

reason and argument

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Notes on a Strange Argument

Let's say someone offers you the following argument on a Friday in September:

Today is Friday.
If today is Friday, then it is September.
Therefore, it is September.

She insists that you understand her claims, including the conditional, truth-functionally. I have shown this argument to many people, on Fridays in September, but I have never found someone who likes it. Everyone seems to think it's valid, but no one thinks it's sound. The problem is that it's hard to say the argument is unsound. It's a bad argument, but it's sound. Understanding this helps clarify a few important points about arguments and validity.

Arguments are series of statements that are intended to establish conclusions. An excellent way of establishing a conclusion is with a valid argument, because in valid arguments the truth of the premises guarantees the truth of the conclusion. Things got a little strange, in three ways, when we started to schematize statements and arguments, and use those schematizations to test for validity.

First, we learned how to schematize a statement by using sentence letters. This is meant to help us see the form of an argument, or how all of the statements in it relate to one another. But the schemata are strictly speaking meaningless! How can you understand how all the parts of an argument relate to one another by using meaningless letters? Don't the meanings of the statements in an argument play an important role in making it a

good argument? Yes and no. A good argument establishes its conclusion, and you can't establish a conclusion with false premises. Nor can you establish a conclusion with irrelevant, but true, premises. So at some point you need to look and see whether the premises are true, and relevant. This can't be determined without consulting their meanings. But if they are true, one way they can establish a conclusion is by being put together in a form that is accurately schematized by a formally valid argument. When an argument is formally valid, one cannot interpret its premises as being true without also interpreting its conclusion as true, regardless of what they mean. That interpretation is forced on you by the structure of the argument.

For example:

$$\begin{array}{r} a \rightarrow b \\ a \\ \hline b \end{array}$$

You interpret a as true, which forces you to interpret b as true, lest $a \rightarrow b$ turn out false. Alternatively, it is impossible to interpret the complex schema $[a \& (a \rightarrow b)] \rightarrow b$ as being false, no matter what truth values you give the components, and no matter what they mean. Notice that formal validity involves nothing but the assignment of truth values to sentence letters. The meanings are not important, but the conditionals are. Without the conditional in the above argument, it would not be formally valid.

Second, these conditionals are strange things. The symbol ' \rightarrow ' picks out a *truth-functional* relationship between two statements, no more. A schema of the form ' $a \rightarrow b$ ' is true if a is false or b is true, and false only if a is true and b false. These conditions say *nothing* about the meaning relationship between a and b. Specifically, they say nothing about whether a and b say something about the same topic, because schemata abstract from the meanings of statements. Because of this, we can invent any number of strange, but true, conditionals:

- If the moon is made of cheese then the grass is green.
- If the leaves fall in autumn, then Saturn has rings.

These conditionals aren't meaningless. They are just not impressively useful. They each say no more, but no less, than that either the antecedent is false or the consequent is true. You learn something when someone truthfully asserts such conditionals, but you don't learn much. Moreover, you do not learn anything that would be particularly useful in any ordinary conversation. We might use truth-functional conditionals sometimes, but we use ones that are useful for achieving our conversational goals. We also use non-truth-functional conditionals. Compare the following four:

If leaves fall in autumn, then Saturn has rings.
(truth-functional)

Necessarily, if leaves fall in autumn, then Saturn has rings.
(This is often called entailment.)

If leaves were to fall in autumn, Saturn would have rings.
(This is a subjunctive conditional.)

Leaves falling in autumn entails that Saturn has rings.
(Another way to express entailment.)

The last three on this list are much beefier claims about the world, requiring much more support. While the first is true, the entailments seem false, and the subjunctive conditional is just confusing. If you were tempted to deny the conditional "If it's Friday, then it's September" my guess is that you thought I was asserting something like one of the last three conditionals above. We often use formulations like the first one on this list to express conditionals like entailments (the second and fourth ones on the list), so it was perfectly reasonable to hear things that way. The problem is that I also insisted that you interpret what I say truth-functionally, and this proves difficult to do. When you interpret the conditional in the strange argument truth-functionally, it is true, because you heard the argument on a Friday in September.

The third strange thing is that sometimes the conditionals seem essential to an argument's validity, while in other cases they do not. Compare:

Normal argument: This is a Rolex.
Rolexes cost \$5k in the store.
I'll sell this for \$500.
Therefore, you get 90% off.

Strange argument: It's Friday.
If it's Friday, then it's September.
Therefore, it's September.

In the normal argument, you don't need a conditional premise at all. The three premises guarantee the truth of the conclusion. You could include a conditional premise, but it would be long, complicated, and unnecessary, so why do it? The only reason to include it is if you want to schematize that argument. Without a conditional in the schematization, the schema will not be *formally valid*. Compare:

a		a
b		b
c	vs.	c
		$(a \& b \& c) \rightarrow d$
Therefore, d		Therefore, d
Not formally valid		Formally valid

The only reason to introduce a truth-functional conditional into the Normal argument is so that its schematization comes out as formally valid. In the Strange Argument, however, without the conditional premise, the argument is invalid. This isn't a matter of formal validity yet. Without the conditional premise, the ordinary English argument is invalid. It is perfectly possible to have Fridays in November, so you can't effectively argue in favor of it being September by claiming that it's Friday, unless you include the conditional premise. Notice, however, that including the conditional premise results in a *sound* argument. That is,

including it gives us a valid argument with true premises. Soundness is typically just what we want in an argument! So why is this sound argument so deeply unsatisfying?

The argument is not unsatisfying because it is invalid, or because its premises are false. There is more to being satisfying than being sound. To see this, consider the following argument:

Boston is north of New York City.
Therefore, Boston is north of New York City.

This argument is sound. Its premise is true and if its premise is true its conclusion must be true as well. But this is an unsatisfying argument. Why? It seems merely to assume the truth of the conclusion, rather than arguing for it. This argument, in some sense, *begs the question*. Begging the question is a tricky thing to understand. For now, we can at least see that what makes this argument bad is that it doesn't seem to give a reason for believing the conclusion, so much as assert the truth of the conclusion. With that in mind, let's return to the Strange Argument:

It's Friday.
If it's Friday, then it's September.
Therefore, it's September.

I asked you to imagine I had introduced this argument on a Friday in September. Under those circumstances, you would have excellent reasons for thinking the first premise is true. Your inclination was to deny the second premise, because, as I said, you were thinking of it as an entailment:

Necessarily, if it's Friday, then it's September.

Or perhaps you thought of it as saying: All Fridays are in September. But it doesn't say either of those things. All it says is that either it is not Friday or it is September. With that in mind, consider this **very important question**: what reason do you have for thinking such a premise is true? What would convince you of its truth, now that we understand it as the truth-functional claim

it is? Well, first you have the fact that it's Friday. What else could count as evidence for this conditional? The only thing left is whether it is September. The conditional just says that it is either not Friday or it is September. So, to figure out whether that's true, you need to ask whether it is September. It is! Now you know the second premise of the argument is true. And you can see that the argument is sound.

But you only came to know the argument is sound by figuring out whether it is September. That is, your confidence in the premise is based on your confidence in whether it is September. But that's just what the conclusion of the argument asserts! You had to figure out whether the conclusion is true in order to learn whether the premise is true. Is this all bass-ackwards? Yes and no. This argument is unsatisfying not because it is unsound, but because it is a bit like the argument about Boston and New York. It seems to assume the truth of the conclusion more than arguing for it. The conditional premise contributes nothing more to the argument than the following premise would contribute:

It's Friday.
It's September.
Therefore, it's September.

This argument differs from the Strange Argument only insofar as it leaves the first premise with no work to do. This argument is just like the Strange Argument, however, in that you need to figure out the same stuff to know whether its premises are true, viz. that it's Friday and that it's September. The Strange Argument is unsatisfying because you need to know whether the conclusion is true in order to know whether one of the premises is true. Incidentally, now you see why we cannot delete the conditional from the Strange Argument.

Finally, notice that the Strange Argument can be modified into a more satisfying argument by switching one of its premises with its conclusion:

It's Friday.

It's September.

Therefore, if it's Friday, then it's September.

This argument is uninteresting, but it's not completely unsatisfying. If I want to know whether the truth-functional conditional in the conclusion is true, I want to know about Friday and September. The one problem with this argument is that the first premise is superfluous. The second premise suffices, because any truth-functional conditional with a true consequent is true.

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