

LINEAR MODELS AND RATES OF CHANGE

Honors Calculus
Kreger J

AVERAGE RATE OF CHANGE

The average rate of change between any two points on the graph of f is the slope of the line through these points.

The average rate of change on the interval $[x_1, x_2]$ is

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

FIND AVERAGE RATE OF CHANGE

Find the average rate of change of $f(x) = -x^2 + 3x$ on each interval:

a) $[-2, -1]$

b) $[0, 1]$

a) $f(-2) = 2$
 $f(-1) = -2$
 $\frac{-2 - 2}{-1 + 2} = \frac{-4}{1} = -4$

b) $f(0) = 0$
 $f(1) = 2$
 $\frac{2 - 0}{1 - 0} = \frac{2}{1} = 2$

FIND AVERAGE RATE OF CHANGE

Find the average rate of change on the interval $[2, 3]$.

$$f(x) = x^3 - 2x^2 - 3x + 2$$

$$f(2) = -4$$

$$f(3) = 2$$

$$\frac{2 + 4}{3 - 2} = \frac{6}{1} = 6$$

FIND AVERAGE RATE OF CHANGE

Find the average rate of change on the interval $[-5, -3]$.

$$f(x) = x^4 - 6x^2 + 4x$$

$$f(-5) = 455$$

$$f(-3) = 15$$

$$\frac{15 - 455}{-3 + 5} = -220$$

FIND AVERAGE RATE OF CHANGE

Find the average rate of change on the interval $[-6, 2]$.

$$f(x) = \frac{x+5}{x-4}$$

$$f(-6) = 1/10$$

$$f(2) = -7/2$$

$$\frac{-7/2 - 1/10}{2 + 6} = \frac{-18/5}{8} = -\frac{9}{20}$$

FIND AVERAGE RATE OF CHANGE

Find the average rate of change on the interval $[-4, 4]$.

$$f(x) = \sqrt{x+8}$$

$$f(-4) = 2$$

$$f(4) = 2\sqrt{3}$$

$$\frac{2\sqrt{3} - 2}{4 + 4} = \frac{2\sqrt{3} - 2}{8} = \frac{\sqrt{3} - 1}{4}$$

WRITING AN EQUATION OF A LINE

Slope-Intercept Form: Given the slope m and the y -intercept b , $y = mx + b$

Point-Slope Form: Given the slope m and a point (x_1, y_1) , $y - y_1 = m(x - x_1)$

Two Points: Given two points (x_1, y_1) and (x_2, y_2) , first find the slope and then use the point-slope form with either of the two points

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

WRITING AN EQUATION OF A LINE

Write the equation of the line that passes through $(2, 3)$ and has a slope of $-\frac{1}{2}$.

$$y - 3 = -\frac{1}{2}(x - 2)$$

$$y - 3 = -\frac{1}{2}x + 1$$

$$y = -\frac{1}{2}x + 4$$

WRITING AN EQUATION OF A LINE

Write the equation of the line that passes through (3,2) and is perpendicular to the line $y = -3x + 2$

$$m = \frac{1}{3}$$

$$y - 2 = \frac{1}{3}(x - 3)$$

$$y - 2 = \frac{1}{3}x - 1$$

$$\boxed{y = \frac{1}{3}x + 1}$$

WRITING AN EQUATION OF A LINE

Write the equation of the line that passes through (3,2) and is parallel to the line $y = -3x + 2$

$$m = -3$$

$$y - 2 = -3(x - 3)$$

$$y - 2 = -3x + 9$$

$$\boxed{y = -3x + 11}$$

WRITING AN EQUATION OF A LINE

Write the equation of the line that passes through (-2, -1) and (3,4).

$$m = \frac{4 - (-1)}{3 - (-2)} = \frac{5}{5} = 1$$

$$y + 1 = 1(x + 2)$$

$$y + 1 = x + 2$$

$$\boxed{y = x + 1}$$

Summary of Equations of Lines

Equation of a line: $y = mx + b$
 Slope: m
 Y-intercept: b
 X-intercept: $-\frac{b}{m}$
 Parallel lines: same slope, different y-intercepts
 Perpendicular lines: opposite reciprocal slopes

PARALLEL AND PERPENDICULAR LINES

Parallel lines have the SAME slope.
 Perpendicular lines have OPPOSITE, RECIPROCAL slopes.

EXAMPLE

Find the general form of the equations of the lines that pass through the point (2, -1) and is parallel to the line $2x - 3y = 5$

$$2x - 3y = 5$$

$$3y = 2x - 5$$

$$y = \frac{2}{3}x - \frac{5}{3}$$

$$m = \frac{2}{3}$$

$$y + 1 = \frac{2}{3}(x - 2)$$

$$y + 1 = \frac{2}{3}x - \frac{4}{3}$$

$$\boxed{y = \frac{2}{3}x - \frac{7}{3}}$$

EXAMPLE

Find the general form of the equations of the lines that pass through the point (2, -1) and is perpendicular to the line $2x - 3y = 5$

$$m = -\frac{3}{2}$$

$$y + 1 = -\frac{3}{2}(x - 2)$$

$$y + 1 = -\frac{3}{2}x + 6$$

$$\boxed{y = -\frac{3}{2}x + 5}$$