

FUNCTIONS AND THEIR GRAPHS

Henon Calculator
Keeper 4

EVALUATING FUNCTIONS

To evaluate a function, substitute the input (the given number or expression) for the function's variable (place holder, x). Replace the x with the number or expression

EXAMPLE: EVALUATING FUNCTIONS

Given: $f(x) = 3x - 5$
Find: $f(4)$

$$3(4) - 5$$

$$12 - 5$$

7

EXAMPLE: EVALUATING FUNCTIONS

Given: $f(x) = x^2 + 7$
Find: $f(3a)$

$$(3a)^2 + 7$$

$9a^2 + 7$

EXAMPLE: EVALUATING FUNCTIONS

Given: $f(x) = x^2 + 7$
Find: $f(b - 1)$

$$(b - 1)^2 + 7$$

$$b^2 - 2b + 1 + 7$$

$b^2 - 2b + 8$

EXAMPLE: EVALUATING FUNCTIONS

Given: $f(x) = x^2 + 7$
Find: $\frac{f(x+h) - f(x)}{h}$

$$\frac{(x+h)^2 + 7 - x^2 - 7}{h}$$

$$\frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$\frac{2xh + h^2}{h} = 2x + h$$

$2x + h$

DOMAIN

Definition:
The domain of a function is the complete set of possible values of the independent variable.

How to find the domain:
The domain is the set of all x values which will cause the function to have a real output (real y value).

How to find the domain:

- The domain and codomain of all functions should be real.
- The function value is a real number, it should be positive in the function.

RANGE

Definition:
The range of a function is the complete set of all possible resulting values of the dependent variable (the set of all possible y values).

How to find the range:

- The range of a function is the set of all possible values of the dependent variable.
- Identify the domain of the function.
- Substitute the domain values into the function.
- Identify the range of the function.

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$f(x) = \sqrt{x - 1}$

Domain: $[1, \infty)$
Range: $[0, \infty)$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = \tan x$$

Domain: \mathbb{R} except
 $x = \pi/2k$

Range: \mathbb{R}

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = 4 - x$$

D: \mathbb{R}
R: \mathbb{R}

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = \frac{4}{x}$$

D: $(-\infty, 0) \cup (0, \infty)$
R: $(-\infty, 0) \cup (0, \infty)$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = \sqrt{x-1}$$

D: $[1, \infty)$
R: $[0, \infty)$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = \frac{1}{2}x^3 + 2$$

D: \mathbb{R}
R: \mathbb{R}

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = \sqrt{9-x^2}$$

D: $[-3, 3]$
R: $[0, 3]$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = x + \sqrt{4-x^2}$$

D: $[-2, 2]$
R: $[-2, 2\sqrt{2}]$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = 2 \sin \pi t$$

D: \mathbb{R}
R: $[-2, 2]$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = -5 \cos \frac{\theta}{2}$$

D: \mathbb{R}
R: $[-5, 5]$

PIECEWISE FUNCTIONS

A function that is defined using two or more equations for different intervals of the domain is called a **piecewise function**.

EVALUATING PIECEWISE FUNCTIONS

Evaluate $f(x)$ when (a) $x = 0$ (b) $x = 2$ and (c) $x = 4$

$$f(x) = \begin{cases} x+2, & \text{if } x < 2 \\ 2x+1, & \text{if } x \geq 2 \end{cases}$$

EVALUATING PIECEWISE FUNCTIONS

Evaluate $f(x)$ when (a) $x = 67$ (b) $x = 72$ and (c) $x = 65$

$$f(x) = \begin{cases} 1.6x - 41.6, & \text{if } 63 < x < 66 \\ 3x - 132, & \text{if } 66 \leq x \leq 68 \\ 2x - 66, & \text{if } x > 68 \end{cases}$$

a) $0+2 = 2$
 b) $2(2)+1 = 5$
 c) $2(4)+1 = 9$

a) $3(67) - 132 = 69$
 b) $2(72) - 66 = 78$
 c) $1.6(65) - 41.6 = 62.4$

GRAPHING PIECEWISE FUNCTIONS

$$f(x) = \begin{cases} -3x+2, & x \leq 2 \\ \frac{1}{2}x-4, & x > 2 \end{cases}$$

GRAPHING PIECEWISE FUNCTIONS

$$f(x) = \begin{cases} 4, & x \leq -2 \\ x^2, & -2 < x < 2 \\ 4, & x \geq 2 \end{cases}$$

GRAPHING PIECEWISE FUNCTIONS

$$f(x) = \begin{cases} 3x+12, & \text{if } x < -3 \\ |x|, & \text{if } -3 \leq x < 5 \\ -3x+12, & \text{if } x \geq 5 \end{cases}$$

GRAPHING PIECEWISE FUNCTIONS

$$f(x) = \begin{cases} x^2-4, & \text{if } x < -2 \\ \sqrt{x+6}, & \text{if } -2 \leq x \leq 3 \\ \frac{2}{3}x-5, & \text{if } x > 3 \end{cases}$$

Example. Write the equation for the graph below.

$y = \frac{1}{3}(x-2) + 3$
 $\frac{1}{3}x - \frac{2}{3} + 3$
 $\frac{1}{3}x + \frac{7}{3}$

Library Function Graphs

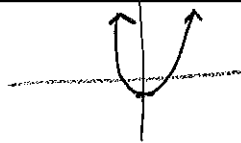
$f(x) = a$ number	Parabola $f(x) = x^2$	$f(x) = \sqrt{x}$	$f(x) = \frac{1}{x}$
Line $f(x) = mx + b$	Circle $f(x) = x^2 + y^2 = r^2$	Power $f(x) = x^n$	Power $f(x) = x^{-n}$
Step function	Exponential $f(x) = e^x$	Logarithmic $f(x) = \ln(x)$	Reciprocal $f(x) = \frac{1}{x}$

$$f(x) = \begin{cases} -x-2 \\ \frac{1}{3}x + \frac{7}{3} \end{cases}$$

RULES FOR TRANSFORMATION OF FUNCTIONS

Transformation	Equation	Graph
Vertical stretch	$g(x) = af(x)$	Stretch by a
Vertical compression	$g(x) = \frac{1}{a}f(x)$	Compression by a
Horizontal stretch	$g(x) = f(ax)$	Stretch by $\frac{1}{a}$
Horizontal compression	$g(x) = f(\frac{x}{a})$	Compression by $\frac{1}{a}$
Vertical shift up	$g(x) = f(x) + a$	Shift up by a
Vertical shift down	$g(x) = f(x) - a$	Shift down by a
Horizontal shift right	$g(x) = f(x - a)$	Shift right by a
Horizontal shift left	$g(x) = f(x + a)$	Shift left by a

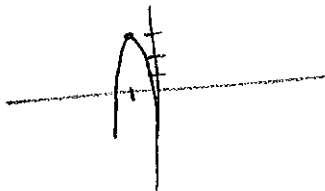
EXAMPLE: DESCRIBE THE TRANSFORMATION AND SKETCH THE GRAPH
 $g(x) = x^2 - 1$
 Down 1



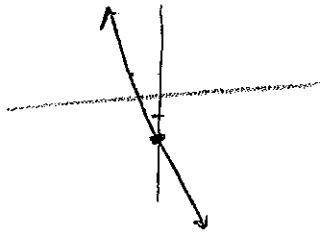
EXAMPLE: DESCRIBE THE TRANSFORMATION AND SKETCH THE GRAPH
 $f(x) = 2|x - 1|$
 Stretch by 2
 Right 1



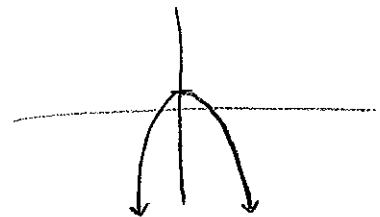
EXAMPLE: DESCRIBE THE TRANSFORMATION AND SKETCH THE GRAPH
 $g(x) = -2(x + 1)^2 + 3$
 X axis ref
 Stretch by 2
 Left 1
 Up 3



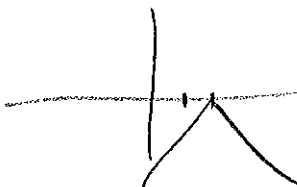
EXAMPLE: DESCRIBE THE TRANSFORMATION AND SKETCH THE GRAPH
 $g(x) = -3x - 2$
 X axis ref
 stretch by 3
 Down 2



EXAMPLE: DESCRIBE THE TRANSFORMATION AND SKETCH THE GRAPH
 $g(x) = -x^2 + 1$
 X axis ref
 up 1



EXAMPLE: DESCRIBE THE TRANSFORMATION AND SKETCH THE GRAPH
 $g(x) = -|x - 2|$
 X axis ref
 Right 2



EXAMPLE: WRITE THE EQUATION DESCRIBED
 Absolute Value - vertical shift up 5,
 horizontal shift right 3
 $y = |x - 3| + 5$

EXAMPLE: WRITE THE EQUATION DESCRIBED
 Linear - vertical stretch/compression by $\frac{2}{5}$
 $y = \frac{2}{5}x$

EXAMPLE: WRITE THE EQUATION DESCRIBED

Quadratic - vertical stretch by 5, horizontal shift left 8, reflected over the x-axis.

$$y = -5(x+8)^2$$

COMBINATION OF FUNCTIONS

Sum: $(f + g)(x) = f(x) + g(x)$
 Difference: $(f - g)(x) = f(x) - g(x)$
 Product: $(fg)(x) = f(x) \cdot g(x)$
 Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

EXAMPLE

$f(x) = x^2 + 4 + 2x$
 $g(x) = -3x + 2$

Find $(g \cdot f)(x)$

$$(-3x+2)(x^2+4+2x)$$

$$-3x^3 - 12x - 6x^2 + 2x^2 + 8 + 4x$$

$$-3x^3 - 4x^2 - 8x + 8$$

EXAMPLE

$f(x) = x^2 + 4 + 2x$
 $g(x) = -3x + 2$

Find $(f + g)(x)$

$$x^2 + 4 + 2x - 3x + 2$$

$$x^2 - x + 6$$

EXAMPLE

$f(x) = x^2 + 4 + 2x$
 $g(x) = -3x + 2$

Find $(f - g)(x)$

$$x^2 + 4 + 2x + 3x - 2$$

$$x^2 + 5x + 2$$

EXAMPLE

$f(x) = x^2 + 4 + 2x$
 $g(x) = -3x + 2$

Find $\left(\frac{g}{f}\right)(x)$

$$\frac{-3x+2}{x^2+4+2x}$$

COMPOSITION OF FUNCTIONS

Substituting a function or its value into another function.

Second function: $f(g(x))$
 First function: f (inside parentheses always first)
 OR $f \circ g(x)$

EXAMPLE

$f(x) = x^2 + 4 + 2x$
 $g(x) = -3x + 2$

Find $(g \circ f)(x)$

f into g

$$-3(x^2+4+2x) + 2$$

$$-3x^2 - 12 - 6x + 2$$

$$-3x^2 - 6x - 10$$

EXAMPLE

$f(x) = x^2 + 4 + 2x$
 $g(x) = -3x + 2$

Find $(f \circ g)(x)$

g into f

$$(-3x+2)^2 + 4 + 2(-3x+2)$$

$$9x^2 - 12x + 4 + 4 - 6x + 4$$

$$9x^2 - 18x + 12$$