FUNCTIONS AND THEIR GRAPHS

Honors Calculus
Keeper 4

EVALUATING FUNCTIONS

To evaluate a function, substitute the input (the given number or expression) for the function's variable (place holder, x). Replace the x with the number or expression

EXAMPLE: EVALUATING FUNCTIONS

Given: $f(x) = 3x - 5$
Find: $f(4)$

$3(4) - 5$
$12 - 5$
$\boxed{7}$

EXAMPLE: EVALUATING FUNCTIONS

Given: $f(x) = x^2 + 7$
Find: $f(3a)$

$(3a)^2 + 7$
$\boxed{9a^2 + 7}$

EXAMPLE: EVALUATING FUNCTIONS

Given: $f(x) = x^2 + 7$
Find: $f(b - 1)$

$(b-1)^2 + 7$
$b^2 - 2b + 1 + 7$
$\boxed{b^2 - 2b + 8}$

EXAMPLE: EVALUATING FUNCTIONS

Given: $f(x) = x^2 + 7$
Find: $\frac{f(x+h) - f(x)}{h}$

$$\frac{(x+h)^2 + 7 - x^2 - 7}{h}$$

$$\frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$\frac{2xh + h^2}{h} = \boxed{2x + h}$$

DOMAIN

RANGE

How to find the range

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$f(x) = \sqrt{x - 1}$

Domain: $[1, \infty)$
Range: $[0, \infty)$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = \tan x$$

Domain: $\mathbb{R}$ except $x = \pi/2\, K$

Range: $\mathbb{R}$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = 4 - x$$

D: $\mathbb{R}$

R: $\mathbb{R}$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = \frac{4}{x}$$

D: $(-\infty, 0) \cup (0, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = \sqrt{x-1}$$

D: $[1, \infty)$

R: $[0, \infty)$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = \frac{1}{2}x^3 + 2$$

D: $\mathbb{R}$

R: $\mathbb{R}$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = \sqrt{9 - x^2}$$

D: $[-3, 3]$

R: $[0, 3]$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = x + \sqrt{4 - x^2}$$

D: $[-2, 2]$

R: $[-2, 2\sqrt{2}]$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = 2\sin \pi t$$

D: $\mathbb{R}$

R: $[-2, 2]$

FIND THE DOMAIN AND RANGE OF THE FUNCTION

$$f(x) = -5\cos\frac{\theta}{2}$$

D: $\mathbb{R}$

R: $[-5, 5]$

PIECEWISE FUNCTIONS

A function that is defined using two or more equations for different intervals of the domain is called a **piecewise function**.

EVALUATING PIECEWISE FUNCTIONS

Evaluate $f(x)$ when (a) $x = 0$ (b) $x = 2$ and (c) $x = 4$

$$f(x) = \begin{cases} x + 2, & \text{if } x < 2 \\ 2x + 1, & \text{if } x \geq 2 \end{cases}$$

a) $0 + 2 = 2$

b) $2(2) + 1 = 5$

c) $2(4) + 1 = 9$

EVALUATING PIECEWISE FUNCTIONS

Evaluate $f(x)$ when (a) $x = 67$ (b) $x = 72$ and (c) $x = 65$

$$f(x) = \begin{cases} 1.6x - 41.6, & \text{if } 63 < x < 66 \\ 3x - 132, & \text{if } 66 \leq x \leq 68 \\ 2x - 66, & \text{if } x > 68 \end{cases}$$

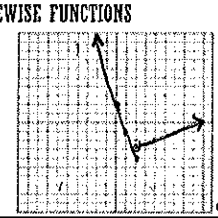
a) $3(67) - 132 = 69$

b) $2(72) - 66 = 78$

c) $1.6(65) - 41.6 = 62.4$

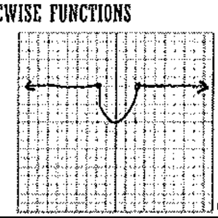
GRAPHING PIECEWISE FUNCTIONS

$$f(x) = \begin{cases} -3x + 2, & x \leq 2 \\ \frac{1}{2}x - 4, & x > 2 \end{cases}$$



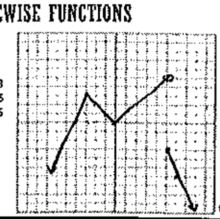
GRAPHING PIECEWISE FUNCTIONS

$$f(x) = \begin{cases} 4, & x \leq -2 \\ x^2, & -2 < x < 2 \\ 4, & x \geq 2 \end{cases}$$



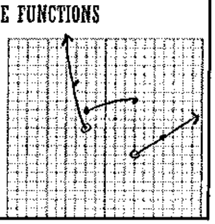
GRAPHING PIECEWISE FUNCTIONS

$$f(x) = \begin{cases} 3x + 12, & \text{if } x < -3 \\ |x|, & \text{if } -3 \leq x < 5 \\ -3x + 12, & \text{if } x \geq 5 \end{cases}$$



GRAPHING PIECEWISE FUNCTIONS

$$f(x) = \begin{cases} x^2 - 4, & \text{if } x < -2 \\ \sqrt{x + 6}, & \text{if } -2 \leq x \leq 3 \\ \frac{2}{3}x - 5, & \text{if } x > 3 \end{cases}$$



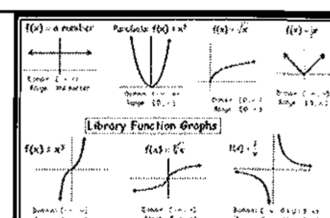
Example. Write the equation for the graph shown.

$y = \frac{1}{3}(x - 2) + 3$

$\frac{1}{3}x - \frac{2}{3} + 3$

$\frac{1}{3}x + \frac{7}{3}$

$$f(x) = \begin{cases} -x - 2 \\ \frac{1}{3}x + \frac{7}{3} \end{cases}$$



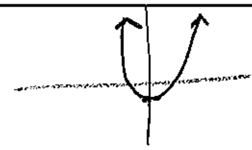
RULES FOR TRANSFORMATION OF FUNCTIONS

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]

EXAMPLE: DESCRIBE THE TRANSFORMATION AND SKETCH THE GRAPH

$g(x) = x^2 - 1$

Down 1

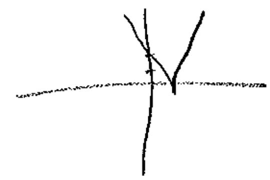


EXAMPLE: DESCRIBE THE TRANSFORMATION AND SKETCH THE GRAPH

$f(x) = 2|x - 1|$

Stretch by 2

Right 1



EXAMPLE: DESCRIBE THE TRANSFORMATION AND SKETCH THE GRAPH

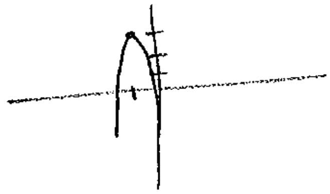
$g(x) = -2(x + 1)^2 + 3$

X axis ref

Stretch by 2

Left 1

Up 3



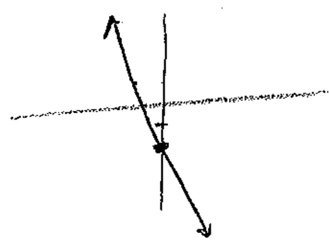
EXAMPLE: DESCRIBE THE TRANSFORMATION AND SKETCH THE GRAPH

$g(x) = -3x - 2$

x axis ref

Stretch by 3

Down 2

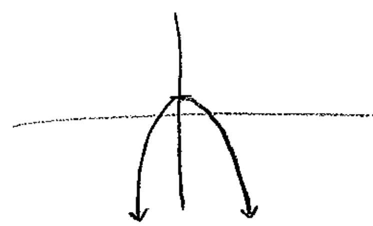


EXAMPLE: DESCRIBE THE TRANSFORMATION AND SKETCH THE GRAPH

$g(x) = -x^2 + 1$

X axis ref

up 1



EXAMPLE: DESCRIBE THE TRANSFORMATION AND SKETCH THE GRAPH

$g(x) = -|x - 2|$

X axis ref

Right 2

EXAMPLE: WRITE THE EQUATION DESCRIBED

Absolute Value – vertical shift up 6, horizontal shift right 3

$y = |x - 3| + 5$

EXAMPLE: WRITE THE EQUATION DESCRIBED

Linear – vertical stretch/compression by $\frac{2}{5}$

$y = \frac{2}{5}x$

EXAMPLE: WRITE THE EQUATION DESCRIBED

Quadratic – vertical stretch by 5, horizontal shift left 8, reflected over the x-axis.

$\boxed{y = -5(x+8)^2}$

COMBINATION OF FUNCTIONS

Sum: $(f+g)(x) = f(x) + g(x)$

Difference: $(f-g)(x) = f(x) - g(x)$

Product: $(fg)(x) = f(x) \cdot g(x)$

Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

EXAMPLE

$f(x) = x^2 + 4 + 2x$
$g(x) = -3x + 2$

Find $(g \cdot f)(x)$

$(-3x+2)(x^2+4+2x)$

$-3x^3 - 12x - 6x^2 + 2x^2 + 8 + 4x$

$\boxed{-3x^3 - 4x^2 - 8x + 8}$

EXAMPLE

$f(x) = x^2 + 4 + 2x$
$g(x) = -3x + 2$

Find $(f+g)(x)$

$x^2 + 4 + 2x - 3x + 2$

$\boxed{x^2 - x + 6}$

EXAMPLE

$f(x) = x^2 + 4 + 2x$
$g(x) = -3x + 2$

Find $(f-g)(x)$

$x^2 + 4 + 2x + 3x - 2$

$\boxed{x^2 + 5x + 2}$

EXAMPLE

$f(x) = x^2 + 4 + 2x$
$g(x) = -3x + 2$

Find $\left(\frac{g}{f}\right)(x)$

$\boxed{\dfrac{-3x+2}{x^2+4+2x}}$

COMPOSITION OF FUNCTIONS

Substituting a function or it's value into another function.

Second

$f(g(x))$

First

(inside parentheses always first)

OR $f \circ g(x)$

EXAMPLE

$f(x) = x^2 + 4 + 2x$
$g(x) = -3x + 2$

Find $(g \circ f)(x)$

f into g

$-3(x^2+4+2x) + 2$

$-3x^2 - 12 - 6x + 2$

$\boxed{-3x^2 - 6x - 10}$

EXAMPLE

$f(x) = x^2 + 4 + 2x$
$g(x) = -3x + 2$

Find $(f \circ g)(x)$

g into f

$(-3x+2)^2 + 4 + 2(-3x+2)$

$9x^2 - 12x + 4 + 4 - 6x + 4$

$\boxed{9x^2 - 18x + 12}$