Limits Assignments

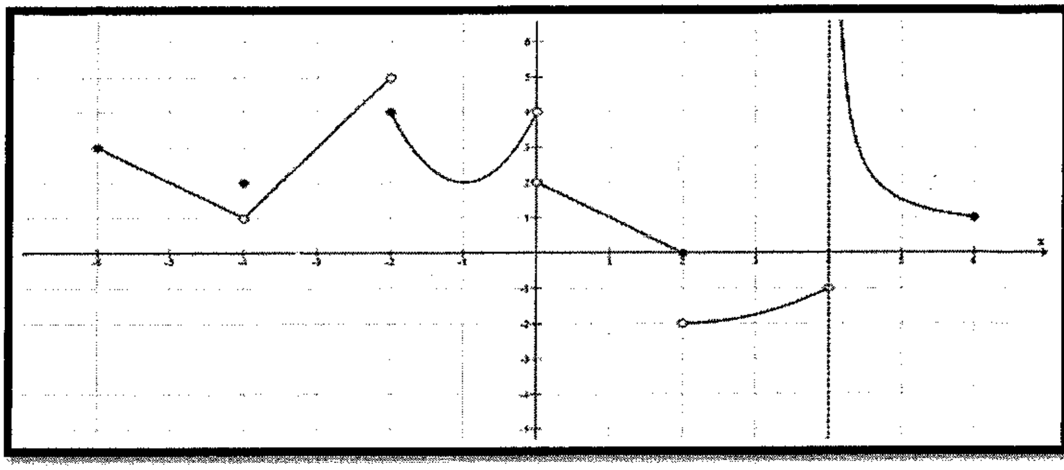
Essential question: *What is a limit and why is it important to calculus?*

Day	Topic	Assignment
Wed Jan 24	EQ: What is a limit? How is the concept of a limit similar to previous concepts learned in other math classes? **Keeper 7 - Finding Limits from Graphs** -Understand the concept of a limit -Use the definition of a limit to estimate limits -Determine whether limits of functions exist -Finding limits from graphs	**Keeper 7 Worksheet - Limits from Graphs Worksheet** **Textbook pg.54 11-20**
Thurs Jan 25	EQ: How can I graph a function given characteristics about the limits of the function? **Keeper 8 - Graphs from Limits** -Understand the concept of a limit -Use the definition of a limit to estimate limits -Determine whether limits of functions exist -Graphing functions given a limit	Quick Homework Quiz **Keeper 8 Worksheet - Graphs from Limits Worksheet**
Fri Jan 26	EQ: How do I calculate a limit from one side of the function at a given point? What makes a function continuous? How can I determine the different types of discontinuity of a function? **Keeper 9 - One-Sided Limits and Continuity** -Understand the concept of a limit -Use the definition of a limit to estimate limits -Determine whether limits of functions exist -Evaluate one-sided limit graphically and algebraically - Continuity at a point -Continuous functions -Algebraic combinations -Composites	Quick Homework Quiz **Keeper 9 Worksheet - One-Sided Limits and Continuity Worksheet**
Mon Jan 29	EQ: How can I find the limit of a function at a given point without graphing? **Keeper 10 - Algebraic Limits** -Use properties of limits and direct substitution to evaluate limits -Using the dividing out technique to evaluate limits of functions -Use rationalizing techniques to evaluate limits of functions -Use technology to approximate limits of functions graphically and numerically	Quick Homework Quiz **Keeper 10 Worksheet - Algebraic Limits Worksheet** **Textbook pg. 64-65 5-27, 41-49 odds**
Tues Jan 30	EQ: What is the intermediate value theorem and why is it important? **Keeper 11 - Intermediate Value Theorem** - Continuity - Intermediate value theorem for continuous functions	Quick Homework Quiz **Keeper 11 Worksheet - Intermediate Value Theorem Worksheet** **Textbook pg. 78 79-82 all**
Wed Jan 31	EQ: How can end behavior and characteristics of asymptotes help me find the limit of a function? How can I use asymptotes and end behavior to determine the limit of a function to negative and positive infinity? **Keeper 12 - Polynomial and Rational Asymptotes** - Exploring end behavior of polynomial functions - Exploring end behavior of rational functions - Calculating vertical, horizontal, and oblique (slant) asymptotes of rational functions - finite Limits as x approaches plus or minus infinity -infinite limits as x approaches a number -End behavior models -visually seeing limits as x approaches plus or minus infinity	Quick Homework Quiz **Keeper 18 Worksheet - Vertical and Horizontal Asymptotes Worksheet** **Keeper 18 Worksheet - Infinite Limits Worksheet**
Thurs 2/1	Review	Complete Limits Review **Textbook pg. 87-88 1-50 all** **Skip 1-2, 21-22, 35, 43-44, 47-48**
Friday 2/2	**Test on Limits**	Turn in all unit 2 homework assignments (worksheets only) for a unit daily grade!

Honors Calculus Name ____________________

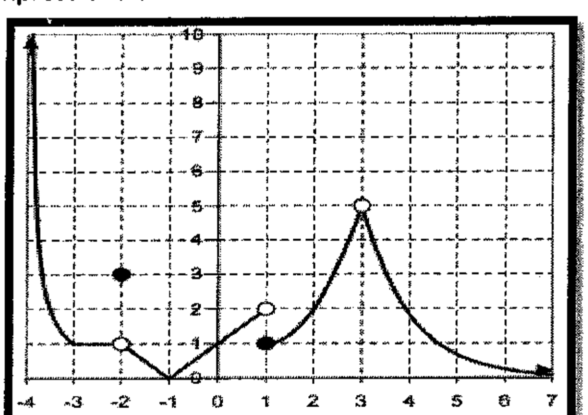
Finding Limits from a Graph

1. Use the graph to evaluate the limits below



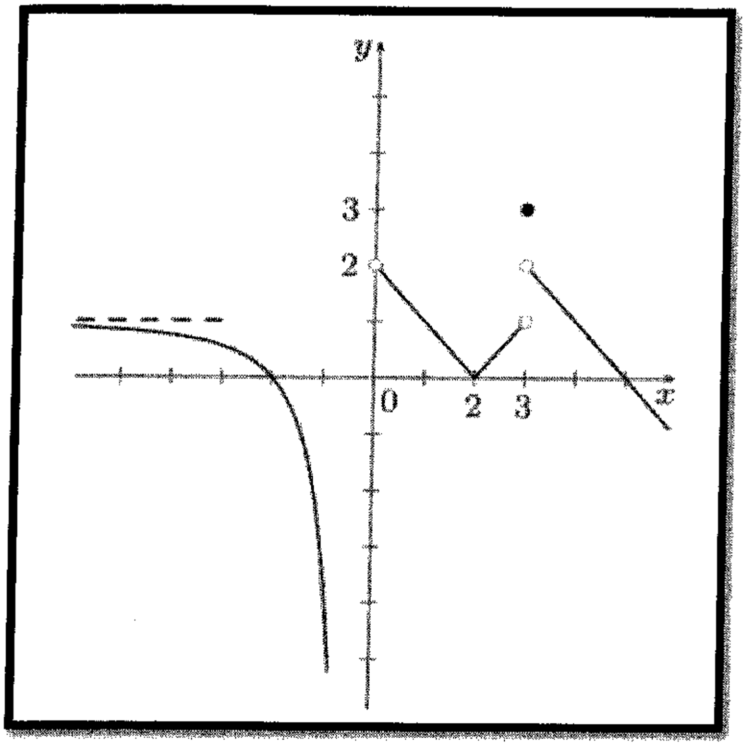
a.	$f(-4)$ 3	b.	$\lim_{x\to -4^-} f(x)$ 1	c.	$\lim_{x\to -4^+} f(x)$ 1	d.	$\lim_{x\to -4} f(x)$ 1
e.	$f(-2)$ 4	f.	$\lim_{x\to -2^-} f(x)$ 5	g.	$\lim_{x\to -2^+} f(x)$ 4	h.	$\lim_{x\to -2} f(x)$ DNE
i.	$f(0)$ DNE	j.	$\lim_{x\to 0^-} f(x)$ 4	k.	$\lim_{x\to 0^+} f(x)$ 2	l.	$\lim_{x\to 0} f(x)$ DNE
m.	$f(2)$ 0	n.	$\lim_{x\to 2^-} f(x)$ 0	o.	$\lim_{x\to 2^+} f(x)$ -2	p.	$\lim_{x\to 2} f(x)$ DNE
q.	$f(4)$ DNE	r.	$\lim_{x\to 4^-} f(x)$ -1	s.	$\lim_{x\to 4^+} f(x)$ ∞	t.	$\lim_{x\to 4} f(x)$ DNE

2. Use the graph to evaluate the expressions below.



a.	$f(-2)$ 3	b.	$\lim_{x\to -2^+} f(x)$ 1	c.	$\lim_{x\to -2} f(x)$ 1
d.	$\lim_{x\to -1^+} f(x)$ 0	e.	$\lim_{x\to -1^-} f(x)$ 0	f.	$\lim_{x\to -1} f(x)$ 0
g.	$\lim_{x\to 1^+} f(x)$ 1	h.	$\lim_{x\to 1^-} f(x)$ 2	i.	$\lim_{x\to 1} f(x)$ DNE
j.	$f(3)$ DNE	k.	$\lim_{x\to 3^+} f(x)$ 5	l.	$\lim_{x\to 3^-} f(x)$ 5
m.	$\lim_{x\to 3} f(x)$ 5	n.	$\lim_{x\to -4^+} f(x)$ ∞	o.	$\lim_{x\to \infty} f(x)$ 0
p.	$f(1)$ 1	q.	$\lim_{x\to -3} f(x)$ 1	r.	$f(-4)$ DNE

3. Use the graph of the function $f(x)$ to answer each question. Use $\infty, -\infty, or\ DNE$ where appropriate.



a. $f(0) =$ DNE

b. $f(2) =$ 0

c. $f(3) =$ 3

d. $\lim_{x \to 0^-} f(x) =$ $-\infty$

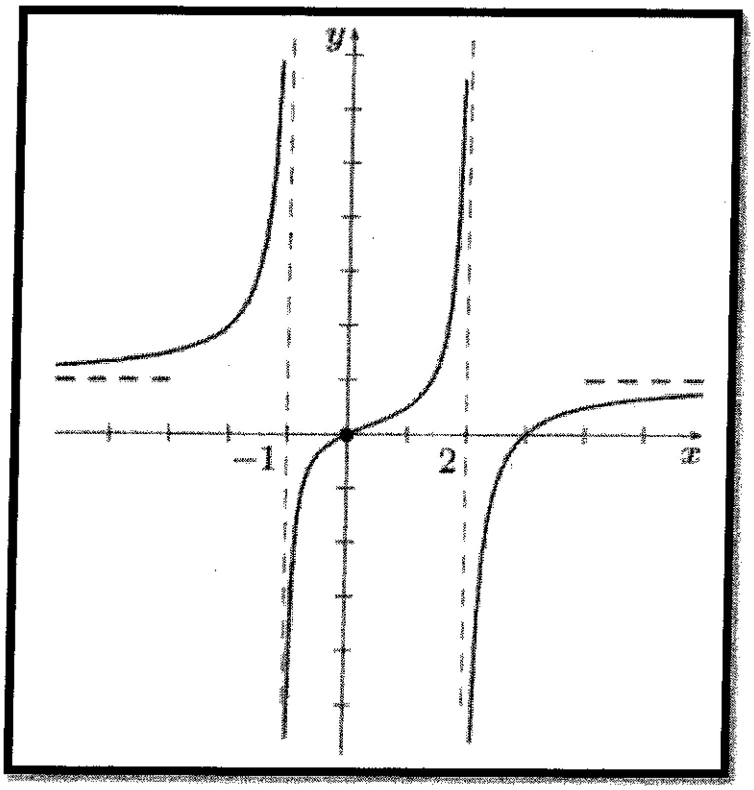
e. $\lim_{x \to 0} f(x) =$ DNE

f. $\lim_{x \to 3^+} f(x) =$ 2

g. $\lim_{x \to 3} f(x) =$ DNE

h. $\lim_{x \to -\infty} f(x) =$ 1

4. Use the graph of the function $f(x)$ to answer each question. Use $\infty, -\infty, or\ DNE$ where appropriate.



a. $f(0) =$ 0

b. $f(2) =$ DNE

c. $f(3) =$ 0

d. $\lim_{x \to -1} f(x) =$ DNE

e. $\lim_{x \to 0} f(x) =$ 0

f. $\lim_{x \to 2^+} f(x) =$ $-\infty$

g. $\lim_{x \to \infty} f(x) =$ 1

Graphs from Limit Worksheet

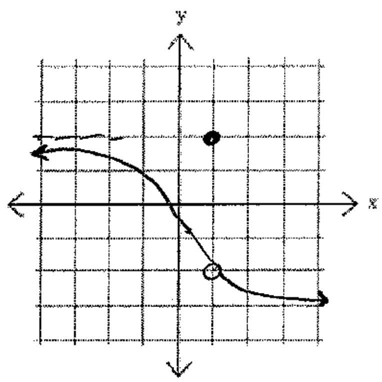
Draw a graph of a function with the give limits.

1. $\lim_{x\to\infty} f(x) = -3$

$\lim_{x\to 1} f(x) = -2$

$\lim_{x\to -\infty} f(x) = 2$

$f(1) = 2$

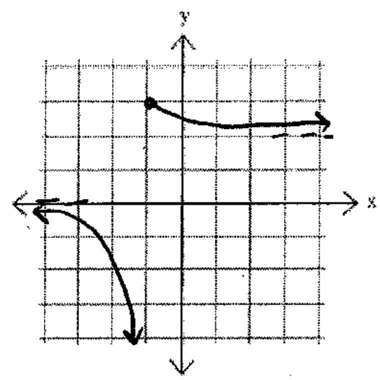


2. $\lim_{x\to\infty} f(x) = 2$

$\lim_{x\to -2^+} f(x) = 3$

$\lim_{x\to -2^-} f(x) = \infty$

$\lim_{x\to -\infty} f(x) = 0$

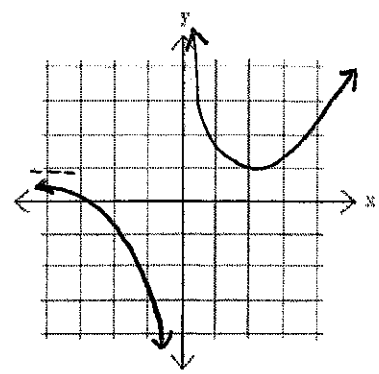


3. $\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to 0^+} f(x) = \infty$

$\lim_{x\to 0^-} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = 1$



4. $\lim_{x\to -\infty} f(x) = -\infty$

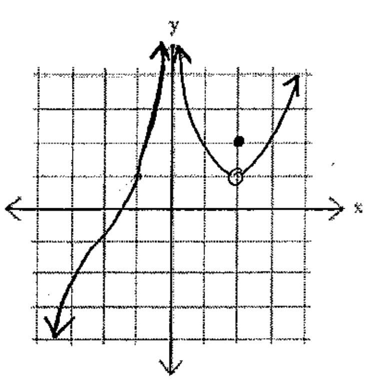
$\lim_{x\to -1} f(x) = 1$

$\lim_{x\to 0} f(x) = \infty$

$\lim_{x\to 2} f(x) = 1$

$f(2) = 2$

$\lim_{x\to\infty} f(x) = \infty$



5. $\lim_{x\to -\infty} f(x) = -\infty$

$\lim_{x\to -2^-} f(x) = \infty$

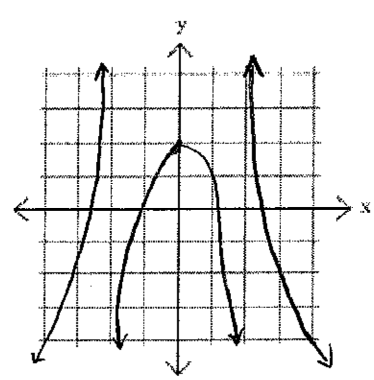
$\lim_{x\to -2^+} f(x) = -\infty$

$\lim_{x\to 0} f(x) = 2$

$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 2^+} f(x) = \infty$

$\lim_{x\to\infty} f(x) = -\infty$



6. $\lim_{x\to -\infty} f(x) = -2$

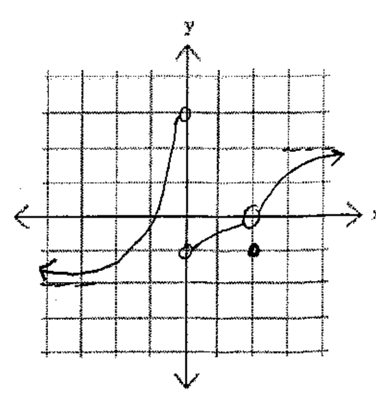
$\lim_{x\to 0^-} f(x) = 3$

$\lim_{x\to 0^+} f(x) = -1$

$\lim_{x\to 2} f(x) = 0$

$\lim_{x\to\infty} f(x) = 2$

$f(2) = -1$



One-sided Limits Worksheet

Evaluate each limit.

1. $\lim\limits_{x\to 2^+} \frac{x}{x-2}$ $\quad \frac{2.01}{2.01-2} = +\infty$

2. $\lim\limits_{x\to 3^+} \frac{x+1}{x^2-6x+9}$ $\quad \frac{3.01+1}{3.01^2-6\cdot 3.01+9} = \infty$

3. $\lim\limits_{x\to -3^-} \frac{x+2}{x^2+6x+9}$ $\quad -\infty$

4. $\lim\limits_{x\to -2^+} \frac{x-2}{x^2+4x+4}$ $\quad -\infty$

5. $\lim\limits_{x\to -3^-} \frac{x^2}{3x+9}$ $\quad -\infty$

6. $\lim\limits_{x\to 2^+} \frac{x^2}{2x-4}$ $\quad \infty$

7. $\lim\limits_{x\to -2^+} \frac{1}{x^2-4}$ $\quad \infty$

8. $\lim\limits_{x\to 1^-} -\frac{2}{x^2-1}$ $\quad -\infty$

9. $\lim\limits_{x\to 3^-} f(x), f(x) = \begin{cases} -x+4, & x<3 \\ \frac{x}{2}+1, & x\geq 3 \end{cases}$

   $-3+4 = \boxed{1}$

10. $\lim\limits_{x\to -1^+} f(x), f(x) = \begin{cases} x+3, & x\leq -1 \\ -x-1, & x>-1 \end{cases}$

    $-(-1)-1 = \boxed{0}$

11. $\lim\limits_{x\to -2^-} f(x), f(x) = \begin{cases} -x^2-8x-17, & x\leq -2 \\ 2x-1, & x>-2 \end{cases}$

    $-(-2)^2-8(-2)-17$

    $-4+16-17 = \boxed{-5}$

12. $\lim\limits_{x\to 1^-} (|x-1|-2)$

    $-(x-1)-2$

    $\boxed{-2}$

13. $\lim\limits_{x\to 0^+} \frac{2x}{|x|}$ $\quad \frac{2x}{x} = \boxed{2}$

14. $\lim\limits_{x\to 1^-} f(x), f(x) = \begin{cases} -\frac{x}{2}-\frac{3}{2}, & x\leq 1 \\ -x^2+4x-5, & x>1 \end{cases}$

    $-\frac{1}{2}-\frac{3}{2} = -\frac{4}{2} = \boxed{-2}$

15. $\lim\limits_{x\to -3^-} f(x), f(x) = \begin{cases} x+6, & x<-3 \\ 3, & x\geq -3 \end{cases}$

    $-3+6 = \boxed{3}$

16. $\lim\limits_{x\to 0^-} f(x), f(x) = \begin{cases} -2x+3, & x\leq 0 \\ -\frac{x}{2}+3, & x>0 \end{cases}$

    $-2(0)+3 = 3$

Continuity Worksheet

Determine if each function is continuous. If the function is not continuous, find the x-axis location of and classify each discontinuity.

1. $f(x) = -\frac{x}{2x^2+2x+1}$

Continuous

2. $f(x) = \frac{x}{x^2+6x+9}$

(x^2+6x+9)
$(x+3)^2$
$x=-3$

Infinite Disc at $x=-3$

3. $f(x) = \frac{x^2+4x+3}{x+3}$

$\frac{\cancel{(x+3)}(x+1)}{\cancel{x+3}}$

Removable @ $x=3$

4. $f(x) = \frac{x}{x^2-4x}$

$\frac{x}{x(x-4)}$

Removable @ $x=0$
Infinite @ $x=4$

5. $f(x) = \begin{cases} x+4, & x \le -2 \\ -2x-11, & x > -2 \end{cases}$

$-2+4=2$
$4-11=-7$

Jump @ $x=-2$

6. $f(x) = \frac{x+7}{x^2+3x}$

$\frac{x+7}{x(x+3)}$

Infinite @ $x=0,-3$

Find the intervals on which each function is continuous.

7. $f(x) = \begin{cases} x, & x \ne 4 \\ 2, & x = 4 \end{cases}$

$(-\infty,4)\cup(4,\infty)$

8. $f(x) = \begin{cases} -2, & x < 3 \\ -2x+6, & x \ge 3 \end{cases}$

$-2(3)+6=0$

$(-\infty,3)\cup[3,\infty)$

9. $f(x) = \frac{(x-1)}{x^2-4x+3}$

$(x-3)(x-1)$

Remov. @ 1
Inf @ 3

$(-\infty,1)\cup(1,3)\cup(3,\infty)$

10. $f(x) = \frac{x^2}{2} + 4x + 10$

$(-\infty,\infty)$

11. $f(x) = -x^2 - 4x + 2$

$(-\infty,\infty)$

12. $f(x) = -\frac{x-2}{x^2-3x+2}$

$(x-2)(x-1)$

Remov. @ 2
Inf @ 1

$(-\infty,1)\cup(1,2)\cup(2,\infty$

13. $f(x) = -\frac{x-1}{x^2-x}$

$x(x-1)$

$(-\infty,0)\cup(0,1)\cup(1,\infty)$

14. $f(x) = \frac{x}{x^2-6x+9}$

$(x-3)^2$

$(-\infty,3)\cup(3,\infty)$

15. Critical Thinking: Write a function that has an infinite discontinuity at $x = 100$

$f(x) = \frac{1}{x-100}$

16 Critical Thinking: Write a function that is continuous over $(-\infty, 0), (0,1), and\ (1, \infty)$ and discontinuous everywhere else.

$f(x) = \frac{x-1}{x^2-x}$

Creative Factoring and Other Interesting Algebra

Difference of Squares

Example: $x-16=(\sqrt{x}+4)(\sqrt{x}-4)$

1. $x-9$ **2.** x^2-5 **3.** $x^{16}-1$ **4.** $(x+5)^2-25$ **5.** $9y-a^4$

$(\sqrt{x}+3)(\sqrt{x}-3)$ $(x+\sqrt{5})(x-\sqrt{5})$ $(x^8+1)(x^8-1)$ $(x+5+5)(x+5-5)$ $(3\sqrt{y}-a^2)(3\sqrt{y}+a^2)$

$x(x+10)$

Sums or Differences of Cubes **"SOAP"**

Example: $a^3+b^3=(a+b)(a^2-ab+b^2)$ **Example:** $a^3-b^3=(a-b)(a^2+ab+b^2)$

6. $64a^3+125b^3$ **7.** $64a^3x^3-125$ **8.** $(x+1)^3+64$ **9.** $8c^3-(a+b)^3$

$(4a+5b)(16a^2-20ab+25b^2)$

$(4ax-5)(16a^2x^2+20ax+25)$

$(x+1+4)((x+1)^2-4(x+1)+16)$

$(x+5)(x^2-2x+13)$

$(2c-a-b)\cdot$

$(2c-a-b)(4c^2+2c(a+b)+(a+b)^2)$

Factor: x^6-y^6:

10. as a difference of **squares**

$(x^3-y^3)(x^3+y^3)$

$(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$

11. as a difference of **cubes**

$(x^2-y^2)(x^4+x^2y^2+y^4)$

$(x+y)(x-y)(x^4+x^2y^2+y^4)$

Compare these two. Which way will allow you to factor completely most easily?

Rationalize the Numerator

12. $\dfrac{\sqrt{x+2}-\sqrt{2}}{x}\cdot\dfrac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}=\dfrac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})}=\dfrac{1}{\sqrt{x+2}+\sqrt{2}}$

13. $\dfrac{\sqrt{x+3}+\sqrt{3}}{x}\cdot\dfrac{\sqrt{x+3}-\sqrt{3}}{\sqrt{x+3}-\sqrt{3}}=\dfrac{x+3-3}{x(\sqrt{x+3}-\sqrt{3})}=\dfrac{1}{\sqrt{x+3}-\sqrt{3}}$

Factor completely. **Use synthetic division to help find all factors.**

14. $x^3+6x^2+5x-12$

$$\begin{array}{r|rrrr} 1 & 1 & 6 & 5 & -12 \\ & & 1 & 7 & 12 \\ \hline & 1 & 7 & 12 & \end{array}$$

$(x-1)(x+4)(x+3)$

15. $x^3+x^2-8x-12$

$$\begin{array}{r|rrrr} 3 & 1 & 1 & -8 & -12 \\ & & 3 & 12 & 12 \\ \hline & 1 & 4 & 4 & \end{array}$$

$(x-3)(x+2)(x+2)$

16. $x^3+6x^2-9x-14$

$$\begin{array}{r|rrrr} -1 & 1 & 6 & -9 & -14 \\ & & -1 & -5 & 14 \\ \hline & 1 & 5 & -14 & \end{array}$$

$(x+1)(x+7)(x-2)$

Simplify:

17. $\dfrac{2x^3+7x^2+8x+3}{x+1}$

$$\begin{array}{r|rrrr} -1 & 2 & 7 & 8 & 3 \\ & & -2 & -5 & -3 \\ \hline & 2 & 5 & 3 & \end{array}$$

$2x^2+5x+3$

$(x+1)(2x+3)$

18. $\dfrac{2x^3+x^2-13x+6}{x+3}$

$$\begin{array}{r|rrrr} -3 & 2 & 1 & -13 & 6 \\ & & -6 & 15 & -6 \\ \hline & 2 & -5 & 2 & \end{array}$$

$2x^2-5x+2$

$(x-2)(2x-1)$

Algebraic Limits Worksheet

1. $\lim_{x\to3} x^2+2x-7$ $9+6-7=\boxed{8}$	2. $\lim_{x\to-1}\frac{\frac{1}{x}+1}{x+1}$ $\frac{1+x}{x}\cdot\frac{1}{x+1}=\frac{1}{x}$ $\boxed{-1}$	3. $\lim_{x\to1}\frac{(4x^4-5x^2+1)}{x^2+2x-3}$ $\boxed{\frac{3}{2}}$
4. $\lim_{x\to0}\frac{(x+1)^2-1}{x}$ $\frac{\cancel{(x+1-1)}(x+1+1)}{\cancel{x}}$ $\boxed{2}$	5. $\lim_{x\to1}\frac{x^2-2x-15}{x-5}$ $\frac{1-2-5}{1-5}=\frac{-6}{-4}=\boxed{\frac{3}{2}}$	6. $\lim_{x\to-3}\frac{2x^2+2x-12}{x^2+4x+3}$ $\frac{2\cancel{(x+3)}(x-2)}{\cancel{(x+3)}(x+1)}=\frac{2(-5)}{-2}$ $=\boxed{5}$
7. $\lim_{x\to2}\frac{(2x+1)^2-25}{x-2}$ $2(x-2)$ $\frac{(2x+1+5)\cancel{(2x+1-5)}}{\cancel{x-2}}$ $2(2x+6)=\boxed{20}$	8. $\lim_{x\to2}\frac{(3x-2)^2-(x+2)^2}{x-2}$ $\frac{(3x-2+x+2)(3x-2-x-2)}{x-2}$ $8x=16$	9. $\lim_{x\to1}\frac{\frac{2x}{x+1}-1}{x-1}$ $\frac{1}{x+1}$ $\boxed{\frac{1}{2}}$
10. $\lim_{x\to2}\frac{\frac{2}{x^2}-\frac{1}{2}}{x-2}$ $\boxed{-\frac{1}{2}}$	11. $\lim_{x\to2}\frac{x^4-2x^2-8}{x^2-x-6}$ $\frac{0}{-4}=\boxed{0}$	12. $\lim_{x\to1}\frac{x-1}{\sqrt{x}-1}$ $\frac{(\sqrt{x}+1)\cancel{(\sqrt{x}-1)}}{\cancel{\sqrt{x}-1}}$ $\boxed{2}$
13. $\lim_{x\to0}\frac{x^2+7x+6}{x+3}=\frac{6}{3}$ $\boxed{2}$	14. $\lim_{x\to-2}\frac{\frac{x}{x+4}+1}{x+2}$ $\boxed{1}$	15. $\lim_{x\to2}\frac{x^3+x^2-4x-4}{x^2+x-6}$ $\boxed{\frac{12}{5}}$
16. $\lim_{x\to2}\frac{x^2-2x-3}{x+5}$ $\frac{4-4-3}{7}$ $\frac{-3}{7}$	17. $\lim_{x\to2}(x^2-x+1)$ $4-2+1=3$	18. $\lim_{x\to1}\frac{2x+1}{3x-2}$ $\frac{2+1}{3-2}$ $\boxed{3}$
19. $\lim_{x\to1}\sqrt{10x-1}$ $\sqrt{9}=\boxed{3}$	20. $\lim_{x\to1}\frac{x^2-x-2}{x-2}$ $\frac{\cancel{(x-2)}(x+1)}{\cancel{x-2}}$ $\boxed{2}$	21. $\lim_{x\to4}\frac{\sqrt{x}-2}{x-4}$ $\frac{\cancel{\sqrt{x}-2}}{(\sqrt{x}+2)\cancel{(\sqrt{x}-2)}}$ $\frac{1}{2+2}=\boxed{\frac{1}{4}}$

22. $\lim\limits_{x\to3}\frac{x^2-9}{x+3}$ $x-3$ $\boxed{0}$	23. $\lim\limits_{x\to3}\frac{x^2-9}{2x^2+7x+3}$ $\frac{(x+3)(x-3)}{(2x+1)\cancel{(x+3)}}=\frac{0}{7}=\boxed{0}$	24. $\lim\limits_{x\to9}\frac{\sqrt{x}-3}{x-9}$ $\frac{\cancel{\sqrt{x}-3}}{(\sqrt{x}+3)\cancel{(\sqrt{x}-3)}}$ $\boxed{\frac{1}{6}}$
25. $\lim\limits_{h\to0}\frac{(1+h)^2-1^2}{h}$ $\frac{(1+h-1)(1+h+1)}{h}=h+2$ $=\boxed{2}$	26. $\lim\limits_{h\to0}\frac{(3+h)^2-3^2}{h}$ $\frac{(3+h-3)(3+h+3)}{h}=6+h$ $=\boxed{6}$	27. $\lim\limits_{h\to0}\frac{(x+h)^2-x^2}{h}$ $\frac{(x+h-x)(x+h+x)}{h}$ $=h+2x=\boxed{2x}$
28. $\lim\limits_{x\to3}(5x^2-6)$ $5(9)-6=\boxed{39}$	29. $\lim\limits_{x\to-1}\frac{x-2}{x^2+4x-3}$ $\frac{-3}{1-4-3}$ $=\frac{-3}{-6}$ $=\boxed{\frac{1}{2}}$	30. $\lim\limits_{x\to4}\frac{x^2-16}{x-4}=x+4$ $=\boxed{8}$
31. $\lim\limits_{x\to0}\frac{6x-9}{x^3-12x+3}=\frac{-9}{3}$ $=\boxed{-3}$	32. $\lim\limits_{x\to2}\frac{x^2-4x+4}{x^2+x-6}$ $\frac{(x-2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$ $\frac{0}{5}=\boxed{0}$	33. $\lim\limits_{x\to-2}\frac{x^3+8}{x+2}=x^2-2x+4$ $=4+4+4$ $=\boxed{12}$

Given $\lim\limits_{x\to a}f(x)=-3$, $\lim\limits_{x\to a}g(x)=0$, *and* $\lim\limits_{x\to a}h(x)=8$, find each limit if it exists.

34. $\lim\limits_{x\to a}[f(x)+h(x)]$ $-3+8=\boxed{5}$	35. $\lim\limits_{x\to a}[f(x)]^2$ $=(-3)^2=\boxed{9}$	36. $\lim\limits_{x\to a}\sqrt[3]{h(x)}=\sqrt[3]{8}$ $=\boxed{2}$
37. $\lim\limits_{x\to a}\frac{1}{f(x)}=\boxed{-\frac{1}{3}}$	38. $\lim\limits_{x\to a}\frac{g(x)}{h(x)}$ $\frac{0}{8}=\boxed{0}$	39. $\lim\limits_{x\to a}\frac{h(x)}{g(x)}$ $\frac{8}{0}$ $\boxed{\text{DNE}}$
40. $\lim\limits_{x\to a}\frac{2f(x)}{h(x)-f(x)}=\frac{2(-3)}{8+3}$ $=\boxed{\frac{-6}{11}}$	41. $\lim\limits_{x\to a}[f(x)h(x)]$ $-3(8)=\boxed{-24}$	42. $\lim\limits_{x\to a}\left[\frac{g(x)+h(x)}{f(x)}\right]$ $\frac{0+8}{-3}=\boxed{-\frac{8}{3}}$

Intermediate Value Theorem Worksheet

1. Verify the conditions of the IVT and find the guaranteed c value over [2,6] for
$$f(x) = x^2 + 2x - 11$$
when $f(c) = 4$.

$f(2) = -3$
$f(6) = 37$
$-3 < 4 < 37$

$x^2 + 2x - 11 = 4$
$x^2 + 2x - 15 = 0$
$(x+5)(x-3) = 0$
$x = -5, 3$
$\boxed{c = 3}$

2. Verify the conditions of the IVT and find the guaranteed c value over $[-1,3]$ for
$$f(x) = 2x^2 + x - 4$$
when $f(c) = 2$.

$f(-1) = -3$
$f(3) = 17$
$-3 < 2 < 17$

$2x^2 + x - 4 = 2$
$2x^2 + x - 6 = 0$
$(2x-3)(x+2) = 0$
$x = 3/2, -2$
$\boxed{c = 3/2}$

3. Use the IVT to show that
$$f(x) = x^3 - 3x^2 - 7x + 1$$
has a root in the interval (4,5)

$f(4) = -11$
$f(5) = 16$

$-11 < 0 < 16$

4. Use the IVT to show that
$$f(x) = x^4 + 3x^2 - 6$$
has a root in the interval $(1,2)$ *and* $(-2,-1)$

$f(1) = -2$
$f(2) = 22$
$-2 < 0 < 22$

$f(-2) = 22$
$f(-1) = -2$
$-2 < 0 < 22$

5. Verify the conditions of the IVT and find the guaranteed c value over [0,5] for
$$f(x) = x^2 + x - 1$$
when $f(c) = 11$.

$f(0) = -1$
$f(5) = 29$
$-1 < 11 < 29$

$x^2 + x - 1 = 11$
$x^2 + x - 12 = 0$
$(x+4)(x-3) = 0$
$x = 3, -4$
$\boxed{c = 3}$

6. Verify the conditions of the IVT and find the guaranteed c value over [0,3] for
$$f(x) = x^2 - 6x + 8$$
when $f(c) = 0$.

$f(0) = 8$
$f(3) = -1$
$-1 < 0 < 8$

$x^2 - 6x + 8 = 0$
$(x-4)(x-2) = 0$
$x = 4, 2$
$\boxed{c = 2}$

7. Verify the conditions of the IVT and find the guaranteed c value over [0,3] for
$$f(x) = x^3 - x^2 + x - 2$$
when $f(c) = 4$.

$f(0) = -2$
$f(3) = 19$
$-2 < 4 < 19$

$x^3 - x^2 + x - 2 = 4$
$x^3 - x^2 + x - 6 = 0$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & 1 & -6 \\ & & 2 & 2 & 6 \\ \hline & 1 & 1 & 3 & \end{array}$$

$\boxed{c = 2}$

8. Verify the conditions of the IVT and find the guaranteed c value over $[\frac{5}{2}, 4]$ for
$$f(x) = \frac{x^2 + x}{x - 1}$$
when $f(c) = 6$.

$f(5/2) = \frac{35}{6}$
$f(4) = \frac{20}{3}$
$\frac{35}{6} < 6 < \frac{20}{3}$

$x^2 + x = 6x - 6$
$x^2 - 5x + 6 = 0$
$(x-3)(x-2) = 0$
$x = 2, 3$
$\boxed{c = 3}$

9. Use the IVT to show that
$$f(x) = x^3 + x - 1$$
has a root in the interval [0,1]

$f(0) = -1$
$f(1) = 1 + 1 - 1 = 1$

$-1 < 0 < 1$

10. Use the IVT to show that
$$f(x) = x^3 + 3x - 2$$
has a root in the interval [0,1]

$f(0) = -2$
$f(1) = 2$

$-2 < 0 < 2$

Infinite Limits Worksheet

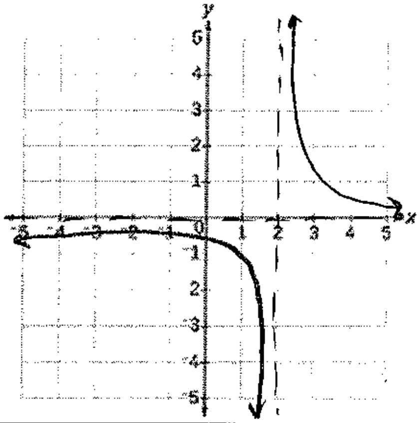
Find the Limit.

1. $\lim_{x\to\infty} 3$ $\boxed{3}$	2. $\lim_{x\to-\infty} 3$ $\boxed{3}$	3. $\lim_{x\to-\infty} (-3)$ $\boxed{-3}$
4. $\lim_{x\to\infty} (-2x)$ $\boxed{-\infty}$	5. $\lim_{x\to\infty} (3-x)$ $\boxed{-\infty}$	6. $\lim_{x\to\infty} \sqrt{x}$ $\boxed{\infty}$
7. $\lim_{x\to-\infty} (4-x)$ $\boxed{\infty}$	8. $\lim_{x\to\infty} \frac{8}{5-3x}$ $\boxed{0}$	9. $\lim_{x\to\infty} \frac{1}{x-12}$ $\boxed{0}$
10. $\lim_{x\to-\infty} \frac{3}{x+4}$ $\boxed{0}$	11. $\lim_{x\to\infty} (1+2x-3x^5)$ $\boxed{-\infty}$	12. $\lim_{x\to\infty} (2x^3-110x+5)$ $\boxed{\infty}$
13. $\lim_{x\to\infty} \frac{(3+2x^2)}{4+5x}$ $\boxed{\infty}$	14. $\lim_{x\to\infty} \frac{x^2+x}{3-x}$ $\boxed{\infty}$	15. $\lim_{x\to\infty} \frac{x+4}{x^2-2x+5}$ $\boxed{0}$
16. $\lim_{x\to-\infty} -\frac{x-2}{x^2+2x+1}$ $\boxed{0}$	17. $\lim_{x\to\infty} \frac{7-6x^5}{x+3}$ $\boxed{-\infty}$	18. $\lim_{x\to\infty} \frac{6-x^3}{7x^3+3}$ $\boxed{-\frac{1}{7}}$
19. $\lim_{x\to\infty} \frac{1}{x^2+1}$ $\boxed{0}$	20. $\lim_{x\to\infty} \frac{x^4+x^2}{x^4+1}$ $\boxed{1}$	21. $\lim_{x\to\infty} \frac{1+x^2}{2-x^2}$ $\boxed{-1}$
22. $\lim_{x\to\infty} \frac{2x^2}{x^2+1}$ $\boxed{2}$	23. $\lim_{x\to-\infty} \frac{x+4}{3x^2-5}$ $\boxed{0}$	24. $\lim_{x\to\infty} \frac{3x^3+25x^2-x+1}{4x^3-7x^2+2x+2}$ $\boxed{\frac{3}{4}}$

Vertical and Horizontal Asymptotes Worksheet

State the vertical, horizontal, or slant asymptotes for the following. Sketch the graph and find the end behavior.

1. $f(x) = \frac{3}{x-2}$



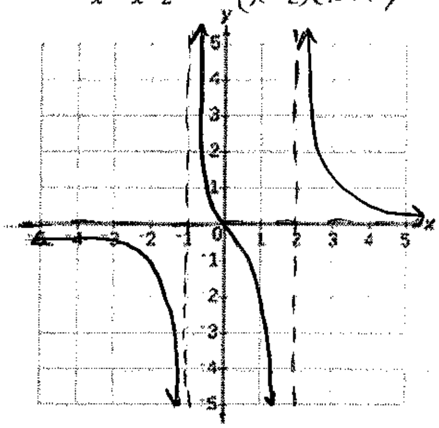
Vertical Asymptote: $x = 2$

Horizontal Asymptote: $y = 0$

Slant Asymptote: —

End Behavior: $\lim_{x\to\infty} f(x) =$ 0
$\lim_{x\to-\infty} f(x) =$ 0

2. $f(x) = \frac{3x}{x^2-x-2}$ $\quad \frac{3x}{(x-2)(x+1)}$



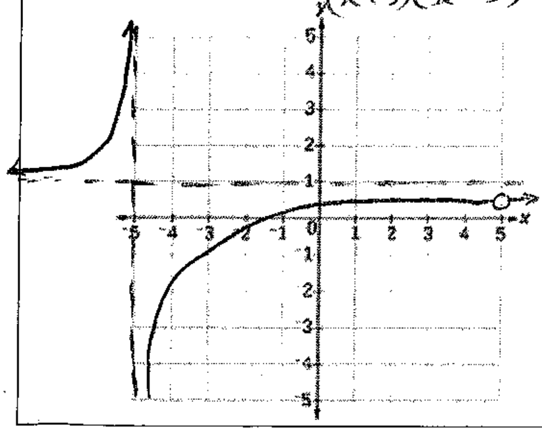
Vertical Asymptote: $x = 2, -1$

Horizontal Asymptote: $y = 0$

Slant Asymptote: —

End Behavior: $\lim_{x\to\infty} f(x) =$ 0
$\lim_{x\to-\infty} f(x) =$ 0

3. $f(x) = \frac{x^2-5x}{x^2-25}$ $\quad \frac{x\cancel{(x-5)}}{(x+5)\cancel{(x-5)}}$



Vertical Asymptote: $x = -5$

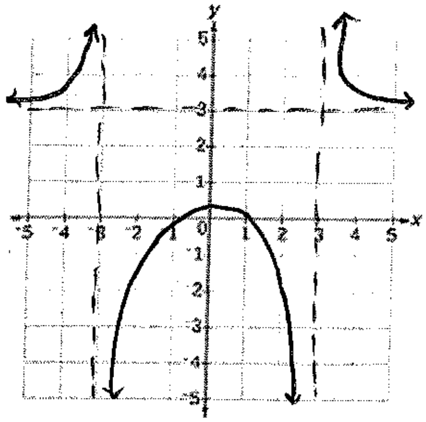
Horizontal Asymptote: $y = 1$

Slant Asymptote: —

Removable disc $(5, 1/2)$

End Behavior: $\lim_{x\to\infty} f(x) =$ 1
$\lim_{x\to-\infty} f(x) =$ 1

4. $f(x) = \frac{3x^2-4}{x^2-9}$



Vertical Asymptote: $x = 3 \quad x = -3$

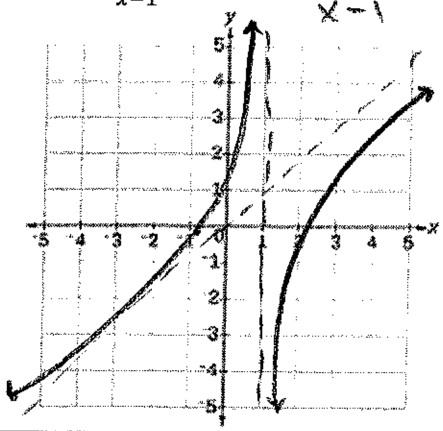
Horizontal Asymptote: $y = 3$

Slant Asymptote: —

End Behavior: $\lim_{x \to \infty} f(x) = 3$

$\lim_{x \to -\infty} f(x) = 3$

5. $f(x) = \frac{x^2-x-2}{x-1}$ $\frac{(x-2)(x+1)}{x-1}$



Vertical Asymptote: $x = 1$

$$\begin{array}{r} x \\ x-1 \overline{\smash{)}x^2-x-2} \\ x^2-x \\ \hline -2 \end{array}$$

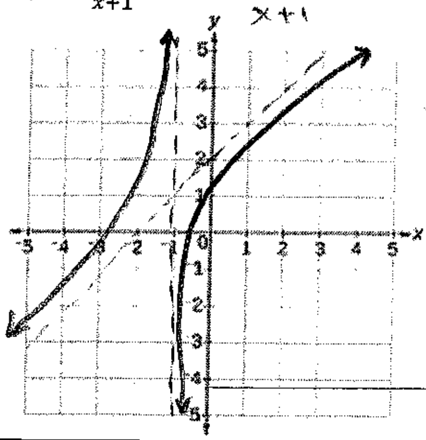
Horizontal Asymptote: —

Slant Asymptote: $y = x$

End Behavior: $\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -\infty$

6. $f(x) = \frac{x^2+3x}{x+1}$ $\frac{x(x+3)}{x+1}$



Vertical Asymptote: $x = -1$

$$\begin{array}{r|rrr} -1 & 1 & 3 & 0 \\ & & -1 & -2 \\ \hline & 1 & 2 & -2 \end{array}$$

Horizontal Asymptote: —

or

$$\begin{array}{r} x+2 \\ x+1 \overline{\smash{)}x^2+3x+0} \\ x^2+x \\ \hline 2x+0 \end{array}$$

Slant Asymptote: $y = x + 2$

End Behavior: $\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -\infty$

Limits Review

Limit of a constant is a constant.

1. $\lim_{x \to e} \sqrt{7}$ $\boxed{\sqrt{7}}$

2. $\lim_{x \to \sqrt{5}} \pi$ $\boxed{\pi}$

Direct Substitution – ALWAYS try direct substitution first!

3. $\lim_{x \to 5} \left(2x^2 - x + 3\right)$

$2(25) - 5 + 3$

$50 - 5 + 3$

$\boxed{48}$

4. $\lim_{y \to 2^-} \frac{y^2 - 3y + 2}{y + 1}$ $\frac{4 - 6 + 2}{2 + 1}$

$\frac{0}{3} = \boxed{0}$

5. $\lim_{x \to 4} \frac{|5 - 3x|}{2x + 1}$ $\frac{5 - 3(4)}{2(4) + 1} = \frac{5 - 12}{9} = \boxed{\frac{-7}{9}}$

6. $\lim_{x \to 4} \cos\left(\frac{3\pi}{x}\right)$ $\cos \frac{3\pi}{4} = \boxed{-\frac{\sqrt{2}}{2}}$

If substitution results in $\frac{0}{0}$, Factor, reduce, and substitute again.

7. $\lim_{x \to 1} \frac{x^4 - 1}{x - 1}$ $\frac{(x^2-1)(x^2+1)}{x-1} = (x+1)(x^2+1)$

$= 2 \cdot 2 = \boxed{4}$

8. $\lim_{x \to 1} \frac{x - 1}{x^3 - x^2 + x - 1}$ $\frac{x-1}{(x^2+1)(x-1)}$

$\frac{1}{x^2+1} = \boxed{\frac{1}{2}}$

9. $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$ $\frac{\sqrt{x} - 3}{(\sqrt{x} - 3)(\sqrt{x} + 3)} = \frac{1}{\sqrt{x} + 3}$

$= \boxed{\frac{1}{6}}$

10. $\lim_{x \to -2} \frac{x + 2}{x^3 + 8}$ $= \frac{x+2}{(x+2)(x^2 - 2x + 4)} = \frac{1}{x^2 - 2x + 4}$

$= \frac{1}{4 + 4 + 4}$

$= \boxed{\frac{1}{12}}$

Multiply by the conjugate.

11. $\lim_{x \to 2} \frac{\sqrt{5x + 6} - 4}{x - 2}$

$\frac{5x + 6 - 16}{(x-2)(\sqrt{5x+6} + 4)}$

$\frac{5}{\sqrt{5x+6} + 4} = \boxed{\frac{5}{8}}$

12. $\lim_{x \to 4} \frac{3 - \sqrt{x + 5}}{x - 4}$

$\frac{9 - x - 5}{(x-4)(3 + \sqrt{x+5})}$

$\frac{-1}{3 + \sqrt{x+5}} = \boxed{-\frac{1}{6}}$

13. $\lim_{x \to 0} \frac{\sqrt{x + 3} - \sqrt{3}}{x}$ $= \frac{x + 3 - 3}{x(\sqrt{x+3} + \sqrt{3})}$

$= \frac{1}{\sqrt{x+3} + \sqrt{3}} = \frac{1}{2\sqrt{3}} = \boxed{\frac{\sqrt{3}}{6}}$

Complex fractions – clear the "little denominators"

14. $\lim_{h \to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$

$\frac{2 - 2 - h}{2(2+h)} \cdot \frac{1}{h}$

$\frac{-1}{2(2+h)} = \boxed{-\frac{1}{4}}$

15. $\lim_{x \to 10} \frac{\frac{x}{5} - 2}{x - 10}$

$\frac{x - 10}{5} \cdot \frac{1}{x - 10}$

$\frac{1}{5}$

16. $\lim_{h \to -2} \frac{(h+5)^{-1} - 3^{-1}}{h + 2}$ $= \frac{\frac{1}{h+5} - \frac{1}{3}}{h+2}$

$= \frac{3 - h - 5}{3(h+5)} \cdot \frac{1}{h+2}$

$= \frac{-1}{3(h+5)} = \boxed{\frac{-1}{9}}$

Rewrite the absolute value.

Reminder, if the inside is positive when you substitute in use the positive of the inside, if the inside is negative when you substitute in use the negative of the inside.

17. $\lim_{x\to 5^-} \frac{|2x-10|}{3x-15}$ $= -\frac{2}{3}$

18. $\lim_{x\to 7^-} \frac{3x-21}{|7-x|}$ $= -3$

19. $\lim_{x\to 2} \frac{|x-2|}{x-2}$ $=$ DNE

One sided limits when you get $\frac{\#}{0}$, do you get ∞ or $-\infty$? Reason it out!

20. $\lim_{x\to 3^-} \frac{5}{x-3}$ $-\infty$

21. $\lim_{x\to 3^+} \frac{-4}{x-3}$ $-\infty$

22. $\lim_{x\to 6^+} \frac{x+6}{x^2-36}$ ∞

Limits to infinity. You can do a **behaves like** only in limits to infinity. You can also divide by the highest power in the denominator, simplify, and then take the limit.

$\lim_{x\to\pm\infty}(\text{polynomial})$ The highest power controls the behavior!

23. $\lim_{x\to\infty}(3x^2-4x+2)$ ∞

24. $\lim_{x\to-\infty}(5x^3-2x^2+1)$ $-\infty$

$$\lim_{x\to\pm\infty} \frac{\text{degree smaller}}{\text{DEGREE LARGER}} = 0$$

25. $\lim_{x\to\infty} \frac{3x-5}{x^2+1}$ 0

26. $\lim_{x\to-\infty} \frac{4x^2-3x}{6x^5-3x+1}$ 0

$$\lim_{x\to\pm\infty} \frac{\text{degree} =}{\text{degree} =} = \text{ratio of the leading coefficients}$$

27. $\lim_{x\to\infty} \frac{5x-11}{4-3x}$ $-\frac{5}{3}$

28. $\lim_{x\to-\infty} \frac{4x^2-5x+2}{3x^2+1}$ $\frac{4}{3}$

29. $\lim_{x\to\infty} \frac{2x^2+1}{\cancel{(2-x)(2+x)}}$ $= -2$

$4-x^2$

$$\lim_{x\to\pm\infty} \frac{\text{DEGREE LARGER}}{\text{degree smaller}} = \infty \text{ or } -\infty$$

30. $\lim_{x\to\infty} \frac{7-6x^5}{x+3}$ $-\infty$

31. $\lim_{x\to-\infty} \frac{7-6x^5}{x+3}$ ∞

32. $\lim_{x\to\infty} \frac{5+x^3-3x^4}{2x-1}$ $-\infty$

33. $\lim_{x\to-\infty} \frac{5+x^3-3x^4}{2x-1}$ ∞

$\lim\limits_{x\to\pm\infty}$ involving square roots: Use the **behaves like** method and remember that $\sqrt{x^2}=|x|$!

34. $\lim\limits_{x\to\infty}\dfrac{\sqrt{4x^2-2}}{x+1}$ 2

35. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{4x^2-2}}{x+1}$ -2

36. $\lim\limits_{x\to\infty}\dfrac{2-x}{\sqrt{7+9x^2}}$ $-\frac{1}{3}$

37. $\lim\limits_{x\to-\infty}\dfrac{2-x}{\sqrt{7+9x^2}}$ $\frac{1}{3}$

Write the equations of all vertical and horizontal asymptotes.

38. $y=\dfrac{2x^2-5x-3}{x^2-2x-3}$ $\dfrac{(2x+1)(x-3)}{(x-3)(x+1)}$ VA: $x=-1$ HA: $y=2$

39. $y=\dfrac{3-x}{9-x^2}$ $(3-x)(3+x)$ $x=-3$ $y=0$

Continuity: Limit from right = limit from left = value of $f(x)$ at the point.

Is $f(x)$ continuous? Why?

40. $f(x)=\begin{cases}-5-x, & x>-1\\ 6x+2, & x\le -1\end{cases}$ yes

$-5+1=-4$

$6(-1)+2=-4$

$\lim\limits_{x\to-1^+}f(x)=\lim\limits_{x\to-1^-}f(x)=f(-1)$

41. $f(x)=\dfrac{|x+2|}{x+2}$ No

$\lim\limits_{x\to-2^+}f(x)\neq\lim\limits_{x\to-2^-}f(x)$

Intermediate Value Theorem.

42. Verify the conditions of the Intermediate Value Theorem, and **find** c guaranteed by the theorem when $f(x)=x^2-6x+7$ over the interval $[0,3]$ and $f(c)=-1$. $f(x)$ is cont.

$f(0)=7$

$f(3)=9-18+7=-2$

$-2<-1<7$

$x^2-6x+7=-1$

$x^2-6x+8=0$

$(x-4)(x-2)=0$

$x=2,4$

$\boxed{c=2}$

Finding values that make a function continuous.

43. Find the value of a that would make the function continuous.

$$f(x)=\begin{cases}3xa+5 & \text{if } x\le -1\\ -2x+5a & \text{if } x>-1\end{cases}$$

$3(-1)a+5=-2(-1)+5a$

$-3a+5=2+5a$

$3=8a$

$\boxed{a=3/8}$

44. Find the value of *m* and *n* that would make the function continuous.

$$g(x)=\begin{cases}3mx-4n & \text{if } x\le -1\\ 4+nx-mx^2 & \text{if } -1<x<2\\ x^2-mx+7n & \text{if } x\ge 2\end{cases}$$

$-3m-4n=4-n-m$

$-2m=4+3n$

$m=-2-\frac{3}{2}n$

$4+2n-4m=4-2m+7n$

$2n-4m=-2m+7n$

$-2m=5n$

$m=-\frac{5}{2}n$

$-2-\frac{3}{2}n=-\frac{5}{2}n$

$-2=-n$

$\boxed{n=2}$

$m=-\frac{5}{2}(2)=-5$

$\boxed{m=-5}$