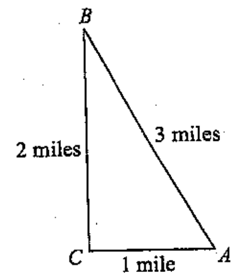
FOR STUDENTS

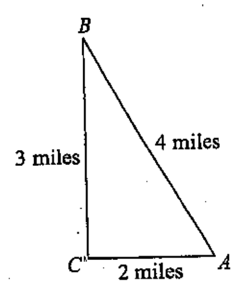
The Pythagorean Triples Review

Look at the right triangle below.



Do you notice anything? Imagine you were at home at point A and walked to school at point B, three miles away. On the way back home, first you walked two miles, from school at point B to your friend's house at point C, and then back home (point A) for another mile. The return trip also totals three miles. Does this make sense? Can these two trips be the same distance? Of course, not! Remember that "the shortest distance between two points is a straight line." This triangle is not even possible!

Now, look at the next figure.



Can we have a right triangle with sides 2, 3, 4? If we use the same thinking from the first triangle, $2 + 3 > 4$, it certainly could be a triangle. But, can it truly be a *right* triangle? The famous Greek mathematician and philosopher, Pythagorus, gave us one of the most important formulas in all of mathematics, for *right* triangle:

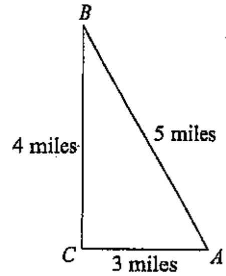
The Pythagorean theorem: $a^2 + b^2 = c^2$

where a and b are the lengths of the legs and c is the length of the hypotenuse: Clearly,

$$2^2 + 3^2 \neq 4^2$$
$$4 + 9 \neq 16$$
$$13 \neq 16$$

Therefore, although the figure with sides 2, 3, 4 is a triangle, it is *not* a right triangle.

Last, let's examine the triangle shown below with sides of 3, 4, and 5.



Check to see if it follows the Pythagorean theorem.

$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$
$$25 = 25$$

The "3-4-5" triangle is definitely a right triangle! This is one of an infinite set of famous right triangles called the *Pythagorean triples.* These right triangles are special because the sides are integers. Although there are many of these kinds of triangles, we will focus on four of the most common ones, shown in the following chart. Verify in the space provided, that these triangles do indeed satisfy the Pythagorean theorem.

THE PYTHAGOREAN TRIPLES			
(3, 4, 5)	________	________	________
(5, 12, 13)	________	________	________
(8, 15, 17)	________	________	________
(7, 24, 25)	________	________	________

Knowing these commonly used right triangles well can help in *quickly* solving many right triangle problems on the SAT. This could save you valuable time. For example, try the following question.

If you drive north 8 miles and then west 15 miles, how far are you from the starting point?

Because north and west form a right angle, this is simply a right triangle with sides 8, 15, 17. Therefore, you are 17 miles from the starting point. Although you could use the Pythagorean theorem to solve this problem, knowing the Pythagorean triples can save time.

But this is not all there is to it. Imagine you cut out a (3, 4, 5) right triangle from a piece of paper and went to a copying machine. You put the triangle on the glass and press the 200% button. What would you get? You'd get a larger triangle in the same proportions, namely a (6, 8, 10) triangle. This is known as a *similar* triangle. Does it follow the Pythagorean theorem?

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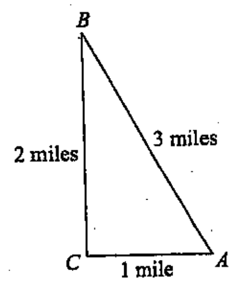

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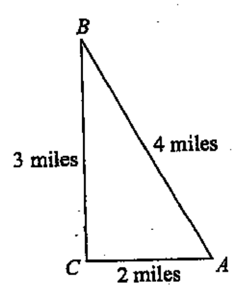

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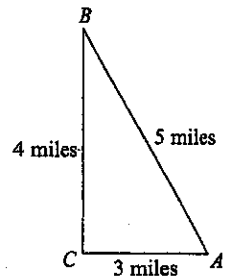

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THE PYTHAGOREAN TRIPLES

(3, 4, 5)	6, 8, 10	9, 12, 15	12, 16, 20
(5, 12, 13)	10, 24, 26	15, 36, 39	20, 48, 52
(8, 15, 17)	16, 30, 34	24, 45, 51	32, 60, 68
(7, 24, 25)	14, 48, 50	21, 72, 75	28, 96, 100

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