DIFFERENTIALS IN THE $\rho$-BOCKSTEIN SPECTRAL SEQUENCE

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1. $\rho$-Bockstein table

The accompanying table displays differentials in the $\rho$-Bockstein spectral sequence
\[ \text{Ext}^{**}_{A^*_2}(M^C_2, M^C_2)[\rho] \Rightarrow \text{Ext}^{**}_{A^R_2}(M^R_2, M^R_2) \]
discussed in [1]. It is a more complete version of [1, Table 5]. For an explanation of the general strategy of the computation, see [1, §5].

The $d_1$ differentials are omitted from the table because there is a large number of them, and they are simple to describe: in the range of $s + f - w$ degrees considered in the table, the $d_1$ differentials are described entirely by multiplicative relations applied to the following $d_1$ differentials.

<table>
<thead>
<tr>
<th>source</th>
<th>target</th>
<th>s</th>
<th>f</th>
<th>w</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>$\rho h_0$</td>
<td>0</td>
<td>0</td>
<td>$-1$</td>
</tr>
<tr>
<td>$\tau g$</td>
<td>$\rho h_0 g$</td>
<td>20</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>$\tau g^2$</td>
<td>$\rho h_0 g^2$</td>
<td>40</td>
<td>8</td>
<td>23</td>
</tr>
<tr>
<td>$\Delta c_0 d_0$</td>
<td>$\rho h_0 d_0 l$</td>
<td>46</td>
<td>11</td>
<td>25</td>
</tr>
<tr>
<td>$\Delta c_0 e_0$</td>
<td>$\rho h_0 e_0 l$</td>
<td>49</td>
<td>11</td>
<td>27</td>
</tr>
</tbody>
</table>

Description of the columns: A Bockstein differential $d_r(x) = \rho^r y$ is recorded by a line in the table as follows.

- **source**: $x$
- **target**: $y$
- **s**: stem of $x$
- **f**: Adams filtration of $x$
- **s-w**: coweight of $x$
- **s+f-w**: see $f$, $s - w$
- **diff length**: $r$, unless the class is a permanent cycle, in which case this field is 0

Some notes about shorthand employed in the table:

- Names of elements correspond to names in $\mathbb{C}$-motivic Ext. See https://s.wayne.edu/isaksen/files/2020/04/Adamscharts.pdf
for the names and degrees of C-motivic elements. In particular, t means \( \tau \) and D means \( \Delta \).

- If an element has \texttt{diff length} = 0, this means that the element is \( \rho \)-local.
- Starting in degree \( s + f - w = 18 \), the differentials are presented in the order they were computed. (This is relevant because there are many process of elimination arguments used in the computation.)

References