Definition: A Series is the sum of the terms of a sequence.

That is, given the sequence: \( a_n = 1, 3, 5, 7, 9, 11, 13, 15, 17, 19 \)

The series is: \( S_n = 1+3+5+7+9+11+13+15+17+19 = 100 \)

(This is called the \( 10^{th} \) partial sum because it is the sum of the first 10 terms of the sequence if it continued)

Part I. We are going to investigate Arithmetic Series today.

As the story goes, when Carl was 8 years old, he got in trouble in class for talking and being off task. His teacher sent him to the back of the room and told him to not come back until he had added all of the numbers from 1 to 100. He didn’t have a calculator (this was in the 1700’s). To be continued...

YOU add up all the numbers from 1 to 100. Use a calculator if you wish. Time yourself.

Sum = _______________            How long did it take you? _________________

Remember, Carl was only 8 years old... Well his teacher became perturbed when he came back to the group after less than a minute. Obviously, little Carl hadn’t gotten the answer yet and was lying that he was finished. She asked for his answer and he was correct! The teacher came to the conclusion that he must have done this task before and had memorized the answer. But NO! Carl had devised a formula (noticed a pattern) in the span of a few seconds that allowed him to quickly devise the answer. Upon explaining to his teacher how he got his answer, she was astonished. Little Carl’s full name was Carl Friedrich Gauss, and he during his life was regarded the world’s greatest mathematician.

So, how did little Carl come up with his answer? Let’s look.

First, let’s examine the SEQUENCE of numbers from 1 to 100.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ..., 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100

A) What is the first term? _____ What is the last term? _____

B) What is the second term? _____ What is the next to last term? _____

C) What is the third term? _____ What is the \( 2\text{nd} \) to last term? _____

Do you notice anything particular with lines A, B, and C above? If so, what is it?
Hopefully you noticed that if you added the terms listed in either line A, line B, or line C that your total was the same.

What is that total? ________ Will this be true for every pair of terms in the sequence? ________
How do you know?

Next, let’s think about how many of these “pairs” exist in our sequence from 1 to 100.

How many will there be? ________ Again, how do you know?

So now, if there are _______ pairs of numbers that all add up to _______, how might we get our sum more quickly than adding them all up in our calculator? Try it and see if it works.

Write the formula (in words) that Carl used to find the sum:

**Part II. Now, let’s see if it will work for other series…**

1. Sequence: 1, 2, 3, 4, 5, ..., 95, 96, 97, 98, 99
   Think about what’s different compared to before? Will it matter?

2. Sequence: 1, 3, 5, 7, 9,... 77, 79, 81, 83

3. Sequence: 1, 2, 3, 4, 5, ..., (n-4), (n-3), (n-2), (n-1), n
   This is the sum of the first n positive integers. (Gauss’ Formula)

4. Sequence: 96, 92, 88, 84, 80, ..., -8, -12, -16

**HOMEWORK:** Page 605 #39-45, 54-56