CLASSICAL AND C-MOTIVIC ADAMS CHARTS

DANIEL C. ISAKSEN, GUOZHEN WANG, AND ZHOULI XU

Abstract. This document contains large-format Adams charts that compute 2-complete stable homotopy groups, both in the classical context and in the C-motivic context. The charts are essentially complete through the 90-stem and contain partial results to the 95-stem.

This document contains large-format Adams charts that compute 2-complete stable homotopy groups, both in the classical context and in the C-motivic context. The charts are essentially complete through the 90-stem and contain partial results to the 95-stem.

The charts are intended to be viewed electronically. The authors can supply versions that are suitable for printing.

Justifications for these computations appear in [3], [6], [7], [9], [14], [15], and [16]. Older references that justify many of the classical computations include [1], [2], [4], [10], [11], [12], and [13].

This document supersedes [5], which included Adams charts for the cofiber of \( \tau \). The charts associated to the cofiber of \( \tau \) now appear in the separate manuscript [8].

1. Cohomology of the classical Steenrod algebra

This chart shows the cohomology of the classical Steenrod algebra, i.e., the \( E_2 \)-page of the classical Adams spectral sequence, through the 110-stem.

(1) Black dots indicate copies of \( \mathbb{F}_2 \).
(2) Vertical lines indicate \( h_0 \) multiplications.
(3) Lines of slope 1 indicate \( h_1 \) multiplications.
(4) Lines of slope 1/3 indicate \( h_2 \) multiplications.

2. The classical Adams spectral sequence

This chart shows the classical Adams spectral sequence. The chart is complete to the 90-stem, with partial results through the 95-stem.

(1) Black dots indicate copies of \( \mathbb{F}_2 \).
(2) Vertical lines indicate \( h_0 \) multiplications.
(3) Lines of slope 1 indicate \( h_1 \) multiplications.
(4) Lines of slope 1/3 indicate \( h_2 \) multiplications.

2010 Mathematics Subject Classification. 55T15, 55Q45, 14F42.

Key words and phrases. Adams spectral sequence, stable homotopy group, motivic stable homotopy group, cohomology of the Steenrod algebra.

The first author was supported by NSF grant DMS-1606290. The second author was supported by grant NSFC-11801082. The third author was supported by NSF grant DMS-1810638. Many of the associated machine computations were performed on the Wayne State University Grid high performance computing cluster.
3. The $E_{\infty}$-page of the classical Adams spectral sequence

This chart indicates the $E_{\infty}$-page of the classical Adams spectral sequence. The chart is complete to the 90-stem, with partial results through the 95-stem. Beyond the 64-stem, not all hidden extensions have been resolved; see [7] for more details. See Section 2 for instructions on interpreting the chart. In addition:

1. Unknown Adams differentials are indicated as dashed lines.
2. Red lines indicate hidden 2 extensions. The dashed red lines in the 54-stem indicate that there is a hidden 2 extension, but its target is not known precisely.
3. Blue lines indicate hidden $\eta$ extensions. The dashed blue lines in the 77-stem indicate that there is a hidden $\eta$ extension, but its source is not known precisely.
4. Green lines indicate hidden $\nu$ extensions.

4. The $E_2$-page of the motivic Adams spectral sequence

This chart indicates the cohomology of the Steenrod algebra, i.e., the $\mathbb{C}$-motivic Adams $E_2$-page, through the 110-stem. Adams $d_2$ differentials are shown through the 95-stem. For legibility, the chart is divided into two pages with different scales.

1. Black dots indicate copies of $M_2$.
2. Red dots indicate copies of $M_2/\tau$.
3. Blue dots indicate copies of $M_2/\tau^2$.
4. Green dots indicate copies of $M_2/\tau^3$.
5. Purple dots indicate copies of $M_2/\tau^4$.
6. Vertical lines indicate $h_0$ multiplications. These lines might be black, red, blue, or green, depending on the $\tau$ torsion of the target.
7. Lines of slope 1 indicate $h_1$ multiplications. These lines might be black, red, blue, or green, depending on the $\tau$ torsion of the target.
8. Lines of slope 1/3 indicate $h_2$ multiplications. These lines might be black, red, blue, or green, depending on the $\tau$ torsion of the target.
9. Red arrows indicate infinite towers of $h_1$ multiplications, all of which are annihilated by $\tau$.
10. Magenta lines indicate that an extension hits $\tau$ times a generator. For example, $h_0 \cdot h_0 h_2 = \tau h_1^3$ in the 3-stem.
11. Orange lines indicate that an extension hits $\tau^k$ times a generator, for some $k \geq 2$. For example, $h_0 \cdot h_0^3 x = \tau^2 h_0 c_0 g$ in the 37-stem.
12. Blue lines of slope $-2$ indicate Adams $d_2$ differentials.
13. Magenta lines of slope $-2$ indicate that an Adams $d_2$ differential hits $\tau$ times a generator. For example, $d_2(h_0 c_2) = \tau h_1^2 e_1$ in the 40-stem.
14. Orange lines of slope $-2$ indicate that an Adams $d_2$ differential hits $\tau^2$ times a generator. For example, $d_2(h_0 y) = \tau^2 h_0 c_0 g$ in the 37-stem.
15. Dashed lines indicate possible Adams $d_2$ differentials.
The use of color is well-illustrated by the element $h_2 g^2$ in the 43-stem. The dot is green, indicating that $\tau^3 h_2 g^2$ is zero. The outgoing blue lines indicate that $h_0 \cdot h_2 g^2$ and $h_2 \cdot h_2 g^2$ are annihilated by $\tau^2$. The incoming magenta line indicates that $h_2 \cdot \tau g^2$ equals $\tau h_2 g^2$, and the incoming orange line indicates that $h_1 \cdot Ph_3^2 h_5$ equals $\tau^3 h_2 g^2$.

5. The $E_3$-Page of the Motivic Adams Spectral Sequence

This chart indicates the Adams $d_3$ differentials on the $E_3$-page of the motivic Adams spectral sequence. The chart is complete through the 95-stem, with indicated exceptions. See Section 4 for instructions on interpreting the chart. In addition:
1. Blue lines of slope $-3$ indicate Adams $d_3$ differentials.
2. Magenta lines of slope $-3$ indicate that an Adams $d_3$ differential hits $\tau$ times a generator.
3. Orange lines of slope $-3$ indicate that an Adams $d_3$ differential hits $\tau^k$ times a generator for some $k \geq 2$.
4. Dashed lines indicate possible Adams $d_3$ differentials.

6. The $E_4$-Page of the Motivic Adams Spectral Sequence

This chart indicates the Adams $d_4$ differentials on the $E_4$-page of the motivic Adams spectral sequence. The chart is complete through the 95-stem. See Section 4 for instructions on interpreting the chart. In addition:
1. Blue lines of slope $-4$ indicate Adams $d_4$ differentials.
2. Magenta lines of negative slope indicate that an Adams differential hits $\tau$ times a generator.
3. Orange lines of negative slope indicate that an Adams differential hits $\tau^k$ times a generator for some $k \geq 2$.
4. Dashed lines indicate possible Adams differentials.

7. The $E_5$-Page of the Motivic Adams Spectral Sequence

This chart indicates the Adams $d_5$ differentials on the $E_5$-page of the motivic Adams spectral sequence. The chart is complete through the 95-stem. See Section 4 for instructions on interpreting the chart. In addition:
1. Blue lines of slope $-5$ indicate Adams $d_5$ differentials.
2. Magenta lines of negative slope indicate that an Adams differential hits $\tau$ times a generator.
3. Orange lines of negative slope indicate that an Adams differential hits $\tau^k$ times a generator for some $k \geq 2$.
4. Dashed lines indicate possible Adams differentials.

8. The $E_6$-Page of the Motivic Adams Spectral Sequence

This chart indicates the higher Adams differentials on the $E_6$-page of the motivic Adams spectral sequence. The chart is complete to the 90-stem, with partial results through the 95-stem. See Section 4 for instructions on interpreting the chart. In addition:
1. Blue lines of negative slope indicate Adams $d_r$ differentials for some $r \geq 6$. 
(2) Magenta lines of negative slope indicate that an Adams differential hits $\tau$ times a generator.
(3) Orange lines of negative slope indicate that an Adams differential hits $\tau^k$ times a generator for some $k \geq 2$.
(4) Dashed lines indicate possible Adams differentials.

9. The $E_\infty$-Page of the Motivic Adams Spectral Sequence

This chart indicates the $E_\infty$-page of the motivic Adams spectral sequence. The chart is complete through the 90-stem, with partial results through the 95-stem.

For clarity, hidden extensions by 2, $\eta$, and $\nu$ are not shown on this chart. See Section 4 for instructions on interpreting the chart. In addition:

(1) Green vertical lines indicate hidden $\tau$ extensions.
(2) Dashed lines of negative slope indicate unknown Adams differentials.

References
