The Derivative– Spring 2018

Essential question: ***How can I apply derivatives to the real world?***

Day	Topic	Assignment
Mon Feb 5	Keeper 13 – The Definition of a Derivative - Find the Derivative using the definition of a derivative - Interpreting Derivatives	**Keeper 13 Worksheet – The Definition of a Derivative**
Tue Feb. 6	Keeper 14 – Secant, Tangent, and Normal Lines - Using the derivative to find tangent lines - Using the derivative to find normal lines - Finding secant lines	**Quick Homework Quiz** **Keeper 14 Worksheet – Secant, Tangent, and Normal Lines**
Wed Feb 7	Keeper 15 – Power and Sum and Difference Rule -Power Rule -Power Rule With Sums -Power Rule With Differences	**Quick Homework Quiz** Keeper 15 Worksheet – Power and Sum and Difference Rule
Thurs Feb 8	Keeper 16 – The Product and Quotient Rules -The product rule -The quotient rule -The power rule with the product rule -The power rule with the quotient rule	**Quick Homework Quiz** Keeper 16 Worksheet – The Product and Quotient Rules
Fri Feb 9	Keeper 17 – The Chain Rule - Chain Rule and Power Rule - Chain Rule and Product Rule - Chain Rule and Quotient Rule	**Quick Homework Quiz** Keeper 17 Worksheet – The Chain Rule
Mon Feb 12	Keeper 18 – The Derivative of logarithmic, Exponential, and Trigonometric Functions -Differentiate: a^x -Differentiate: ln(a) -Differentiate: e^x - Differentiate all Trig Functions	**Quick Homework Quiz** Keeper 18 Worksheet – The Derivative of logarithmic, Exponential, and Trigonometric Functions
Tues Feb 13	Keeper 19 – Implicit Differentiation - Finding first and second derivatives of variables in terms of x	Keeper 19 Worksheet – Implicit Differentiation
Wed Feb 14	Keeper 20 – L'Hopital's Rule - Finding Limits using derivatives - Finding Limits in indeterminate forms	Keeper 20 Worksheet – L'Hopital's Rule
Thurs 2/15	Review	***Complete Homework Packet***
Fri 2/16	**Test on The Derivative**	**Turn in all unit 3 homework assignments (worksheets only) for a unit daily grade!**

Precalculus Name________________________________ ID: 1

Assignment

Evens ONLY

Date________________ Period____

Use the definition of the derivative to find the derivative of each function with respect to x.

1) $f(x) = -3x^2 + 3x + 3$

$\lim_{h \to 0} \frac{-3(x+h)^2 + 3(x+h) + 3 + 3x^2 - 3x - 3}{h}$

$\lim_{h \to 0} \frac{-3(x^2 + 2xh + h^2) + 3x + 3h + 3 + 3x^2 - 3x - 3}{h}$

$\lim_{h \to 0} \frac{-6xh - 3h^2 + 3h}{h} = \lim_{h \to 0} -6x - 3h + 3$

$= \boxed{-6x + 3}$

2) $y = 3x^2 + 5x - 4$

$\boxed{y' = 6x + 5}$

3) $y = 5x^2 + 4x + 3$

$\lim_{h \to 0} \frac{5(x+h)^2 + 4(x+h) + 3 - 5x^2 - 4x - 3}{h}$

$\lim_{h \to 0} \frac{\cancel{5x^2} + 5xh + 5h^2 + \cancel{4x} + 4h + \cancel{3} - \cancel{5x^2} - \cancel{4x} - \cancel{3}}{h}$

$\lim_{h \to 0} \frac{5xh + h^2 + 4h}{h} = \lim_{h \to 0} 5x + h + 4 = \boxed{5x + 4}$

4) $f(x) = -3x^2 - 5x + 3$

$\boxed{f'(x) = -6x - 5}$

5) $f(x) = \sqrt{4x + 4}$

$\lim_{h \to 0} \frac{\sqrt{4x+4h+4} - \sqrt{4x+4}}{h} \cdot \frac{\sqrt{4x+4h+4} + \sqrt{4x+4}}{\sqrt{4x+4h+4} + \sqrt{4x+4}}$

$\lim_{h \to 0} \frac{\cancel{4x} + 4\cancel{h} + \cancel{4} - \cancel{4x} - \cancel{4}}{\cancel{h}(\sqrt{4x+4h+4} + \sqrt{4x+4})}$

$\lim_{h \to 0} \frac{4}{\sqrt{4x+4h+4} + \sqrt{4x+4}} = \frac{4}{2\sqrt{4x+4}} = \boxed{\frac{2}{\sqrt{4x+4}} \text{ or } \frac{1}{\sqrt{x+1}}}$

6) $y = \sqrt{x + 1}$

$\boxed{y' = \frac{1}{2\sqrt{x+1}}}$

7) $f(x) = 2x^2 - 5x + 1$

$\lim_{h \to 0} \frac{2(x+h)^2 - 5(x+h) + 1 - 2x^2 + 5x - 1}{h}$

$\lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-5x} - 5h \cancel{+1} \cancel{-2x^2} \cancel{+5x} \cancel{-1}}{h}$

$\lim_{h \to 0} \frac{4xh + 2h^2 - 5h}{h} = \lim_{h \to 0} 4x + 2h - 5 = \boxed{4x - 5}$

8) $f(x) = x + 4$

$\boxed{f'(x) = 1}$

9) $y = -\frac{2}{x + 5}$

$\lim_{h \to 0} \frac{\frac{-2}{x+h+5} + \frac{2}{x+5}}{h}$

$\lim_{h \to 0} \frac{\cancel{-2x} \cancel{-10} + \cancel{2x} + 2\cancel{h} \cancel{+10}}{(x+h+5)(x+5)} \cdot \frac{1}{\cancel{h}}$

$\lim_{h \to 0} \frac{2}{(x+5)(x+h+5)} = \boxed{\frac{2}{(x+5)^2}}$

10) $f(x) = 2x^2 + 3x + 1$

$\boxed{f'(x) = 4x + 3}$

11) $y = -3x^2 + x + 4$

$$\lim_{h \to 0} \frac{-3(x+h)^2 + \cancel{x} + h + \cancel{4} + 3x^2 - \cancel{x} - \cancel{4}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{-3x^2} - 6xh - 3h^2 + h + \cancel{3x^2}}{h}$$

$$\lim_{h \to 0} -6x - 3h + 1 = \boxed{-6x + 1}$$

12) $f(x) = \sqrt{2x + 2}$

$$\boxed{f'(x) = \frac{1}{\sqrt{2x+2}}}$$

13) $y = \sqrt{3x - 1}$

$$\lim_{h \to 0} \frac{\sqrt{3(x+h)-1} - \sqrt{3x-1}}{h} \cdot \frac{\sqrt{3(x+h)-1} + \sqrt{3x-1}}{\sqrt{3(x+h)-1} + \sqrt{3x-1}}$$

$$\lim_{h \to 0} \frac{3(x+h) - 1 - 3x + 1}{h\left(\sqrt{3(x+h)-1} + \sqrt{3x-1}\right)}$$

$$\lim_{h \to 0} \frac{3}{\sqrt{3(x+h)-1} + \sqrt{3x-1}} = \frac{3}{\sqrt{3x-1} + \sqrt{3x-1}} = \boxed{\frac{3}{2\sqrt{3x-1}}}$$

14) $y = 4x^2 + 3x + 1$

$$\boxed{y' = 8x + 3}$$

15) $y = 2x^2 + 5$

$$\lim_{h \to 0} \frac{2(x+h)^2 + 5 - 2x^2 - 5}{h}$$

$$\lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{+5} \cancel{-2x^2} \cancel{-5}}{h}$$

$$\lim_{h \to 0} 4x + 2h = \boxed{4x}$$

16) $y = 4x + 4$

$$\boxed{y' = 4}$$

17) $f(x) = \sqrt{3x + 2}$

$$\lim_{h \to 0} \frac{\sqrt{3(x+h)+2} - \sqrt{3x+2}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{3x} + 3h \cancel{+2} \cancel{-3x} \cancel{-2}}{h\left(\sqrt{3(x+h)+2} + \sqrt{3x+2}\right)}$$

$$\lim_{h \to 0} \frac{3}{\sqrt{3(x+h)+2} + \sqrt{3x+2}} = \boxed{\frac{3}{2\sqrt{3x+2}}}$$

18) $y = 2x - 4$

$$\boxed{y' = 2}$$

19) $y = \sqrt{3x + 5}$

$$\lim_{h \to 0} \frac{\sqrt{3(x+h)+5} - \sqrt{3x+5}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{3x} + 3h \cancel{+5} \cancel{-3x} \cancel{-5}}{h\left(\sqrt{3(x+h)+5} + \sqrt{3x+5}\right)}$$

$$\lim_{h \to 0} \frac{3}{\sqrt{3(x+h)+5} + \sqrt{3x+5}} = \frac{3}{2\sqrt{3x+5}}$$

20) $y = 3x - 5$

$$\boxed{y' = 3}$$

Evens ONLY

Honors Calculus Name________________________________

Secant, Tangent and Normal Lines

Date________________ Period____

Use the definition of the derivative to find the derivative of each function with respect to x.

1) $y = -5x^2 - 2x + 5$

$\lim_{h \to 0} \frac{-5(x+h)^2 - 2(x+h) + 5 + 5x^2 + 2x - 5}{h} = \boxed{-10x - 2}$

2) $y = x^2 + x + 4$

$\boxed{y' = 2x + 1}$

3) $y = 2x - 3$

$\lim_{h \to 0} \frac{2(x+h) - 3 - 2x + 3}{h} = \boxed{2}$

4) $y = 4x + 1$

$\boxed{y' = 4}$

5) $y = 3x + 5$

$\lim_{h \to 0} \frac{3(x+h) + 5 - 3x - 5}{h} = \boxed{3}$

6) $y = -\frac{2}{x+4}$

$\boxed{y' = \frac{2}{(x+4)^2}}$

7) $y = -5x^2 + 5$

$\lim_{h \to 0} \frac{-5(x+h)^2 + 5 + 5x^2 - 5}{h} = \boxed{-10x}$

8) $y = \sqrt{3x+3}$

$\boxed{y' = \frac{3}{2\sqrt{3x+3}}}$

9) $y = -\frac{1}{x+3}$

$\lim_{h \to 0} \frac{-\frac{1}{x+h+3} + \frac{1}{x+3}}{h} = \boxed{\frac{1}{(x+3)^2}}$

10) $y = x^2 + 2x - 5$

$\boxed{y' = 2x + 2}$

11) $y = \frac{2}{x+1}$

$\lim_{h \to 0} \frac{\frac{2}{x+h+1} - \frac{2}{x+1}}{h} = \boxed{-\frac{2}{(x+1)^2}}$

12) $y = x^2 + 4x - 5$

$\boxed{y' = 2x + 4}$

13) $y = \frac{2}{2x+3}$

$\lim_{h \to 0} \frac{\frac{2}{2(x+h)+3} - \frac{2}{2x+3}}{h} = -\frac{4}{(2x+3)^2}$

14) $y = \sqrt{x-2}$

$\boxed{y' = \frac{1}{2\sqrt{x-2}}}$

15) $y = -\frac{1}{x+4}$

$\lim_{h \to 0} \frac{-\frac{1}{x+h+4} + \frac{1}{x+4}}{h} = \frac{1}{(x+4)^2}$

16) $y = \sqrt{2x+5}$

$\boxed{y' = \frac{1}{\sqrt{2x+5}}}$

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form. – $y = mx + b$

17) $y = \frac{x^2}{2} - 4$ at $\left(-3, \frac{1}{2}\right)$

$\lim_{x \to -3} \frac{\frac{x^2}{2} - 4 - \frac{1}{2}}{x+3} = \lim_{x \to -3} \frac{\frac{x^2}{2} - \frac{9}{2}}{x+3} = \frac{(x+3)(x-3)}{2(x+3)} = \lim_{x \to -3} \frac{x-3}{2} = \frac{-6}{2} = -3$

$y - \frac{1}{2} = -3(x+3)$

$y - \frac{1}{2} = -3x - 9$

$y = -3x - 9 + \frac{1}{2}$

$\boxed{y = -3x - \frac{17}{2}}$

18) $y = -\frac{x^2}{2} - 3x - \frac{3}{2}$ at $\left(-2, \frac{5}{2}\right)$

$y' = -x - 3$

$y'(-2) = 2 - 3 = -1$

$y - \frac{5}{2} = -1(x+2)$

$y = -x - 2 + \frac{5}{2} \Rightarrow \boxed{y = -x - \frac{1}{2}}$

19) $y=-2x^2+16x-31$ at $(2,-7)$

$\lim_{x\to 2} \frac{-2x^2+16x-31+7}{x-2}$

$\lim_{x\to 2} \frac{-2(x^2-8x+12)}{x-2}$

$\lim_{x\to 2} -2(x-6) = -2(-4) = 8$

$y+7=8(x-2)$

$y+7=8x-16$

$\boxed{y=8x-23}$

20) $y=\frac{1}{x^2-1}$ at $\left(2,\frac{1}{3}\right)$

$\boxed{y=-\frac{4}{9}x+\frac{11}{9}}$

21) $y=2x^2+12x+14$ at $(-2,-2)$

$\lim_{x\to -2} \frac{2x^2+12x+14+2}{x+2}$

$\lim_{x\to -2} \frac{2(x^2+6x+8)}{x+2} = \lim_{x\to -2} 2(x+4) = 4$

$y+2=4(x+2)$

$\boxed{y=4x+6}$

22) $y=-x^3+x^2+3$ at $(-1,5)$

$y'=-3x^2+2x$

$y'(-1)=-3-2=-5$

$y-5=-5(x+1)$

$y-5=-5x-5$

$\boxed{y=-5x}$

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.

23) $y=-2x^2-16x-26$ at $(-3,4)$

$y'=-4x-16$

$y'(-3)=12-16=-4$

$y-4=\frac{1}{4}(x+3)$

$y=\frac{1}{4}x+3/4+4$

$\boxed{y=\frac{1}{4}x+\frac{19}{4}}$

24) $y=x^2-6x+9$ at $(2,1)$

$y'=2x-6$

$y'(2)=4-6=-2$

$y-1=\frac{1}{2}(x-2)$

$y=\frac{1}{2}x-1+1$

$\boxed{y=\frac{1}{2}x}$

25) $y=-\frac{5}{x^2+5}$ at $\left(-1,-\frac{5}{6}\right)$

$y'=\frac{10x}{(x^2+5)^2}$

$y'(-1)=\frac{-10}{36}=\frac{-5}{18}$

$y+\frac{5}{6}=\frac{18}{5}(x+1)$

$y=\frac{18}{5}x+\frac{18}{5}-\frac{5}{6}$

$\boxed{y=\frac{18}{5}x+\frac{83}{30}}$

26) $y=\frac{1}{x+1}$ at $(-2,-1)$

$y'=-\frac{1}{(x+1)^2}$

$y'(-2)=-\frac{1}{1}=-1$

$y+1=1(x+2)$

$y=x+2-1$

$\boxed{y=x+1}$

27) $y=(-x+2)^{\frac{1}{2}}$ at $(-2,2)$

$y'=\frac{-1}{2\sqrt{-x+2}}$

$y'(-2)=-\frac{1}{4}$

$y-2=4(x+2)$

$y=4x+8+2$

$\boxed{y=4x+10}$

28) $y=x^3-4x^2+7$ at $(1,4)$

$y'=3x^2-8x$

$y'(1)=3-8=-5$

$y-4=\frac{1}{5}(x-1)$

$y-4=\frac{1}{5}x-\frac{1}{5}$

$\boxed{y=\frac{1}{5}x+\frac{19}{5}}$

For each problem, find the slope of the function at the given value.

29) $y=x^3-4x^2+2$ at $x=3$

$f(3)=27-4(9)+2$
$=-7$

$\lim_{x\to 3} \frac{x^3-4x^2+2+7}{x-3}$

$$\begin{array}{r|rrrr} 3 & 1 & -4 & 0 & 9 \\ & & 3 & -3 & -9 \\ \hline & 1 & -1 & -3 & \end{array}$$

$\lim_{x\to 3} x^2-x-3 = 9-3-3 = \boxed{3}$

30) $y=\frac{25x}{x^2+25}$ at $x=2$

$y'=\frac{-25x^2+625}{(x^2+25)^2}$

$y'(2)=\frac{-100+625}{(29)^2}=\boxed{\frac{525}{841}}$

31) $y=-(x+2)^3$ at $x=3$

$f(3)=-(5)^3$
$=-125$

$\lim_{x\to 3} \frac{-(x+2)^3+125}{x-3} = \lim_{x\to 3} \frac{-x^3-6x^2-12x-8+125}{(x-3)}$

$= \lim_{x\to 3} \frac{-x^3-6x^2-12x+117}{x-3}$

$= \lim_{x\to 3} -x^2-9x-39$

$= -9-27-39$

$= \boxed{-75}$

32) $y=-x^3+3x^2-4$ at $x=-1$

$y'=-3x^2+6x$

$y'(-1)=-3-6=\boxed{-9}$

33) $y=-\frac{x^2}{2}-x-\frac{3}{2}$ at $x=-1$

$y'=-x-1$

$y'(-1)=1-1=\boxed{0}$

34) $y=x^2-8x+11$ at $x=3$

$y'=2x-8$

$y'(3)=2(3)-8=\boxed{-2}$

Honors Calculus All Name______________________

Power Rule for Derivatives

Date______________ Period____

Differentiate each function with respect to *x*.

1) $y = -4x^2$

$\boxed{y' = -8x}$

2) $y = 3x^5$

$\boxed{y' = 15x^4}$

3) $y = x^3$

$\boxed{y' = 3x^2}$

4) $y = 2x^3$

$\boxed{y' = 6x^2}$

5) $y = 3x^2$

$\boxed{y' = 6x}$

6) $y = 2x$

$\boxed{y' = 2}$

7) $y = -\frac{3}{5}x - \frac{4}{5}x^{-2}$

$y' = -\frac{3}{5} + \frac{8}{5}x^{-3}$

$\boxed{y' = -\frac{3}{5} + \frac{8}{5x^3}}$

8) $y = -\frac{2}{3}x^4 + \frac{4}{3}x$

$\boxed{y' = -\frac{8}{3}x^3 + \frac{4}{3}}$

9) $y = x^4 + 2x^{-3}$

$y' = 4x^3 - 6x^{-4}$

$\boxed{y' = 4x^3 - \frac{6}{x^4}}$

10) $y = -\frac{3}{5}x^5 + \frac{2}{5}x^4$

$\boxed{y' = -3x^4 + \frac{8}{5}x^3}$

11) $y = \frac{1}{4}x^5 + \frac{1}{5}x^{-3}$

$y' = \frac{5}{4}x^4 - \frac{3}{5}x^{-4}$

$\boxed{y' = \frac{5}{4}x^4 - \frac{3}{5x^4}}$

12) $y = -\frac{5}{4}x^{-3} - \frac{2}{3}x^{-5}$

$y' = \frac{15}{4}x^{-4} + \frac{10}{3}x^{-6}$

$\boxed{y' = \frac{15}{4x^4} + \frac{10}{3x^6}}$

13) $y = 3x^3 + \frac{3}{4}x^{\frac{2}{3}} + \frac{4}{5}x^{-1}$

$y' = 9x^2 + \frac{1}{2}x^{-1/3} - \frac{4}{5}x^{-2}$

$\boxed{y' = 9x^2 + \frac{1}{2x^{1/3}} - \frac{4}{5x^2}}$

14) $y = \frac{1}{2}x^2 - 4x + \frac{1}{2}x^{\frac{3}{5}}$

$y' = x - 4 + \frac{3}{10}x^{-2/5}$

$\boxed{y' = x - 4 + \frac{3}{10x^{2/5}}}$

15) $y = 4x^5 + 3x^{-2} - \frac{2}{3}x^{-4}$

$y' = 20x^4 - 6x^{-1} + \frac{8}{3}x^{-3}$

$\boxed{y' = 20x^4 - \frac{6}{x} + \frac{8}{3x^3}}$

16) $y = -\frac{1}{2}x^{\frac{2}{3}} - \frac{3}{2}x^{-3} + \frac{3}{2}x^{-4}$

$y' = -\frac{1}{3}x^{-1/3} + \frac{9}{2}x^{-4} - 6x^{-5}$

$\boxed{y' = \frac{-1}{3x^{1/3}} + \frac{9}{2x^4} - \frac{6}{x^5}}$

17) $y = 2x^4 + 2x^{\frac{5}{2}} - x^{-5}$

$y' = 8x^3 + 5x^{3/2} + 5x^{-6}$

$\boxed{y' = 8x^3 + 5x^{3/2} + \frac{5}{x^6}}$

18) $y = 4x^3 + 5x^{-2} - \frac{1}{2}x^{-3}$

$y' = 12x^2 - 10x^{-3} + \frac{3}{2}x^{-4}$

$\boxed{y' = 12x^2 - \frac{10}{x^3} + \frac{3}{2x^4}}$

Honors Calculus Name________________________________

Product and Quotient Rules

Date________________ Period____

Differentiate each function with respect to *x*.

1) $y = (5x^4 + 2)(2x^3 - 3)$

$(5x^4+2)(6x^2) + (2x^3-1)(20x^3)$

$30x^6+12x^2+40x^6-20x^3$

$\boxed{70x^6-20x^3+12x^2}$

2) $y = (x^5 + 5)(-3x^5 + 1)$

$(x^5+5)(-15x^4) + (-3x^5+1)(5x^4)$

$-15x^9-75x^4-15x^9+5x^4$

$\boxed{-30x^9-70x^4}$

3) $y = (5x^2 - 5)(2x^2 + 1)$

$(5x^2-5)(4x) + (2x^2+1)(10x)$

$20x^3-20x+20x^3+10x$

$\boxed{40x^3-10x}$

4) $y = (x^2 + 1)(2x^2 - 1)$

$(x^2+1)(4x) + (2x^2-1)(2x)$

$4x^3+4x+4x^3-2x$

$\boxed{8x^3+2x}$

5) $y = (x^4 - 1)(5x^5 + 4)$

$(x^4-1)(25x^4) + (5x^5+4)(4x^3)$

$25x^8-25x^4+20x^8+16x^3$

$\boxed{45x^8-25x^4+16x^3}$

6) $y = (2x^5 - 2)(x^5 + 3)$

$(2x^5-2)(5x^4) + (x^5+3)(10x^4)$

$10x^9-10x^4+10x^9+30x^4$

$\boxed{20x^9+20x^4}$

7) $y = (5x^3 + 3)(-x^2 + 3)$

$(5x^3+3)(-2x) + (-x^2+3)(15x^2)$

$-10x^4-6x-15x^4+45x^2$

$\boxed{-25x^4+45x^2-6x}$

8) $y = (2x^4 - 1)(2x^4 + 1)$

$(2x^4-1)(8x^3) + (2x^4+1)(8x^3)$

$16x^7-8x^3+16x^7+8x^3$

$\boxed{32x^7}$

9) $y = (x^3 + 5)(2x^3 + 5)$

$(x^3+5)(6x^2) + (2x^3+5)(3x^2)$

$6x^5+30x^2+6x^5+15x^2$

$\boxed{12x^5+45x^2}$

10) $y = (-3x^3 + 5)(-2x^3 + 1)$

$(-3x^3+5)(-6x^2) + (-2x^3+1)(-9x^2)$

$18x^5-30x^2+18x^5-9x^2$

$\boxed{36x^5-39x^2}$

11) $y = \left(2x^{\frac{5}{3}} - 3\right)\left(-x^5 + 3\right)$

$(2x^{5/3} - 3)(-5x^4) + (-x^5+3)\left(\frac{10}{3}x^{2/3}\right)$

$-10x^{17/3} + 15x^4 - \frac{10}{3}x^{17/3} + 10x^{2/3}$

$\boxed{\frac{40}{3}x^{17/3} + 15x^4 + 10x^{2/3}}$

12) $y = \left(x^{\frac{2}{5}} + 2\right)\left(5x^5 + 2\right)$

$(x^{2/5}+2)(25x^4) + (5x^5+2)(2/5 x^{-3/5})$

$25x^{22/5} + 25x^4 + 5x^{22/5} + \frac{4}{5}x^{-3/5}$

$\boxed{30x^{22/5} + 25x^4 + \frac{4}{5x^{3/5}}}$

13) $y = \left(-4 + \frac{4}{x^4}\right)\left(3x^3 + 4\right)$

$\left(-4 + \frac{4}{x^4}\right)(9x^2) + (3x^3+4)(-16x^{-5})$

$-36x^2 + 36x^{-2} - 48x^{-2} - 64x^{-5}$

$\boxed{-36x^2 - \frac{12}{x^2} - \frac{64}{x^5}}$

14) $y = \left(4 + x^{-4}\right)\left(2x^4 + 5\right)$

$(4+x^{-4})(8x^3) + (2x^4+5)(-4x^{-5})$

$32x^3 + 8x^{-1} - 8x^{-1} - 20x^{-5}$

$\boxed{32x^3 - \frac{20}{x^5}}$

15) $y = \frac{3x^4 + 5x^2}{x^5 + 2}$

$\frac{(x^5+2)(12x^3+10x) - (3x^4+5x^2)(5x^4)}{(x^5+2)^2}$

$\frac{12x^8 + 10x^6 + 24x^3 + 20x - 15x^8 - 25x^6}{(x^5+2)^2} = \boxed{\frac{-3x^8 - 15x^6 + 24x^3 + 20x}{(x^5+2)^2}}$

16) $y = \frac{5x^4 - 4x^3}{3x^5 - 3}$

$\frac{(3x^5-3)(20x^3-12x^2) - (5x^4-4x^3)(15x^4)}{(3x^5-3)^2}$

$\frac{60x^8 - 36x^7 - 60x^3 + 36x^2 - 75x^8 + 60x^7}{(3x^5-3)^2}$

$\boxed{\frac{-15x^8 + 24x^7 - 60x^3 + 36x^2}{(3x^5-3)^2}}$

17) $y = \frac{5x^4 + 1}{x^2 - 4}$

$\frac{(x^2-4)(20x^3) - (5x^4+1)(2x)}{(x^2-4)^2}$

$\frac{20x^5 - 80x^3 - 10x^5 - 2x}{(x^2-4)^2}$

$\boxed{\frac{10x^5 - 80x^3 - 2x}{(x^2-4)^2}}$

18) $y = \frac{5x^4 + 4x^2}{3x^4 + 2}$

$\frac{(3x^4+2)(20x^3+8x) - (5x^4+4x^2)(12x^3)}{(3x^4+2)^2}$

$\boxed{\frac{-24x^5 + 40x^3 + 16x}{(3x+2)^2}}$

19) $y = \frac{5x^2 - 4}{3x^3 + 5}$

$\frac{(3x^3+5)(10x) - (5x^2-4)(9x^2)}{(3x^3+5)^2}$

$\frac{30x^4 + 50x - 45x^4 + 36x^2}{(3x^3+5)^2}$

$\boxed{\frac{-15x^4 + 36x^2 + 50x}{(3x^3+5)^2}}$

20) $y = \frac{4x^2 + 5}{2x^2 - 4}$

$\frac{(2x^2-4)(8x) - (4x^2+5)(4x)}{(2x^2-4)^2}$

$\frac{16x^3 - 32x - 16x^3 - 20x}{(2x^2-4)^2}$

$\boxed{\frac{-52x}{(2x^2-4)^2}}$

Honors Calculus Name________________________

Chain Rule

Date________________ Period____

Differentiate each function with respect to x.

1) $y = (5x^5 - 4)^5$

$5(5x^5-4)^4(25x^4)$

$\boxed{125x^4(5x^5-4)^4}$

2) $y = (x^2 + 2)^3$

$3(x^2+2)^2(2x)$

$\boxed{6x(x^2+2)^2}$

3) $y = (x + 4)^5$

$5(x+4)^4(1)$

$\boxed{5(x+4)^4}$

4) $y = (4x^5 + 1)^2$

$2(4x^5+1)(20x^4)$

$\boxed{80x^4(4x^5+1)}$

5) $y = (4x^4 + 5)^4$

$4(4x^4+5)^3(16x^3)$

$\boxed{64x^3(4x^4+5)^3}$

6) $y = (4x^4 + 5)^5$

$5(4x^4+5)^4(16x^3)$

$\boxed{80x^3(4x^4+5)^4}$

7) $y = (-3x^2 - 4)^4$

$4(-3x^2-4)^3(-6x)$

$\boxed{-24x(-3x^2-4)^3}$

8) $y = (x^2 + 2)^2$

$2(x^2+2)(2x)$

$4x(x^2+2)$

$\boxed{4x^3+8x}$

9) $y = (-4x^2 - 3)^2$

$2(-4x^2-3)^1(-8x)$

$-16x(-4x^2-3)$

$\boxed{64x^3+48x}$

10) $y = (3x^3 - 2)^2$

$2(3x^3-2)(9x^2)$

$18x^2(3x^3-2)$

$\boxed{54x^5-36x^2}$

11) $y = ((3x^2 + 4)^2 + 2)^5$

$5((3x^2+4)^2+2)^4 \cdot 2(3x^2+4) \cdot 6x$

$\boxed{60x(3x^2+4)[(3x^2+4)^2+2]}$

12) $y = ((2x^5 - 3)^2 - 4)^4$

$4((2x^5-3)^2-4)^3 \cdot 2(2x^5-3)(10x^4)$

$\boxed{80x^4(2x^5-3)((2x^5-3)^2-4)^3}$

13) $y = ((-4x+5)^2+5)^5$

$5((-4x+5)^2+5)^4\,(2(-4x+5))\,(-4)$

$\boxed{-40(-4x+5)\,((-4x+5)^2+5)}$

14) $y = ((5x^4-3)^5+3)^4$

$4((5x^4-3)^5+3)^3 \cdot 5(5x^4-3)^4 \cdot 20x^3$

$\boxed{400x^3((5x^4-3)^5+3)^3 \cdot (5x^4-3)^4}$

15) $y = ((2x^5+5)^3-1)^2$

$2((2x^5+5)^3-1) \cdot 3(2x^5+5)^2 \cdot 10x^4$

$60x^4(2x^5+5)^2((2x^5+5)^3-1)$

16) $y = \sqrt[4]{x^2-2}(4x^3+1)^{\frac{1}{3}}$

$(x^2-2)^{1/4} \cdot \frac{1}{3}(4x^3+1)^{-2/3} \cdot 12x^2 + (4x^3+1)^{1/3} \cdot \frac{1}{4}(x^2-2)^{-3/4} 2x$

$= \boxed{\dfrac{x(12x^3-16x+1)}{2(4x^3+1)^{2/3} \cdot (x^2-2)^{3/4}}}$

17) $y = \sqrt[3]{2x+1} \cdot \sqrt{5x^4-1}$

$(2x+1)^{1/2} \cdot \frac{1}{2}(5x^4-1)^{-1/2} \cdot 20x^3 + (5x^4-1)^{1/2} \cdot \frac{1}{3}(2x+1)^{-2/3} \cdot 2$

$= \dfrac{2(35x^4+15x^3-1)}{3(5x^4-1)^{1/2} \cdot (2x+1)^{2/3}}$

18) $y = (4x^2-5)^{\frac{1}{3}} \cdot (x+5)^4$

$(4x^2-5)^{1/3} \cdot 4(x+5)^3 + (x+5)^4 \cdot \frac{1}{3}(4x^2-5)^{-2/3} \cdot 8x$

$\dfrac{4(x+5)^3(14x^2-15+10x)}{3(4x^2-5)^{2/3}}$

19) $y = \sqrt{\sqrt[5]{3x+2}-1}$

$y = ((3x+2)^{1/5}-1)^{1/2}$

$1/2((3x+2)^{1/5}-1)^{-1/2} \cdot \frac{1}{5}(3x+2)^{-4/5} \cdot 3$

$= \dfrac{3}{10((3x+2)^{1/2}-1)^{1/2} \cdot (3x+2)^{4/5}}$

20) $y = \sqrt[5]{5x+4}(3x^5-4)^3$

$(5x+4)^{1/4} \cdot 3(3x^5-4)^2 \cdot 15x^4 + (3x^5-4)^3 \cdot \frac{1}{5} \cdot 5(5x+4)^{-4/5}$

$\dfrac{4(5x+4)^{1/2}(57x^5+45x^4+\ldots)}{(5x+4)^{4/5}}$

Honors Calculus

Name________________

Log, Exponential, and Trig Derivatives

Date________________ Period____

Differentiate each function with respect to *x*.

1) $y = e^{3x^3}$ $\boxed{9x^2 \cdot e^{3x^3}}$

2) $y = e^{3x^5}$ $\boxed{15x^4 e^{3x^5}}$

3) $y = \ln x^2$ $\boxed{\frac{2}{x}}$

4) $y = e^{5x^2}$ $\boxed{10x e^{5x^2}}$

5) $y = \ln x^4$ $\boxed{\frac{4}{x}}$

6) $y = \ln\left(\frac{x^4}{5x^5+3}\right)^3$ $= 3\ln x^4 - 3\ln(5x^5+3)$

$\frac{12}{x} - \frac{3}{5x^5+3} \cdot 25x^4$

$\boxed{\frac{12}{x} - \frac{75x^4}{5x^5+3}}$

7) $y = \ln\left(\frac{5x^2}{5x^5+1}\right)^3$ $\frac{3}{5x^2} \cdot 10x - \frac{3}{5x^5+1} \cdot 25x^4$

$\boxed{\frac{6}{x} - \frac{75x^4}{5x^5+1}}$

8) $y = \ln\left(\frac{5x^3}{3x^4-1}\right)^3$ $3 \cdot \frac{15x^2}{5x^3} - \frac{3 \cdot 12x^3}{3x^4-1}$

$\boxed{\frac{9}{x} - \frac{36x^3}{3x^4-1}}$

9) $y = \ln\left(\frac{x^5}{3x^3-5}\right)^2$ $2\left(\frac{1}{x^5} \cdot 5x^4 - \frac{1}{3x^3-5} \cdot 9x^2\right)$

$= \frac{2(6x^3-25)}{x(3x^3-5)}$

10) $y = \ln\left(\frac{4x^5}{3x^3+5}\right)^5$ $5\left(\frac{1}{4x^5} \cdot 20x^4 - \frac{1}{3x^3+5} \cdot 9x^2\right)$

$= \boxed{\frac{5(6x^3+25)}{x(3x^3+5)}}$

11) $y = \ln 4x^2 \cdot \sqrt{3x^3+2}$ $\boxed{\frac{9x^3 \ln 4x^2 + 12x^3 + 8}{2x\sqrt{3x^3+2}}}$

12) $y = \ln \ln 4x^2$ $\frac{1}{\ln 4x^2} \cdot \frac{1}{4x^2} \cdot 8x$

$\boxed{\frac{2}{x \ln 4x^2}}$

13) $y = (2 + e^{2x^5})^5$ $5(2+e^{2x^5})^4 \cdot e^{2x^5} \cdot 10x^4$

$50x^4(2+e^{2x^5})^4 \cdot e^{2x^5}$

14) $y = \ln \ln 4x^3$ $\boxed{\frac{3}{x \ln 4x^3}}$

15) $y = \ln \ln x^4$ $\frac{1}{\ln x^4} \cdot \frac{1}{x^4} \cdot 4x^3$

$\frac{4}{x \ln x^4}$

16) $y = \sqrt{e^{2x^5}+3}$ $\frac{1}{2}(e^{2x^5}+3)^{-1/2} \cdot e^{2x^5} \cdot 10x^4$

$= \boxed{\frac{5x^4 e^{2x^5}}{\sqrt{e^{2x^5}+3}}}$

17) $y = e^{4x^2}\sqrt{x^4+1}$ $\frac{2xe^{4x^2}(4x^4+4+x^2)}{\sqrt{x^4+1}}$

18) $y = e^{4 + \ln x^3}$ $e^{4+\ln x^3} \cdot \frac{1}{x^3} \cdot 3x^2$ $= \frac{3e^{4+\ln x^3}}{x}$

19) $y = e^{\ln 2x^2 + 3}$ $\frac{2e^{\ln(2x^2+3)}}{x}$

20) $y = e^{3x^5} \ln 2x^2$ $\frac{e^{3x^5}(15x^5 \ln 2x^2 + 2)}{x}$

21) $y = \cos 2x^4$ $-8x^3 \sin 2x^4$

22) $y = \cos 4x^3$ $-12x^2 \sin 4x^3$

23) $y = \cos 3x^3$ $-9x^2 \sin(3x^3)$

24) $y = \sin 2x^5$ $10x^4 \cos 2x^5$

25) $y = \cos 4x^4$ $-16x^3 \sin 4x^4$

26) $y = \cos 5x^3$ $-15 \sin 5x^3$

27) $y = \csc 2x^3$ $-6x^2 \csc 2x^3 \cot 2x^3$

28) $y = \cos 2x^5$ $-10x^4 \sin 2x^5$

29) $y = \tan 5x^5$ $25x^4 \sec^2 5x^5$

30) $y = \tan 5x^4$ $20x^3 \sec^2 5x^4$

31) $y = \sec \sqrt{5x^3 - 1}$ $\frac{1}{2}(5x^3-1)^{-1/2} \cdot 15x^2 \sec\sqrt{5x^3-1} \tan\sqrt{5x^3-1}$ $= \frac{15x^2 \sec\sqrt{5x^3-1} \tan\sqrt{5x^3-1}}{2\sqrt{5x^3-1}}$

32) $y = \tan(\sin 2x^5)$ $\sec^2(\sin(2x^5)) \cdot \cos 2x^5 \cdot 10x = 10x \cos(2x^5) \sec^2(\sin(2x^5))$

33) $y = \sec \sqrt[3]{-2x^3 + 3}$ $\sec\sqrt[3]{-2x^3+3} \cdot \tan\sqrt[3]{-2x^3+3} \cdot \frac{1}{3}(-2x^3+3)^{-2/3} \cdot -6x^2$ $\frac{2x^2 \sec\sqrt[3]{-2x^3+3} \tan\sqrt[3]{-2x^3+3}}{(-2x^3+3)^{2/3}}$

34) $y = \cos(\csc x^5)$ $+\sin(\csc x^5) \cdot \csc x^5 \cdot \cot x^5 \cdot 5x^2 = 5x^2 \csc x^5 \cot x^5 \sin(\csc x^5)$

35) $y = \cos(-5x^5 + 4)^3$ $-\sin(-5x^5+4)^3, 3(-5x^5+4)^2, -25x^4$ $75x^4 \sin(-5x^5+4)^3 \cdot (-5x^5+4)^2$

36) $y = \cos \sqrt[3]{-x^5 + 3}$ $-\sin\sqrt[3]{-x^5+3} \cdot \frac{1}{3}(-x^5+3)^{-2/3} \cdot -5x^4$ $\frac{-5x^4 \sin\sqrt[3]{-x^5+3}}{3(-x^5+3)^{2/3}}$

Honors Calculus Name______________________

Implicit Differentiation

Date________________ Period____

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

1) $x = -5y^2 + 1$

$1 = -10y \frac{dy}{dx}$

$-\frac{1}{10y} = \frac{dy}{dx}$

2) $-5y^3 + 2 = 4x^2$

$\frac{dy}{dx} = \frac{-8x}{15y^2}$

3) $3 = 4x^2 + 3y^3$

$0 = 8x + 27y^2 \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{-8x}{9y^2}$

4) $x = -5y^3 + 3$

$\frac{dy}{dx} = -\frac{1}{15y^2}$

5) $x^2 + 3y^3 = -y^2 + 4$

$2x + 9y^2 \frac{dy}{dx} = -2y \frac{dy}{dx}$

$\frac{dy}{dx} = -\frac{2x}{(2y + 9y^2)}$

6) $-4y^3 - 5y^2 + 5 = 5x$

$\frac{dy}{dx} = \frac{5}{-12y^2 - 10y}$

7) $x^3 = -3y^3 - 4y^2 + 5$

$3x^2 = -9y^2 \frac{dy}{dx} - 8y \frac{dy}{dx}$

$\frac{3x^2}{-9y^2 - 3y} = \frac{dy}{dx}$

8) $-5y^2 + 5 = 4x + 3y^3$

$\frac{dy}{dx} = \frac{4}{-10y - 9y^2}$

9) $2x^3 + 3x^3y = -3y^2 + 2$

$\frac{dy}{dx} = \frac{-2x^2 - 3x^2y}{x^3 + 2y}$

10) $4x^2 + x^2y + 2x^2y^2 = 1$

$\frac{dy}{dx} = \frac{-8 - 2y - 4y^2}{x + 4xy}$

11) $-2xy + 1 = 2x^3 + 2x^2y^2$

$\frac{dy}{dx} = \frac{3x^2 + 2xy^2 + y}{-x - 2x^2y}$

12) $4x^2 + 5x^2y^2 + 5y^3 = 4$

$\frac{dy}{dx} = \frac{-8x - 10xy^2}{10yx^2 + 15y^2}$

Honors Calculus Name________________________

L'Hopital's Rule

Date________________ Period____

Evaluate each limit using L'Hôpital's Rule.

1) $\lim_{x \to 0} \frac{e^x - 1}{2x^3}$ $\frac{e^x}{6x^2} \rightarrow \frac{e^x}{12x} \rightarrow \frac{e^x}{12}$

∞

2) $\lim_{x \to 1} \frac{\ln x^2}{x^2 - 1}$ $\frac{\frac{2}{x}}{2x} = \frac{2}{2} = 1$

3) $\lim_{x \to -2} -\frac{x+2}{x^2 + 5x + 6}$ -1

$-\frac{1}{2x+5}$

4) $\lim_{x \to -1} \frac{2(x^2 - 1)}{\ln x^2}$ $\frac{4x}{\frac{2}{x}} = \frac{-4}{-2} = 2$

5) $\lim_{x \to 0} \frac{\sin(3x)}{x}$

$\frac{3 \cos 3x}{1} = 3$

6) $\lim_{x \to 0} \frac{4(e^x - e^{-x})}{\sin x}$ $\frac{4e^x + 4e^{-x}}{\cos x} = \frac{4+4}{1}$

$= 8$

7) $\lim_{x \to 0} \frac{5x}{\tan(2x)}$ $\frac{5}{2\sec^2(2x)} = \frac{5}{2 \cdot 1} = \frac{5}{2}$

8) $\lim_{x \to 0} \frac{4x}{\ln(x+1)}$ $\frac{4}{\frac{1}{x+1}} = \frac{4}{\frac{1}{1}} = 4$

9) $\lim_{x \to 1} \frac{5(x-1)}{\ln x}$ $\frac{5}{\frac{1}{x}} = 5$

10) $\lim_{x \to 0} \frac{3(e^x - e^{-x})}{x}$ 6

11) $\lim_{x \to \infty} \frac{e^x}{x}$

∞

12) $\lim_{x \to 0} \frac{e^x - 1}{x}$

1

13) $\lim_{x \to 0^+} \frac{2 \ln \sin x}{\ln \tan x}$

2

14) $\lim_{x \to 0^+} \frac{\ln \tan x}{\ln \sin x}$

1

15) $\lim_{x \to 0} \frac{\sin(3x)}{\sin(4x)}$

$\frac{3}{4}$

16) $\lim_{x \to 0} \frac{e^{2x} - 1}{\sin(3x)}$

$\frac{2}{3}$

17) $\lim_{x \to 3} \frac{\sqrt{x+6} - 3}{x - 3}$

$\frac{1}{6}$

18) $\lim_{x \to \infty} \frac{x}{\ln x}$

∞

19) $\lim_{x \to \infty} \frac{x}{e^{2x}}$

0

20) $\lim_{x \to \infty} \frac{\ln x}{x}$

0

Derivatives Review

1. Find the average rate of change of $f(x)=5\sqrt{x^2+9}-2$ over [0, 4].

$f(0)=5\sqrt{9}-2=5\cdot 3-2=13$ $\quad \frac{23-13}{4-0}=\frac{10}{4}=\boxed{\frac{5}{2}}$

$f(4)=5\cdot\sqrt{25}-2=23$

2. Find the Equation of the tangent line: $f(x)=4x^2-5x+2$ at $x=3$

$f'(x)=8x-5$ $\quad f(3)=4\cdot 9-15+2=23$

$f'(3)=8\cdot 3-5=19$

$y-23=19(x-3)$

3. Find the Equation of the normal line: $f(x)=4x^2-5x+2$ at $x=3$

$y-23=-\frac{1}{19}(x-3)$

4. Find the Derivative using the definition: $f(x)=\sqrt{2x+3}$

$\lim_{h\to 0}\frac{\sqrt{2(x+h)+3}-\sqrt{2x+3}}{h}=\frac{2(x+h)+3-2x-3}{h\left(\sqrt{2(x+h)+3}+\sqrt{2x+3}\right)}$

$=\frac{2}{\sqrt{2(x+h)+3}+\sqrt{2x+3}}$

$=\frac{2}{2\sqrt{2x+3}}=\boxed{\frac{1}{\sqrt{2x+3}}}$

5. Find the Derivative using the definition: $f(x)=\frac{3}{2x-1}$

$\lim_{h\to 0}\frac{\frac{3}{2(x+h)-1}-\frac{3}{2x-1}}{h}=\frac{\cancel{6x}-\cancel{3}-\cancel{6x}-6h+\cancel{3}}{(2x-1)(2x+2h-1)}\cdot\frac{1}{h}$

$=\frac{-6}{(2x-1)^2}$

6. Find the Derivative using the definition: $f(x)=2x^2+3x-5$

$\lim_{h\to 0}\frac{2(x+h)^2+3(x+h)-5-2x^2-3x+5}{h}$

$=4x+3$

7. Find the Derivative using the definition: $f(x)=2-\frac{3}{x}$

$\lim_{h\to 0}\frac{2-\frac{3}{x+h}-2+\frac{3}{x}}{h}$

$=\frac{3}{x^2}$

Find the Derivative. PUT A BOX around your answers.

8. $f(x)=\frac{x^2+3x+2}{x^2-1}=\frac{(x+2)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}=\frac{x+2}{x-1}$

$\frac{(x-1)(1)-(x+2)(1)}{(x-1)^2}=\boxed{\frac{-1}{(x-1)^2}}$

9. $f(x)=\frac{x+1}{\sqrt{x}}=\frac{x}{x^{1/2}}+\frac{1}{x^{1/2}}$

$=x^{1/2}+x^{-1/2}$

$\frac{1}{2}x^{-1/2}-\frac{1}{2}x^{-3/2}=\frac{1}{2\sqrt{x}}-\frac{1}{2x^{3/2}}$

10. $f(x)=\sin^2 3x$

$2\sin(3x)\cdot\cos(3x)\cdot 3$

$6\sin(3x)\cos(3x)$

11. $f(x)=\frac{x(x^2-1)}{x+1}=\frac{x(x-1)\cancel{(x+1)}}{\cancel{x+1}}$

$=x(x-1)$

$=x^2-x$

$\boxed{2x-1}$

12. $y = (x^2 - x)(x^2 + 1)(x^2 + x + 1)$

$y' = (2x-1)(x^2+1)(x^2+x+1)$
$+ (x^2-x)(2x)(x^2+x+1)$
$+ (x^2-x)(x^2+1)(2x+1$

13. $y = x \cdot \sin x + \cos x$

$x\cos x + \cos x - \sin x$

14. $y = 5\sec x + \tan x$

$5\sec x \tan x + \sec^2 x$

15. $f(x) = \sqrt{x} + 4\csc x$

$\frac{1}{2\sqrt{x}} - 4\csc x \cot x$

16. $f(x) = \frac{1}{x} - 10\sec x$

$-\frac{1}{x^2} - 10\sec x \tan x$

17. $y = \left(\frac{x+1}{x+2}\right)(2x-5)$

$\left(\frac{x+1}{x+2}\right) \cdot 2 + (2x-5) \cdot \frac{x+2-x-1}{(x+2)^2}$

$\frac{2x+2}{x+2} + \frac{2x-5}{(x+2)^2}$

18. $y = (2x-7)^3$

$3(2x-7)^2(2)$

$6(2x-7)^2$

19. $f(x) = (9 - x^2)^{\frac{2}{3}}$

$\frac{2}{3}(9-x^2)^{-1/3}(-2x)$

$\frac{-4x}{3(9-x^2)^{1/3}}$

20. $y = \sqrt{\frac{1}{4x^2}} = \frac{1}{2x}$

$-\frac{1}{2x^2}$

21. $f(x) = \frac{x^2}{x^2+3}$

$\frac{(x^2+3)(2x) - x^2(2x)}{(x^2+3)^2}$

$\frac{6x}{(x^2+3)^2}$

22. $y = \frac{\cos \pi x + 1}{x}$

$\frac{-\pi x \sin \pi x - (\cos \pi x + 1)}{x^2}$

$\frac{-\pi x \sin(\pi x) - \cos(\pi x) - 1}{x^2}$

23. $f(x) = 3\tan 4x$

$3 \cdot \sec^2(4x) \cdot 4$

$12\sec^2(4x)$

24. $y = \sin(\cos x)$

$-\cos(\cos x) \cdot \sin x$

25. $y = 3x - 5\cos(\pi x)^2$

$3 + 5\sin(\pi x)^2 \cdot 2(\pi x) \cdot \pi$

$3 + 10\pi^2 x \sin(\pi x)^2$

26. $f(x) = \sqrt{x^2 + 2x + 8}$

$\frac{1}{2}(x^2+2x+8)^{-1/2}(2x+2)$

$\frac{x+1}{\sqrt{x^2+2x+8}}$

27. $y = \frac{1}{x} + \sqrt{\cos x}$

$-\frac{1}{x^2} + \frac{1}{2}(\cos x)^{-1/2}(-\sin x)$

$-\frac{1}{x^2} - \frac{\sin x}{2\sqrt{\cos x}}$

28. Find the first TWO Derivatives:

$$y = 2(x^2-1)^3$$

$y' = 6(x^2-1)^2(2x)$

$= 12x(x^2-1)^2$

$y'' = 24x(x^2-1)\cdot 2x$

$= 48x^2(x^2-1)$

29. Find the $f', f'', and\ f'''$

$$f(x) = x^3 + 2x^2 - 4x + 5$$

$f'(x) = 3x^2 + 4x - 4$

$f''(x) = 6x + 4$

$f'''(x) = 6$

Evaluate the Limit

30. $\lim_{x\to 3} \frac{x^2+4x-21}{x^2-7x+12}$ $= \frac{2x+4}{2x-7} = \frac{6+4}{6-7} = -10$

31. $\lim_{x\to 0} \frac{\tan 3x}{\ln(1+x)}$ $\frac{3\sec^2 3x}{\frac{1}{1+x}} = 3$

32. $\lim_{x\to 1} \frac{x^2-1}{x-1}$ $2x = 2$

33. $\lim_{x\to 3} \frac{\sqrt{x+6}-3}{x-3}$ $= \frac{1}{2}(x+6)^{-1/2} = \frac{1}{2\sqrt{x+6}}$

$\frac{1}{2\cdot 3} = \frac{1}{6}$

34. $\lim_{x\to 0} \frac{x}{\frac{1}{3+x}-\frac{1}{3}}$ $\frac{1}{\frac{(3+x)(0)-1}{(3+x)^2}} = \frac{(3+x)^2}{-1}$

$= -9$

35. $\lim_{x\to -1} -\frac{x-4}{x^2-6x+8}$ $\frac{-5}{1+6+8} = \frac{-5}{15} = -\frac{1}{3}$

36. $\lim_{x\to 2} x^2-6x+8$ $4-12+8 = 0$

37. $\lim_{x\to \infty} \frac{x-4}{x^2-6x+8}$ 0

Find the Derivative

38. $y = (x^2+1)e^{3x}$

$3(x^2+1)e^{3x} + e^{3x}\cdot 2x$

$3e^{3x}(x^2+1) + 2xe^{3x}$

39. $y = x \cdot 5^{3x}$

$x\cdot 5^{3x}\ln 5\cdot 3 + 5^{3x}$

$3x\ln(5)5^{3x}\cdot 5^{3x}$

40. $y = \sin(e^{2x})$

$\cos(e^{2x})\, e^{2x} \cdot 2$

$2e^{2x} \cos(e^{2x})$

41. $y = e^{e^{5x}}$

$e^{e^{5x}} \cdot e^{5x} \cdot 5$

$5e^{5x}\, e^{e^{5x}}$

42. $y = 3^{x^2+3x}$

$3^{x^2+3x} \cdot 2x+3 \cdot \ln 3$

43. $y = (\ln x)^x$

Do Not Do

44. $y = 3^{5x}$

$3^{5x} \cdot \ln 3 \cdot 5$

45. $y = e^{\ln(5x^2)} = 5x^2$

$10x$

46. Differentiate implicitly to find $\frac{dy}{dx}$:

$$2xy + 3 = 0$$

$2xy' + 2y = 0$

$y' = \frac{-y}{x}$

47. Differentiate implicitly to find $\frac{dy}{dx}$:

$$x^2 - y^2 = 1$$

$2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{2x}{2y} = \frac{x}{y}$

48. Differentiate implicitly to find $\frac{dy}{dx}$:

$$2y^2 = \frac{5x-3}{5x+3}$$

$4y\, y' = \frac{(5x+3)5 - (5x-3)5}{(5x+3)^2}$

$y' = \frac{15}{2y(5x+3)^2}$

49. Differentiate implicitly to find $\frac{d^2y}{dx^2}$:

$$3x^3 - y^3 = 7$$

$27x^2 - 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{27x^2}{3y^2}$

$\frac{dy}{dx} = \frac{9x^2}{y^2}$

$y'' = \frac{18xy^2 - 18y^2 y\, y'}{y^4} = \frac{18xy^2 - \frac{162x^4}{y}}{y^4}$