EXPONENTIALS AND LOGARITHMS

Honors Calculus
Keeper 5

Properties of Exponents	Let a and b be real #'s and let m and n be integers.
Product of Powers	$a^m \cdot a^n = a^{m+n}$
Power of a Power	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^m = a^m b^m$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$
Zero Exponent	$a^0 = 1, a \neq 0$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$

WHAT DOES IT MEAN TO SIMPLIFY?

* Apply the property(s) of exponents.

* Rewrite rational exponents as radicals and simplify if possible.

* We can **NEVER** leave negative exponents or rational exponents/radicals in the denominator!

HELPFUL HINTS:

* Negative exponents have to move to the "opposite" side of the fraction to become positive.

* If you end up with a rational exponent in the denominator, rewrite in radical form and then rationalize the denominator.

EXAMPLE 1: SIMPLIFY THE EXPRESSION COMPLETELY.

$$x^{\frac{5}{3}} \cdot x^{\frac{1}{3}}$$

Note: If the exponent is a whole number...STOP, that is as far as that place will go!

$x^{5/3 + 1/3} = x^{6/3}$

$= x^2$

EXAMPLE 2: SIMPLIFY THE EXPRESSION COMPLETELY.

$$(16x^4)^{\frac{1}{2}}$$

$4x^2$

EXAMPLE 3: SIMPLIFY THE EXPRESSION COMPLETELY.

$$3x^4 \cdot 6x^{\frac{1}{8}}$$

$18x^{33/8}$

EXAMPLE 4: SIMPLIFY THE EXPRESSION COMPLETELY.

$$(x^5y^{-1})^{\frac{1}{2}}$$

$x^{5/2}y^{-1/2}$

$\frac{x^{5/2}}{\sqrt{y}} = \frac{x^{5/2}y^{1/2}}{y}$

EXAMPLE 5: SIMPLIFY THE EXPRESSION COMPLETELY.

$$\left(\frac{27x^{12}}{y^{15}}\right)^{\frac{1}{3}}$$

$\frac{3x^4}{y^5}$

EXAMPLE 6: SIMPLIFY THE EXPRESSION COMPLETELY.

$$\frac{4z^{\frac{5}{3}}}{2\sqrt[3]{z^2}}$$

$2z^{3/3}$

$\boxed{2z}$

EXAMPLE 7: SIMPLIFY THE EXPRESSION COMPLETELY.

$$\frac{2x^{-\frac{1}{4}} \cdot 2y^{\frac{3}{4}}}{xy^{-\frac{1}{2}}}$$

$4\,x^{-1/4-1}\,y^{3/4+1/2}$

$4\,x^{-5/4}\,y^{5/4}$

$\frac{4\,y^{5/4}}{x^{5/4}} \cdot \frac{x^{3/4}}{x^{3/4}}$

$\boxed{\frac{4x^{3/4}y^{5/4}}{x^2}}$

EXAMPLE 8: SIMPLIFY THE EXPRESSION COMPLETELY.

$$\frac{x^{-1}y}{xy^{-2}}$$

$x^{-1-1}\,y^{1+2}$

$x^{-2}\,y^{3}$

$\boxed{\frac{y^3}{x^2}}$

EXAMPLE 9: SIMPLIFY THE EXPRESSION COMPLETELY.

$$(4x^2y^5)^{-2}$$

$4^{-2}x^{-4}y^{-10}$

$\boxed{\frac{1}{16x^4y^{10}}}$

EXAMPLE 10: SIMPLIFY THE EXPRESSION COMPLETELY.

$$\frac{2x^2y}{6xy^{-1}}$$

$\frac{1}{3}\,x\,y^{1+1}$

$\boxed{\frac{1}{3}xy^2}$

or

$\boxed{\frac{xy^2}{3}}$

EXAMPLE 11: SIMPLIFY THE EXPRESSION COMPLETELY.

$$\frac{5x^3y^9}{20x^2y^{-2}}$$

$\boxed{\frac{1}{4}x\,y^{11}}$

or

$\boxed{\frac{xy^{11}}{4}}$

EXAMPLE 12: SIMPLIFY THE EXPRESSION COMPLETELY.

$$\frac{xy^9}{3y^{-2}} \cdot \frac{-7y}{21x^5}$$

$\frac{-7}{63}\,x^{1-5}\,y^{9+1+2}$

$-\frac{1}{9}\,x^{-4}\,y^{12}$

$\boxed{-\frac{y^{12}}{9x^4}}$

EXAMPLE 13: SIMPLIFY THE EXPRESSION COMPLETELY.

$$\frac{y^{10}}{2x^3} \cdot \frac{20x^{14}}{xy^6}$$

$10\,x^{14-3-1}\,y^{10-6}$

$10x^{10}y^4$

EXAMPLE 14: SIMPLIFY THE EXPRESSION COMPLETELY.

$$\frac{12xy}{7x^4} \cdot \frac{7x^5y^2}{4y}$$

$3\,x^{1+5-4}\,y^{1+2-1}$

$3x^2y^2$

WARM UP

$$\frac{(x^5y)^{\frac{1}{4}}\cdot z^{\frac{5}{4}}}{(x^8y^4z)^{\frac{1}{4}}}$$

$x^{5/4-2}\ y^{1/4-4}\ z^{5/4-1/4}$

$x^{-3/4}\ y^{-15/4}\ z$

$\frac{z}{x^{3/4}\ y^{15/4}} \cdot \frac{x^{1/4}\ y^{1/4}}{x^{1/4}\ y^{1/4}}$

$\boxed{\frac{z\,x^{1/4}y^{1/4}}{x\,y^4}}$

SOLVING EQUATIONS WITH COMMON BASES:

If $b^x = b^y$
Then $x = y$

EXAMPLE 1: SOLVE THE EQUATION

$2^x = 2^{2x-3}$

$x = 2x - 3$

$-x = -3$

$\boxed{x = 3}$

EXAMPLE 2: SOLVE THE EQUATION

$5^x = 5^1$

$x = 1$

EXAMPLE 3: SOLVE THE EQUATION

$3^{x+4} = 3^{x-1}$

$x + 4 = x - 1$

$4 = -1$

$\boxed{\text{No solution}}$

EXAMPLE 4: SOLVE THE EQUATION

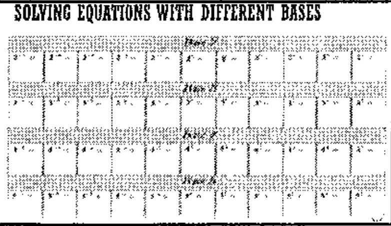
$\left(\frac{1}{3}\right)^{-x+7} = \left(\frac{1}{3}\right)^{3x-1}$

$-x + 7 = 3x - 1$

$8 = 4x$

$\boxed{x = 2}$

SOLVING EQUATIONS WITH DIFFERENT BASES



HELPFUL TIPS:

*Check to see if the larger base can be rewritten as the smaller base.

*Check to see if both bases can be rewritten as the same number.

*Don't forget to distribute the "new" exponent to all of the "old" exponent.

EXAMPLE 5: SOLVE THE EQUATION

$2^x = 4^x$

$2^x = 2^{2x}$

$x = 2x$

$\boxed{x = 0}$

EXAMPLE 6: SOLVE THE EQUATION

$$8^{x+2} = 16^{2x+7}$$

$2^{3(x+2)} = 2^{4(2x+7)}$

$3x+6 = 8x+28$

$-22 = 5x$

$x = \frac{-22}{5}$

EXAMPLE 7: SOLVE THE EQUATIONS

$$3^{2x} = 27^{x-1}$$

$3^{2x} = 3^{3(x-1)}$

$2x = 3x-3$

$\boxed{x=3}$

EXAMPLE 8: SOLVE THE EQUATIONS

$$\left(\frac{1}{9}\right)^{-x+5} = 3^x$$

$3^{-2(-x+5)} = 3^x$

$2x-10 = x$

$\boxed{x=10}$

EXAMPLE 9: SOLVE THE EQUATIONS

$$4^{x+7} = 8^{x+3}$$

$2^{2(x+7)} = 2^{3(x+3)}$

$2x+14 = 3x+9$

$x=5$

EXAMPLE 8: SOLVE THE EQUATIONS

$$49^{x+4} = 7^{18-x}$$

$7^{2(x+4)} = 7^{18-x}$

$2x+8 = 18-x$

$3x = 10$

$\boxed{x=\frac{10}{3}}$

EXAMPLE 8: SOLVE THE EQUATIONS

$$\left(\frac{9}{16}\right)^{3x-2} = \left(\frac{3}{4}\right)^{5x+4}$$

$\left(\frac{3}{4}\right)^{2(3x-2)} = \left(\frac{3}{4}\right)^{5x+4}$

$6x-4 = 5x+4$

$\boxed{x=8}$

EXAMPLE 8: SOLVE THE EQUATIONS

$$25^{\frac{x}{3}} = 5^{x-4}$$

$\frac{2x}{3} = x-4$

$-\frac{1}{3}x = -4$

$\boxed{x=12}$

SOLVING EXPONENTIAL EQUATIONS

$$4^x = 21$$

$x = \log_4 21$

$\boxed{x \approx 2.196}$

REWRITING EQUATIONS TO SOLVE

$$3e^{4x} = 45$$

$e^{4x} = 15$

$4x = \ln 15$

$x = \frac{\ln 15}{4}$

$\boxed{x \approx .677}$

SOLVING EXPONENTIAL EQUATIONS

$2(5^{2x})-1=47$

$2(5^{2x})=48$

$5^{2x}=24$

$2x=\log_5 24$

$x=\frac{\log_5 24}{2}$

$\boxed{x=.987}$

SOLVING EXPONENTIAL EQUATIONS

$2(5^{4x-1})+8=24$

$2(5^{4x-1})=16$

$5^{4x-1}=8$

$4x-1=\log_5 8$

$4x=(\log_5 8)+1$

$x=\frac{(\log_5 8)+1}{4}$

$\boxed{x\approx .573}$

SOLVING EXPONENTIAL EQUATIONS

$0.75e^{3.4x}-0.3=80.1$

$e^{3.4x}=107.2$

$3.4x=\ln 107.2$

$x=\frac{\ln 107.2}{3.4}$

$\boxed{x\approx 1.375}$

SOLVING EXPONENTIAL EQUATIONS

$e^{-x}=6$

$-x=\ln 6$

$x=-\ln 6$

$\boxed{x\approx -1.792}$

SOLVING EXPONENTIAL EQUATIONS

$4-2e^{x}=-23$

$e^{x}=13.5$

$x=\ln(13.5)$

$\boxed{x\approx 2.603}$

SOLVING EXPONENTIAL EQUATIONS

$2e^{4x-1}=30$

$e^{4x-1}=15$

$4x-1=\ln 15$

$4x=(\ln 15)+1$

$x=\frac{\ln(15)+1}{4}$

$\boxed{x\approx .927}$

SOLVING LOGARITHMIC EQUATIONS

- Isolate the logarithm.
- Write in exponential form (inverse property).
- Solve for the variable.

REMEMBER YOUR LOGARITHM PROPERTIES!!!

The Produce Rule: $\log_a MN=\log_a M+\log_a N$

The Power Rule: $\log_a M^p=p\cdot\log_a M$

The Quotient Rule: $\log_a \frac{M}{N}=\log_a M-\log_a N$

SOLVING LOGARITHMIC EQUATIONS

$\log_3(5x-1)=\log_3(x+7)$

$5x-1=x+7$

$4x=8$

$\boxed{x=2}$

*Be sure to check your Answer

* Check your Answers!

SOLVING LOGARITHMIC EQUATIONS

$\ln(2x+1)=\ln(4-5x)$

$2x+1=4-5x$

$7x=3$

$\boxed{x=3/7}$

SOLVING LOGARITHMIC EQUATIONS

$1+\log_3 x=5$

$\log_3 x=4$

$x=3^4$

$\boxed{x=81}$

SOLVING LOGARITHMIC EQUATIONS

$\log_5(3x+1)=2$

$3x+1=5^2$

$3x+1=25$

$3x=24$

$\boxed{x=8}$

SOLVING LOGARITHMIC EQUATIONS

$3\ln(x+2)=6$

$\ln(x+2)=2$

$x+2=e^2$

$\boxed{x\approx 5.389}$

SOLVING LOGARITHMIC EQUATIONS

$\frac{1}{2}\log_4(x+2)=-1$

$\log_4(x+2)=-2$

$(x+2)=\frac{1}{16}$

$\boxed{x=\frac{-31}{16}}$

SOLVING LOGARITHMIC EQUATIONS

$1-2\ln x=-4$

$\ln x=2.5$

$x=e^{2.5}$

$\boxed{x\approx 12.182}$

SOLVING LOGARITHMIC EQUATIONS

$\log x+\log 2=2$

$\log 2x=2$

$2x=10^2$

$2x=100$

$\boxed{x=50}$

SOLVING LOGARITHMIC EQUATIONS

$\log_2 4x+\log_2(x+3)=4$

$\log_2 4x(x+3)=4$

$4x^2+12x=2^4$

$4x^2+12x=16$

$x^2+3x=4$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$x=1, \cancel{-4}$

SOLVING LOGARITHMIC EQUATIONS

$\log 2x+\log(5x+15)=2$

$\log 2x(5x+15)=2$

$10x^2+30x=100$

$x^2+3x-10=0$

$(x+5)(x-2)=0$

$x=\cancel{-5}, 2$

YOU TRY!!!

$\log(5x^2+4) = 2\log 3x^2 - \log(2x^2-1)$

$5x^2+4 = \frac{9x^4}{2x^2-1}$

$10x^4 - 5x^2 + 8x^2 - 4 = 9x^4$

$x^4+3x^2-4=0$

$(x^2+4)(x^2-1)$

$x = \cancel{\pm 2i}, \boxed{\pm 1}$

YOU TRY!!!

$\log(x+6) = \log(8x) - \log(3x+2)$

$x+6 = \frac{8x}{3x+2}$

$3x^2+12x+12=0$

$x^2+4x+4=0$

$(x+2)^2=0$

$x = \cancel{-2}$

$\boxed{\text{No solution}}$

YOU TRY!!!

$\ln(4x^2-3x) = \ln(16x-12) - \ln x$

$4x^2-3x = \frac{16x-12}{x}$

$4x^3-3x^2-16x+12=0$

$x^2(4x-3)-4(4x-3)=0$

$(x^2-4)(4x-3)=0$

$\boxed{x=2}, \cancel{-2}, \cancel{3/4}$

YOU TRY!!!

$\ln(3x^2-4) + \ln(x^2+1) = \ln(2-x^2)$

$(3x^2-4)(x^2+1) = 2-x^2$

$3x^4+3x^2-4x^2-4 = 2-x^2$

$3x^4-6=0$

$3x^4=6$

$x^4=2$

$x = \pm\sqrt[4]{2}$