DEFINITION OF A DERIVATIVE

Keeper 13
Honors Calculus

DERIVATIVE

- Slope of a tangent line at $x = a$
- Rate of change at $x = a$
- Instantaneous rate of change or derivative
- Denoted by y', $f'(x)$, or $\frac{dy}{dx}$

DEFINITION OF A DERIVATIVE

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

FIND THE DERIVATIVE

$$f(x) = x^2 + 2$$

$$\lim_{h \to 0} \frac{(x+h)^2 + 2 - x^2 - 2}{h}$$

$$\lim_{h \to 0} \frac{x^2 + 2xh + 2 - x^2 - 2}{h}$$

$$\lim_{h \to 0} \frac{2xh}{h}$$

$$\lim_{h \to 0} 2x = \boxed{2x}$$

FIND THE DERIVATIVE

$$f(x) = 2\sqrt{x+3}$$

$$\lim_{h \to 0} \frac{2\sqrt{x+h+3} - 2\sqrt{x+3}}{h} \cdot \frac{2\sqrt{x+h+3} + 2\sqrt{x+3}}{2\sqrt{x+h+3} + 2\sqrt{x+3}}$$

$$\lim_{h \to 0} \frac{4x + 4h + 12 - 4x - 12}{h\left(2\sqrt{x+h+3} + 2\sqrt{x+3}\right)}$$

$$\lim_{h \to 0} \frac{4h}{h\left(2\sqrt{x+h+3} + 2\sqrt{x+3}\right)}$$

$$\lim_{h \to 0} \frac{4}{2\sqrt{x+h+3} + 2\sqrt{x+3}} = \frac{4}{2\sqrt{x+3} + 2\sqrt{x+3}}$$

$$= \frac{4}{4\sqrt{x+3}} = \frac{1}{\sqrt{x+3}}$$

FIND THE DERIVATIVE

$$f(x) = \frac{3}{x-2}$$

$$\lim_{h \to 0} \frac{\frac{3}{x+h-2} - \frac{3}{x-2}}{h}$$

$$\lim_{h \to 0} \frac{3x - 6 - 3x + 3h + 6}{(x+h-2)(x-2)} \cdot \frac{1}{h}$$

$$\lim_{h \to 0} \frac{-3}{(x+h-2)(x-2)}$$

$$= \frac{-3}{(x-2)(x-2)} = \frac{-3}{(x-2)^2}$$

ALTERNATE DEFINITION OF A DERIVATIVE AT A POINT $(a, f(a))$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

When to use this version:

-you are given a specific x value to evaluate the derivative at.

FIND THE DERIVATIVE

$f(x) = \frac{1}{x}$ $\quad at\ x = 3$

$$\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$$

$$\lim_{x \to 3} \frac{\frac{3 - x}{3x}}{x - 3}$$

$$\lim_{x \to 3} \frac{-1}{3x} = \frac{-1}{9}$$

FIND THE DERIVATIVE

$f(x) = 4x - 3x^2$ $\quad at (2, -4)$

$$\lim_{x \to 2} \frac{4x - 3x^2 + 4}{x - 2}$$

$$\lim_{x \to 2} \frac{-(3x^2 - 4x - 4)}{x - 2}$$

$$\lim_{x \to 2} \frac{-(x - 2)(3x + 2)}{x - 2}$$

$$\lim_{x \to 2} 3x + 2 = 3(2) + 2 = 8$$