classroom and individual interview data, using the Complementary Accounts Methodology (Clarke, 1998). Images from two video cameras were mixed on-site to provide a split-screen record of both teacher and student actions in the classroom. Students interviewed after each lesson, using the video record as stimulus for their reconstruction of classroom event, were given control of the video replay and asked to identify and comment upon classroom events of personal importance. Among the methodologically most interesting aspects of the study has been the collaborative negotiation of the study design, the method of data collection, the general and local analyses, and the process whereby the various complementary accounts can be integrated into a rich and useful portrayal of mathematics classrooms internationally.

Among the most interesting analyses afforded by the data collected in the Learner's Perspective Study are those related to lesson structure. The following sections focus on the analyses related to two aspects of lesson structure. First, a preliminary analysis of the Japanese data reveals a difference of lesson structures between the one identified by the TIMSS Videotape Classroom Study and the one which is common in our data. By the analysis of videotaped sequences of ten consecutive mathematics lessons, we explore the "lesson structure in the sequence" as compared with the lesson pattern identified by the analysis of a set of single lessons. Second, while Japanese teachers may devote considerable effort into the planning and structuring of their lessons, these structures may not be perceived by the students. Discrepancies in perceptions of lesson structures between the teacher and the students will be explored through the analysis of post-lesson interviews with both of them.

Lesson Structure as Embedded in the Sequence of Lessons

In the TIMSS Videotape Classroom Study, certain recurring features that typified many of the lessons within a country and distinguished the lessons among countries were identified as “lesson patterns”. The following sequence of five activities has been described as the Japanese pattern: reviewing the previous lesson; presenting the problems for the day; students working individually or in groups; discussing solution methods; and, highlighting and summarizing the main point (Table 5). The study has shown that, to a significant extent, Japanese lessons can be characterized as structured problem solving. The teacher intends to have the students work on problem and then discuss solution methods, sharing important ideas found in both problem solving processes and the discussion.

Table 5
The Japanese Lesson Pattern (Stigler & Hiebert, 1999, pp.79-80)

- Reviewing the previous lesson
- Presenting the problems for the day
- Students working individually or in groups
- Discussing solution methods
- Highlighting and summarizing the main point

A Japanese lesson can also be regarded as a drama, which has a beginning and leads to a climax. In fact, one of the characteristics of Japanese teachers’ planning of lessons is the deliberate structuring of the lesson around a climax, or “Yamaba” in Japanese. How does the pattern relate to the characteristic “climax” central to each Japanese lesson? In the Learner’s Perspective Study, videotape and interview data were collected in relation to a sequence of ten lessons for each teacher studied. Analysis of the data has the potential to reveal both pattern and variation in the ways in which the teacher and students perceive the climax in each lesson.

A preliminary analysis for exploring the Japanese lesson structure form a different perspective has been conducted with the transcriptions of ten consecutive lessons from a public school in Tokyo, that is one of the three public schools selected for the data collection in the LPS. Transcriptions of ten lessons were coded by three persons independently, using the following thirteen coding categories which derived from the descriptors for the lesson pattern of the three countries (Stigler, & Hiebert, 1999). For example, “Checking Homework (CH)” is the category that was the descriptor for both the German pattern and the US pattern. An earlier version of the coding system was developed by examining the descriptors of lesson pattern in each country, being applied to the transcriptions of the first four lessons in the data set. The current thirteen coding categories were developed as the result of some modifications of them (Table 6).

Table 6
The Thirteen Categories for Analyzing Lesson Pattern

- Reviewing the Previous Lesson (RP)
- Checking Homework (CH)
- Presenting the Topic (PT)
- Formulating the Problem for the Day (FP)
- Presenting the Problems for the Day (PP)
- Working on Sub-problem (WS)
- Working on the Problem Individually or in Groups (WP)
- Presentation by Students (PS)

Discussing Solution Methods (DS)
 Practicing (P)
 Highlighting and Summarizing the Main Point (HS)
 Assigning Homework (AH)
 Announcement of the Next Topic (AN)

Table 7
 Lesson Patterns in Ten Lessons from the First Data

Lesson	Categories
J1-1	CH, (FP), PP, WP, PS, DS, AN, AH
J1-2	CH, (RP), (PS), (WP), PS, DS
J1-3	RP, PP, (WP), (PS), DS, WP, (PS), HS, P, AN
J1-4	PT, FP, PP, WP, PS, AH
J1-5	RP, WP, PS, DS, AH
J1-6	CH, RP, (PP), WP, WS, (DS), (HS), P
J1-7	RP, (PP), (PS), (DS), (HS)
J1-8	RP, PT, (PP), PS, DS, (P), AH
J1-9	PT, WP, PS, (DS), HS, HS, DS
J1-10	P, WP, PS, DS, HS, P, WP, PS, AN

Note: Categories with the full agreement among all the three coders are shown without parenthesis. Categories with the agreement of two coders are shown with parenthesis.

Table 7 shows the lesson patterns found in the first data set of the LPS. In general, each lesson includes “reviewing the previous lesson” and “presenting the problems for the day” in the earlier parts, followed by such activities like “students working individually or in groups” and “discussing solution methods”. As the result shows, both the lesson J1-3 and J1-6, for example, have a similar pattern identified in the TIMSS Videotape Classroom Study.

On the other hand, the analysis of the first data also reveals a difference of lesson structures between the one identified by the first TIMSS Videotape Classroom Study and the one found in our data. For instance, “checking homework” and “assigning homework”, which were regarded as typical activities in both the German and the US lessons, were often found in the LPS Japanese data with a slightly different style. In such cases, homework tasks were treated as the main topic of entire lesson or as important building blocks for the next lesson. Also, we can see four “Assigning Homework” and three “Announcement of the Next Topic”, both of which were not included in the Japanese pattern.

A Closer Look at the Lesson Structure: Discrepancies in Perceptions of Lesson Structure between the Teacher and the Students

While Japanese teachers may devote considerable effort into the planning and structuring of their lessons around a “climax”, these structures may be perceived differently, or even may not be perceived, by the students. The methodology employed in the Learner’s Perspective Study offered students the opportunity in post-lesson video-stimulated interviews to “parse” the lesson they had just experienced. That is, the students were requested to identify for the interviewer those elements in the lesson that they felt to be significant. It is clearly possible that students identify as significant classroom events quite different from those intended by the teachers.

In the post-lesson interview of the lesson J1-5, the teacher identified nine elements in the lesson to be significant, while each of two students interviewed identified eight and seven elements respectively. Although the number of elements identified as felt to be significant are similar between the teacher and two students, the point they identified were different. Only four elements were identical among three of them. As for the lesson J1-7, the teacher identified twelve elements in the lesson to be significant, while one of the two students identified only three elements and the other student did eight. In this case, the numbers of elements are different between them.

Table 8
Elements in the Lessons Felt to Be Significant

Lesson	Teacher	Student 1	Student 2
J1-5	9	8	7
J1-7	12	8	3

As was mentioned earlier, one of the characteristics of Japanese teachers' planning of lessons is the deliberate structuring of the lesson around a climax. It is a striking finding of the analysis of the Japanese data that students in Japanese classrooms seem unaware of the occurrence of these climactic points or their intended significance.

In the Japanese lessons studied in the Learner's Perspective Study, students are expected to discuss solution methods to the problems for the day and share the main point(s) emerging from the discussion. Analysis of student interview data suggests that Japanese students perceive lesson structure differently from their teacher. Even when the perceptions of the lesson structure by the teacher and the students differ, however, the lesson itself proceeded with a minor modification of what the teacher had planned. The LPS data suggest that teacher and students may share invisible rules in the classroom about what should be valued and what should not be permitted.

Discussion

In the TIMSS Videotape Classroom Study the concept of "lesson script" was proposed as commonly accepted and predictable way of structuring a classroom session and sequencing its instructional activities. "The difference in the scripts undoubtedly follow from different instructional goals and are probably based on different assumptions about the nature of mathematics, the ways in which students learn, the appropriate role of the teacher" (Stigler & Hiebert, 1997, P.18). As the preliminary analysis of LPS data suggests, a different script may be used by the teachers depending on the their intention at each phase of the entire unit.

U.S. and German lessons analyzed in the TIMSS Videotape Classroom Study tend to have two phases: an initial acquisition phase and a subsequent application phase. In the acquisition phase, the teacher demonstrates and/or explains how to solve an example problem. In the application phase, students practice solving examples on their own while the teacher helps individual students who are experiencing difficulty. Japanese lessons appear to follow a different script. The cultural script for Japanese lesson was described in the study as follows. First, teacher poses a complex, thought-provoking problem. Then, students struggle with the problem. Various students present ideas or solutions to the class. The teacher summarizes the class' conclusions. Students practice similar problems (Stigler et al., 1999, P.136).

While students are working on the problem, the teacher moves about to observe students' work. During this time period, the teacher gives suggestions or helps individually those who are having difficulty. She or he also watches for students who have good ideas, with the

intention of calling on those students in a certain order in the subsequent discussion. Then, a whole-class discussion begins. In this discussion, students spend the majority of their time listening to the solutions proposed by their classmates, as well as presenting their own ideas. When discussing solutions to the problem, the teacher asks students to present alternative methods to solve the same problem. Presenting an idea, even a wrong one, is strongly encouraged and praised.

A common framework, that has a similar structure to the typical pattern of organization described above, is usually used by teachers when they are writing lesson plans (see Fig. 1). An underlying assumption of using such a framework is that it enables a teacher to give students opportunities for working on problem by themselves, even by individual or in a group, and for communicating ideas with their classmate.

Steps	Main learning activities	Anticipated students' responses	Remarks on teaching
Posing a problem			
Students' problem solving on their own			
Whole-class Discussion			
Summing up (Exercise/Extension)			

Fig 1: A common framework for writing lesson plans

Though they do not write lesson plans for their daily practice, Japanese teachers do have opportunities, at lesson study meetings, for example, for writing and reading lesson plan. Also, they can easily access a sample lesson plan for any particular topic that also includes expected students' responses to the problem to be posed. Actually, both the Ministry of Education and private textbook companies publish support materials for the teacher that include lesson plans. Experienced teachers and mathematics educators are invited to write sample plans for such support materials.

The cultural script for Japanese lesson identified by the TIMSS Video Study seems to fit quite naturally with such a typical framework for writing a lesson plan. It would be safe to say that one of the origins of such a cultural script would be in the tradition of use of such a framework by Japanese teachers in planning and implementing lessons. Again, an underlying assumption of using such a framework is that it enables a teacher to foster mathematical thinking through opportunities for thinking and discussing mathematical ideas with their classmate.

On the other hand, the data in the LPS suggest that an experienced teacher may be more flexible in following the lesson pattern, depending on the phase of the entire unit or on the states of students' understanding of the topic taught. The teacher in our data seemed to be able to follow the lesson pattern of structured problem solving, as was the case in both the lesson J1-3 and J1-6. She could also "break" the structure in order to incorporate homework as the main point or a building block for the next lesson.

The lesson pattern in the sequence of the lessons clearly relates to the connections constructed by teachers between lessons and to the structure of each actual lesson itself. The lesson pattern can be varied within the entire unit of topic, depending on the phase of sequence of lessons.

In other words, the lesson at the introductory phase of the entire unit can be in the structured problem-solving mode, whereas the lesson at the final phase of the unit can be focused on practicing what the students have learned. The analysis of videotaped sequences of ten consecutive mathematics lessons suggests the need to explore the “lesson structure in the sequence of lesson” in more details, as compared with the lesson pattern identified by the analysis of a set of single lessons in the TIMSS Videotape Classroom Study.

Final remarks

In this paper the author discussed some characteristics of Japanese mathematics lessons, based on the findings of two international comparative studies. Focusing on the features that appear to make Japanese lessons different from the other two countries in the TIMSS Videotape Classroom Study, we examined how Japanese teachers promote mathematical thinking in the classroom with the emphasis on thinking and discussing alternative solutions to a problem.

A preliminary analysis of the data from the Learner’s Perspective Study was also presented. The study focused on how Japanese teachers approach to teaching mathematics in the “structured problem solving” mode and how their students perceive such lesson structure. The analysis of videotaped sequences of ten consecutive mathematics lessons suggests the need to explore the “cultural script in the sequence” in more details. The analysis described in this paper has identified the discrepancies in perceptions between the teacher and the students. A further analysis of the LPS data is needed to clarify the discrepancies and associated meanings.

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