ONE-SIDED LIMITS AND CONTINUITY

Keeper 9
Honors Calculus

FINDING ONE-SIDED LIMITS

One-Sided limits are the same as normal limits, we just restrict x so that it approaches from just one side.

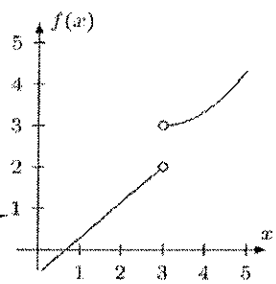
$x \to a^+$ means x us approaching from the right
$x \to a^-$ means x is approaching from the left

FIND THE LIMIT

$\lim_{x\to 3^+} f(x) = 3$

$\lim_{x\to 3^-} f(x) = 2$

$\lim_{x\to 3} f(x) = \text{DNE}$



FIND THE LIMIT

$$\lim_{x\to 5^+} \frac{x-5}{x^2-25}$$

$$\frac{\cancel{x-5}}{\cancel{(x-5)}(x+5)} = \frac{1}{10}$$

FIND THE LIMIT

$$\lim_{x\to -3^-} \frac{x}{\sqrt{x^2-9}} \quad \frac{-3}{\sqrt{9-9}} = \frac{-3}{0}$$

∞

$-\infty$

$$\frac{-2.99}{\sqrt{(-2.99)^2-9}} = -\infty$$

FIND THE LIMIT

$$\lim_{x\to 0^-} \frac{|x|}{x} \quad \frac{-x}{x} = -1$$

FIND THE LIMIT

$$\lim_{x \to 2^+} \frac{|x-2|}{x-2}$$

$\frac{x-2}{x-2} = 1$

FIND THE LIMIT

$$\lim_{x \to 4^-} \frac{\sqrt{x}-2}{x-4}$$

$\frac{1}{\sqrt{x}+2}$

$= \frac{1}{2+2} = \frac{1}{4}$

FIND THE LIMIT

$$\lim_{x \to 2^+} \frac{2-x}{x^2-4}$$

$\frac{-1}{x-2}$

$\frac{-1}{2.01-2} = \frac{-1}{.01}$

$-\infty$

FIND THE LIMITS

$$f(x) = \begin{cases} -x-3, & x < -3 \\ 2, & -3 \le x < 2 \\ -\frac{1}{2}x+3, & x > 2 \end{cases}$$

$\lim_{x \to -3^-} f(x) = 3-3 = 0$

$\lim_{x \to 2^-} f(x) = 2$

$\lim_{x \to -3^+} f(x) = 2$

$\lim_{x \to 1} f(x) = 2$

$\lim_{x \to 2^+} f(x) = 2$

FIND THE LIMITS

$$f(x) = \begin{cases} x^2+1, & x < 0 \\ -2x+4, & 0 \le x < 2 \\ (x-2)^2+1, & x > 2 \end{cases}$$

$\lim_{x \to -0^-} f(x) = 1$

$\lim_{x \to 2^-} f(x) = 0$

$\lim_{x \to 0} f(x) =$ DNE

$\lim_{x \to 1} f(x) = 2$

$\lim_{x \to 2^+} f(x) = 1$

FIND THE LIMIT

$$\lim_{x \to 5^+} \frac{|x-5|}{x-5} = 1$$

FIND THE LIMIT

$$\lim_{x \to 3^-} \frac{1}{x-3} \qquad \frac{1}{2.99-3}$$

$-\infty$

FIND THE LIMIT

$$\lim_{x \to 2^-} \frac{3}{x-2} \qquad \frac{3}{1.99-2}$$

$-\infty$

FIND THE LIMIT

$$\lim_{x \to -3^+} \frac{5}{x+3} \qquad \frac{5}{-2.99+3}$$

∞

FIND THE LIMIT

$$\lim_{x \to 2} \frac{-7}{2-x} = DNE$$

$$\lim_{x \to 2^+} \neq \lim_{x \to 2^-}$$

FIND THE LIMIT

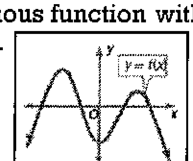
$$\lim_{x \to 5^+} \frac{3x-15}{|4x-20|} \qquad \frac{3x-15}{4x-20}$$

$$\frac{3(x-5)}{4(x-5)}$$

$3/4$

CONTINUOUS FUNCTIONS

The graph of a continuous function has no breaks, holes, or gaps. You can trace the graph of a continuous function without lifting your pencil.

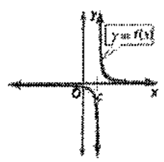


f(x) is continuous for all x.

TYPES OF DISCONTINUITY

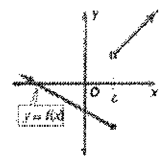
A function has an **infinite discontinuity** at $x = c$ if the function value increases or decreases indefinitely as x approaches c from the left and right.

Example



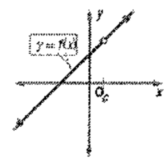
A function has a **jump discontinuity** at $x = c$ if the limits of the function as x approaches c from the left and right exist but have two distinct values.

Example



A function has a **removable discontinuity** if the function is continuous everywhere except for a hole at $x = c$.

Example



CONTINUITY TEST

A function $f(x)$ is continuous at $x = c$ if it satisfies the following conditions.

$$\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x) = \lim_{x\to c} f(x) = f(c)$$

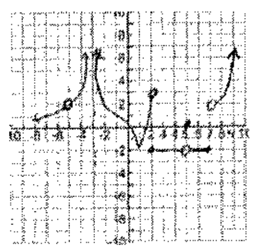
UNDERSTANDING CONTINUITY

a. Does $f(5)$ exist? 0

b. Does $\lim_{x\to 5} f(x)$ exist? -2

c. Is $f(x)$ continuous at $x = 5$? Justify. No

d. What new value should be assigned to $f(5)$ to remove the discontinuity? -2

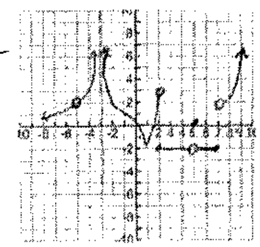


c) $\lim_{x\to 5} \neq f(5)$

UNDERSTANDING CONTINUITY

e. Does $f(2)$ exist? -2

f. Does $\lim_{x\to 2} f(x)$ exist? DNE



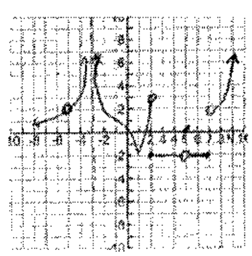
UNDERSTANDING CONTINUITY

g. Does $f(-5)$ exist? NO

h. Does $\lim_{x\to -5} f(x)$ exist? 2

i. Is $f(x)$ continuous at $x = -5$? Justify. NO

j. What new value should be assigned to $f(-5)$ to make $f(x)$ continuous at $x = -5$? 2

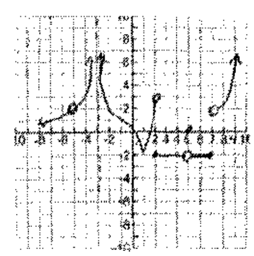


c) $f(-5) \neq \lim_{x\to -5} f(x)$

UNDERSTANDING CONTINUITY

k. Is $f(x)$ right continuous, left continuous, or neither at $x = 2$? How about for $x = 7$?
Right Left

l. List all places where $f(x)$ is discontinuous and state the type of discontinuity.



$X = -5$ Removable
$X = -3$ Infinite
$X = 2$ Jump
$X = 5$ Removable
$X = 7$ Jump

IDENTIFY THE TYPE OF DISCONTINUITY IN THE FOLLOWING EQUATIONS

a. $h(x) = \frac{6}{x-3}$ Infinite

b. $p(x) = \begin{cases} 3x-1, & if\ x \geq 1 \\ 4x-2, & if\ x < 1 \end{cases}$ Cont.

c. $m(x) = \begin{cases} 2x-5, & if\ x \geq 2 \\ 3x, & if\ x < 2 \end{cases}$ Jump

d. $k(x) = \frac{6x-2}{9x-3}$ $\frac{2(3x-1)}{3(3x-1)}$ Removable

e. $j(x) = \frac{2x-4}{x^2-2x}$ $\frac{2(x-2)}{x(x-2)}$ Removable & Infinite

FINDING VALUES FOR DISCONTINUITY

Find a value for a so that $f(x)$ is continuous.

$$f(x) = \begin{cases} 2x+3, & if\ x \leq 2 \\ ax+1, & if\ x > 2 \end{cases}$$

$2(2)+3 = a(2)+1$

$6 = 2a$

$a = 3$

FINDING VALUES FOR DISCONTINUITY

Find a value for k so that $g(x)$ is continuous.

$$g(x) = \begin{cases} 4x-7k, & if\ x \geq -3 \\ 2k+x, & if\ x < -3 \end{cases}$$

$-12 - 7k = 2k - 3$

$-9k = 9$

$k = -1$

COMPLETE THE TABLE

	$f(x)$	Discontinuity at:	Type of discontinuity(Be Specific):
1.	$\frac{4}{x^2-1}$	$x = 1$, $x = -1$	Infinite
2.	$\begin{cases} x^2, & x \geq 0 \\ -3, & x < 0 \end{cases}$	$x = 0$	Jump
3.	$\frac{x^2-x-12}{x-4}$ $(x-4)(x+3)$	$x = 4$	Removable
4.	$\frac{x-3}{x^2-9}$	$x = 3$, $x = -3$	Removable, Infinite