ADAMS-NOVIKOV CHARTS

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ABSTRACT. This document contains large-format Adams-Novikov charts that compute the classical 2-complete stable homotopy groups. The charts are essentially complete through the 90-stem.

This document contains large-format Adams-Novikov charts that compute the classical 2-complete stable homotopy groups. The charts are essentially complete through the 90-stem.

The charts are intended to be viewed electronically. The authors can supply versions that are suitable for printing.

Justifications for these computations appear in [1] and [3]. In essence, they are entirely determined by an analysis of the C-motivic Adams spectral sequences for the sphere spectrum and for the cofiber of τ. See also [4] and [5] for computations in a smaller range.

This document supersedes [2].

1. $v_1$-periodic elements in the Adams-Novikov spectral sequence

This chart shows the $v_1$-periodic part of the Adams-Novikov $E_2$-page through the 110-stem. Traditionally, these elements have names involving the symbol α, but our notation for elements is incompatible with the Greek letter system. Rather, our names for elements are consistent with the names of elements in the C-motivic Adams spectral sequence for the cofiber of τ.

We have separated the $v_1$-periodic elements from the main Adams-Novikov $E_2$-page for legibility, since they are entirely understood.

1. Solid dots indicate copies of $\mathbb{Z}/2$.
2. Open circles indicate copies of $\mathbb{Z}/2^k$, for some $k \geq 2$. The value of $k$ is shown next to each circle.
3. Lines of slope 1 indicate $h_1$ multiplications.
4. Arrows of slope 1 indicate infinitely many $h_1$ multiplications.
5. Lines of slope $1/3$ indicate $h_2$ multiplications.

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2. $v_1$-periodic elements in the Adams-Novikov $E_\infty$-page

This chart shows the $v_1$-periodic part of the Adams-Novikov $E_2$-page through the 110-stem. Traditionally, these elements have names involving the symbol $\alpha$, but our notation for elements is incompatible with the Greek letter system. Rather, our names for elements are consistent with the names of elements in the $\mathbb{C}$-motivic Adams spectral sequence for the cofiber of $\tau$.

We have separated the $v_1$-periodic elements from the main Adams-Novikov $E_\infty$-page for legibility, since they are entirely understood.

See Section 1 for instructions on interpreting the chart. In addition:

1. Green lines indicate hidden 2 extensions.

3. $E_2$-page of the Adams-Novikov spectral sequence

This chart shows the Adams-Novikov $E_2$-page through the 110-stem, excluding the $v_1$-periodic elements. Our names for elements are consistent with the names of elements in the $\mathbb{C}$-motivic Adams spectral sequence for the cofiber of $\tau$. Our notation is incompatible with the Greek letter system.

1. Black dots indicate copies of $\mathbb{Z}/2$.
2. Red dots indicate copies of $\mathbb{Z}/4$.
3. Blue dots indicate copies of $\mathbb{Z}/8$.
4. Green dots indicate copies of $\mathbb{Z}/16$.
5. Purple dots indicate copies of $\mathbb{Z}/32$.
6. Lines of slope 1 indicate $h_1$ multiplications.
7. Lines of slope $1/3$ indicate $h_2$ multiplications.
8. Magenta lines indicate that an extension equals 2 times a generator. For example, $h_2 \cdot h_2 d_0$ equals $2 \cdot h_0 g$ in the 20-stem.
9. With one exception, orange lines indicate that an extension equals 4 times a generator. For example, $h_1 \cdot \tau g$ equals $4 \cdot h_2 g$ in the 23-stem. The exception is that $h_2 \cdot P^2 h_1 h_6 c_0$ equals $8 \cdot h_0 ^2 \cdot \Delta h_2 ^2 h_6$ in the 93-stem.

4. $d_3$ differentials in the Adams-Novikov spectral sequence

This chart shows the Adams-Novikov $d_3$ differentials through the 90-stem, excluding the $v_1$-periodic differentials. Our names for elements are consistent with the names of elements in the $\mathbb{C}$-motivic Adams spectral sequence for the cofiber of $\tau$. Our notation is incompatible with the Greek letter system.

See Section 3 for instructions on interpreting the chart. In addition,

1. Blue lines of slope $-3$ indicate Adams-Novikov $d_3$ differentials.
2. Dashed blue lines indicate possible differentials.

5. $E_4$-page of the Adams-Novikov spectral sequence

This chart shows the Adams-Novikov $E_4$-page through the 90-stem, excluding the $v_1$-periodic elements. The Adams-Novikov $d_3$ differentials are also indicated. Our names for elements are consistent with the names of elements in the $\mathbb{C}$-motivic Adams spectral sequence for the cofiber of $\tau$. Our notation is incompatible with the Greek letter system.

1. Black dots indicate copies of $\mathbb{Z}/2$.
2. Red dots indicate copies of $\mathbb{Z}/4$. 
(3) Blue dots indicate copies of $\mathbb{Z}/8$.
(4) Green dots indicate copies of $\mathbb{Z}/16$.
(5) Lines of slope 1 indicate $h_1$ multiplications.
(6) Lines of slope $1/3$ indicate $h_2$ multiplications.
(7) Magenta lines indicate that an extension equals 2 times a generator. For example, $h_2 \cdot h_2 d_0$ equals $2 \cdot h_0 g$ in the 20-stem.
(8) With one exception, orange lines indicate that an extension equals 4 times a generator. For example, $h_3 \cdot \tau g$ equals $4 \cdot h_2 g$ in the 23-stem. The exception is that $h_2 \cdot Ph_2 h_5$ equals $8 \cdot h_3^2 h_5$ in the 45-stem.
(9) Blue lines of slope $-5$ indicate Adams-Novikov $d_5$ differentials.
(10) Dashed blue lines indicate possible differentials.

For clarity, the chart also shows the possible $d_3$ differentials.

6. $E_6$-PAGE OF THE ADAMS-NOVIKOV SPECTRAL SEQUENCE

This chart shows the Adams-Novikov $E_6$-page through the 90-stem, excluding the $v_1$-periodic elements. The Adams-Novikov $d_r$ differentials for $r \geq 7$ are also indicated. Our names for elements are consistent with the names of elements in the $\mathbb{C}$-motivic Adams spectral sequence for the cofiber of $\tau$. Our notation is incompatible with the Greek letter system.

See Section 5 for instructions on interpreting the chart. In addition,

(1) Blue lines of indicate Adams-Novikov $d_r$ differentials for $r \geq 7$.
(2) Dashed blue lines indicate possible differentials.

For clarity, the chart also shows the possible $d_3$ and $d_5$ differentials.

7. THE $E_\infty$-PAGE OF THE ADAMS-NOVIKOV SPECTRAL SEQUENCE

This chart shows the Adams-Novikov $E_\infty$-page through the 90-stem, excluding the $v_1$-periodic elements. Our names for elements are consistent with the names of elements in the $\mathbb{C}$-motivic Adams spectral sequence for the cofiber of $\tau$. Our notation is incompatible with the Greek letter system.

See Section 5 for instructions on interpreting the chart. In addition,

(1) Red lines indicate hidden extensions by 2. The dashed red lines in the 54-stem indicate that there is a hidden 2 extension, but its target is not known precisely.
(2) Blue lines indicate hidden extensions by $\eta$.
(3) Green lines indicate hidden extensions by $\nu$.
(4) Dashed light blue lines indicate possible differentials.

Beyond the 64-stem, not all hidden extensions have been resolved; see [3] for more details.

For clarity, the chart also shows possible differentials as dashed lines.

REFERENCES

The α family in the Adams-Novikov spectral sequence.
The $\alpha$-family in the Adams-Novikov $E^\infty$-page