Vibration Analysis of Gantry Structures

KSU Bridge Design Workshop

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Outline

- Project Description and Task
- Method of Analysis
- Summary of Findings
Project Description

- The KTA has a goal for open road tolling at their three largest toll plazas: Eastern Terminal, East Topeka, and Southern Terminal

- Open road tolling
  - Allows for vehicle traffic to pass through toll roads at high speeds without stopping with use of a KTAG
  - Provides tollbooths to the right of the main roadway to allow for customers without KTAG’s to pay
  - Implements video reinforcement to ensure payment of tolls

- KDOT was tasked to design the structures that will hold the technology for open road tolling
Task

- Perform a dynamic analysis and check that structures fulfill the following criteria:
  - Vibration should be limited to excursions no greater than 0.001” above 20Hz
  - No combined movement greater than 1” per millisecond
  - Overall sway should be less than 6”
- Performed analysis for Spans of:
  - 50’ gantries
  - 63’ gantries
  - 86’ gantries
  - Some held signs, others did not
Method-Overview

• Research
  ▫ **Code: Standard Specification For Structural Supports For Highway Signs, Luminaires, and Traffic Signals (11.7.4)** to determine the amplitude of truck gust forces
    • Truck Gust = 18.8*C*I = 18.8*1.2*1.0 = 22.56 (PSF)
  ▫ **STAAD analysis and limitations**
    • Can only vary nodal loads w.r.t. time
    • Can only have up to 136 different forcing functions

• Utilized STAAD Finite Element Models to analyze the structures
  ▫ Performs a response time history analysis by modal superposition
    • Uses mass modeling to determine the natural frequencies and mode shapes
    • The natural frequencies and eigen values are then used to solve for displacements under the effects of forcing functions using modal superposition

• Utilized excel to organize and synthesize STAAD output
Method-Mass Modeling

- Natural Frequencies – the frequency at which a system oscillates when not subjected to a continuous or repeated external force

  Resonance:
  - Tacoma Narrows Bridge

- Mode Shapes – the shape of which a structure oscillates at its natural frequency

  Modal Superposition:
Method-STAAD Model

- Used in house program to build gantry structure model.
- Used model’s loads for natural wind, and input truck gust loads
- Assumptions:
  - Truck Gust will act vertically along the bottom chord of the structure directly above the two 12 ft lanes. Applied as a pressure load.
  - The frequency of the wind gust is equal to the length of the truck, 44 ft, divided by the truck speed, 65mph. $\omega = 0.46$ HZ

\[\text{Harmonic Behavior Sinusoidal Wave}\]
Method-STAAD Model

- Used tributary area to idealize the structure such that all wind loads were represented as point loads
  - This allowed us to vary the wind load w.r.t. time
- Placed nodes at each of the camera locations and analyzed the structure’s deflections at those points
STAAD Model of Gantry Structure
Loading of Gantry Structure for Vibration Analysis
Mode Shape of Gantry Structure
SDOF System – Forced Motion

- **Equation of Motion:**

  \[ M\ddot{U} + C\dot{U} + KU = P \sin \bar{\omega}t \]

- **Complementary Solution:**

  \[ U_{complementary}(t) = e^{-\xi \omega t}(A \cos \omega t + B \sin \omega t) \]

- **Particular Solution:**

  \[ U_{particular}(t) = C_1 \cos \bar{\omega}t + C_2 \sin \bar{\omega}t \]

- **General Solution:**

  \[ U(t) = U_{complementary}(t) + U_{particular}(t) \]

- **Transient Response:**

  \[ U(t) = e^{-\xi \omega t}(A \cos \omega t + B \sin \omega t) + \frac{P}{K} \frac{(1-r^2) \sin \bar{\omega}t - 2\xi r \cos \bar{\omega}t}{(1-r^2)^2 + (2\xi r)^2} \]

- **Steady State Response**

  Graph courtesy of Chopra
MDOF System - Modal Superposition

- Equation of Motion:
  \[ \mathbf{M} \ddot{\mathbf{U}} + \mathbf{C} \dot{\mathbf{U}} + \mathbf{K} \mathbf{U} = \mathbf{P}(t) \]
  \[ \mathbf{U} = \mathbf{\phi Z} \]
  \[ \mathbf{M} \mathbf{\phi} \ddot{\mathbf{Z}} + \mathbf{C} \mathbf{\phi} \dot{\mathbf{Z}} + \mathbf{K} \mathbf{\phi Z} = \mathbf{P}(t) \]
- Pre-multiply equation by \( \mathbf{\phi}_i^T \):
  \[ \mathbf{\phi}_i^T \mathbf{M} \mathbf{\phi} \ddot{\mathbf{Z}} + \mathbf{\phi}_i^T \mathbf{C} \mathbf{\phi} \dot{\mathbf{Z}} + \mathbf{\phi}_i^T \mathbf{K} \mathbf{\phi Z} = \mathbf{\phi}_i^T \mathbf{P}(t) \]
- Due to the orthogonality of \( \mathbf{M} \) and \( \mathbf{K} \) while using Rayleigh Damping
  \[ \ddot{Z}_i + (\alpha + \beta \omega_i^2) \dot{Z}_i + \omega_i^2 Z_i = P_{eq i} \]
  \[ Z_i(t) = \frac{P_{eq i}}{\omega_i^2} \frac{(1-r^2) \sin \omega t - 2 \xi r \cos \omega t}{(1-r^2)^2 + (2 \xi r)^2} \]
  \[ Z_{i\text{ max}} = \frac{P_{eq i}}{\omega_i^2} \frac{1}{(1-r_i^2)^2 + (2 \xi r_i)^2} \]
- Using SRSS:
  \[ U_{i\text{ max}} \sqrt{(\phi_{i1} Z_{1\text{ max}})^2 + (\phi_{i2} Z_{2\text{ max}})^2 + \ldots} \]
Summary of Findings-STAAD Output

- Post-processing feature allowed us to see the overall sway of the structure (maximum deflection). All were less than 6”.
- The gantry structures had few to no natural frequencies above 20 HZ.

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<th>Mode</th>
<th>Frequency Hz</th>
<th>Period Seconds</th>
<th>Part. X %</th>
<th>Part. Y %</th>
<th>Part. Z %</th>
<th>Type</th>
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<td>0.000</td>
<td>0.004</td>
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Summary of Findings-STAAD Output

- The post-processor in STAAD allows you to graphically select nodes and then draws the dynamic displacement in three different graphs (x,y,z).
- Defined the time step to be 1 millisecond so that the data from the graphs could be easily compared with the criteria specifying that excursions be less than 1” per millisecond.

<table>
<thead>
<tr>
<th>Time</th>
<th>Displacement X</th>
<th>Displacement Y</th>
<th>Displacement Z</th>
<th>RESULTANT</th>
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<table>
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<tr>
<th>Time</th>
<th>MAX X</th>
<th>MAX Y</th>
<th>MAX Z</th>
<th>MAX RESULTANT</th>
<th>MAIN MODES</th>
<th>ABOVE 20 HZ?</th>
<th>EXCURSION &lt; 0.001&quot;</th>
<th>&lt; 1&quot; PER MILLISEC</th>
<th>&lt; 6&quot; SWAY</th>
<th>CRITERIA MET</th>
<th>Total Sway</th>
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<tr>
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</tbody>
</table>
Summary of Findings-Conclusions

- **50’ Structures**
  - No excursions above 20 HZ
  - Maximum displacement in 1 millisecond = 0.9”
  - Maximum sway = 3.2”

- **63’ Structures**
  - No excursions above 20 HZ
  - Maximum displacement in 1 millisecond = 0.006”
  - Maximum sway = 2.5”

- **86’ Structures**
  - No excursions above 20 HZ
  - Maximum displacement in 1 millisecond = 0.3”
  - Maximum sway = 3.6”

- All structures were found to meet the criteria specified by the KTA equipment.
- The forcing functions used represents truck pairs running in a train while natural wind gust is applied perpendicularly to the sign with a frequency close to that of the truck gust frequency. This situation is highly unlikely to occur, so we conclude that the structures will perform exceptionally for normal traffic and weather conditions.
Questions?

State Bridge Office