Applications of Integration

Essential question: *How can integrals be applied in the real world?*

Day	Topic	Assignment
April 23-24	Keeper 34 - Area Between Two Curves - Area of a region between two curves - Area of a region between intersecting curves	**Keeper 34 Worksheet- Area Between Two Curves**
April 25-27	Keeper 35 - Average Function Value and Mean Value Theorem - Review the fundamental theorem of calculus - The mean value theorem for integrals - The average value theorem for integrals - Review the second fundamental theorem of calculus	**Quick Homework Quiz** **Keeper 35 - Average Function Value and Mean Value Theorem** **Major Quiz - April 27th**
April 30-1	Keeper 36 - Differential Equations - General and particular solutions - Separation of variables - Homogeneous differential equations - Applications	**Quick Homework Quiz** **Keeper 36 - Differential Equations**
May 2-4	Keeper 37 - Volumes of Solids with Known Cross Sections - Finding volumes of solids with known bases - circles, semicircles, triangles, squares - Finding volumes when reflected over the x and y axis	**Quick Homework Quiz** **Keeper 37 - Volumes of Solids with Known Cross Sections** **Major Quiz - May 4th**
May 7-9	Keeper 38 - Volumes of Revolution - Finding volumes using the disk method - Finding volumes using the washer method - Finding volumes using the shell method (*time permitting)	**Quick Homework Quiz** **Keeper 38 - Volumes of Revolution**
May 10th	Review	**Complete Homework Packet**
May 11th	**Test on Integration Applications**	**Turn in all unit 8 homework assignments (worksheets only) for a unit daily grade!**

***May 14th – May 21st – Final Exam Review**

Honors Calculus Name________________________________

Area Between Two Curves

Date________________ Period____

For each problem, find the area of the region enclosed by the curves.

1) $y = -\frac{x^3}{2} - \frac{x^2}{2} + 2x,\ y = -\frac{x^2}{2}$

Intersection:
$-\frac{x^3}{2} - \frac{x^2}{2} + 2x = -\frac{x^2}{2}$
$-\frac{x^3}{2} + 2x = 0$
$x^3 - 4x = 0$
$x(x+2)(x-2)$
$0, -2, 2$

$\left|\int_{-2}^{0} -\frac{x^3}{2} - \frac{x^2}{2} + 2x + \frac{x^2}{2}\,dx\right| + \left|\int_{0}^{2} -\frac{x^2}{2} - \frac{x^2}{2} + 2x + \frac{x^2}{2}\,dx\right|$
$= 2 + 2 = \boxed{4}$

2) $x = -\frac{y^2}{2} + 2y - 1,\ x = \frac{y}{2} - 3,$
$y = 1,\ y = 6$

Intersection:
$-\frac{y^2}{2} + 2y - 1 = \frac{y}{2} - 3$
$y^2 - 4y + 2 = -y + 6$
$y^2 - 3y - 4 = 0$
$(y-4)(y+1) = 0$
$4, -1$

$\left|\int_{1}^{4} \frac{y}{2} - 3 + \frac{y^2}{2} - 2y + 1\,dy\right|$
$\left|\int_{4}^{6} \frac{y}{2} - 3 + \frac{y^2}{2} - 2y + 1\,dy\right|$
$= \frac{157}{12}$

3) $x = \sqrt{y},\ x = 3\sqrt{y},$
$y = 0,\ y = 4$

Intersection
$\sqrt{y} = 3\sqrt{y}$
$y = 9y$
$9y - y =$
$y = 0$

$\left|\int_{0}^{4} \sqrt{y} - 3\sqrt{y}\,dy\right|$

$\boxed{\frac{32}{3}}$

4) $y = -2x^2 + 12x - 14,\ y = 2x^2 - 8x + 2$

Intersection
$-2x^2 + 12x - 14 = 2x^2 - 8x + 2$
$4x^2 - 20x + 16 = 0$
$x^2 - 5x + 4 = 0$
$(x-4)(x-1) = 0$
$4, 1$

$\left|\int_{1}^{4} 2x^2 - 8x + 2 + 2x^2 - 12x + 14\,dx\right|$
$= 18$

5) $y = -2x^2 + 3,\ y = -2x - 1,$
$x = -2,\ x = 2$

Intersection
$-2x^2 + 3 = -2x - 1$
$2x^2 - 2x - 4 = 0$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2, -1$

$\left|\int_{-2}^{-1} -2x^2 + 3 + 2x + 1\,dx\right| + \left|\int_{-1}^{2} -2x^2 + 3 + 2x + 1\,dx\right|$
$= \frac{11}{3} + 9 = \frac{38}{3}$

6) $x = \sqrt[3]{y^2},\ x = \frac{1}{2}y$

Intersection
$y^{2/3} = \frac{1}{2}y$
$y^2 = \frac{1}{8}y^3$
$\frac{1}{8}y^3 - y^2 = 0$
$y^2(y-8) = 0$
$0, 8$

$\left|\int_{0}^{8} y^{2/3} - \frac{1}{2}y\,dy\right|$
$= 3.2$

7) $x = y^3 - 6y,\ x = y^2$

Intersection:
$y^3 - 6y = y^2$
$y^3 - y^2 - 6y = 0$
$y(y^2 - y - 6)$
$y(y-3)(y+2)$
$0, 3, -2$

$\left|\int_{-2}^{0} y^3 - 6y - y^2\,dy\right| + \left|\int_{0}^{3} y^3 - 6y - y^2\,dy\right|$
$= \frac{16}{3} + \frac{63}{4} = \frac{253}{12}$

8) $y = -\frac{x^2}{2} - 4x - 7,\ y = -x^2 - 6x - 7,$
$x = -6,\ x = -1$

Intersection:
$\frac{x^2}{2} + 4x + 7 = x^2 + 6x + 7$
$x^2 + 8x + 14 = 2x^2 + 12x + 14$
$x^2 + 4x = 0$
$x(x+4) = 0$
$0, -4$

$\left|\int_{-6}^{-4} -\frac{x^2}{2} - 4x - 7 + x^2 + 6x + 7\,dx\right| + \left|\int_{-4}^{-1} -\frac{x^2}{2} - 4x - 7 + x^2 + 6x + 7\,dx\right|$
$= \frac{16}{3} + \frac{9}{2} = \frac{59}{6}$

9) $x = -\frac{y^3}{2} + 3y,\ x = -\frac{y^2}{2}$

Intersection:
$-\frac{y^3}{2} + 3y = -\frac{y^2}{2}$
$y^3 - 6y = y^2$
$y(y^2 - y - 6) = 0$
$y(y-3)(y+2) = 0$
$0, 3, -2$

$\left|\int_{-2}^{0} -\frac{y^3}{2} + 3y + \frac{y^2}{2}\,dy\right| + \left|\int_{0}^{3} -\frac{y^3}{2} + 3y + \frac{y^2}{2}\,dy\right|$
$= 10.54$

10) $x = \frac{1}{y^2},\ x = -2,$
$y = -4,\ y = -1$

Intersection:
$\frac{1}{y^2} = -2$
$1 = -2y^2$

$\left|\int_{-4}^{-1} \frac{1}{y^2} + 2\,dy\right|$
$= \frac{27}{4}$

11) $y = x^2 - 8x + 12,\ y = -x + 2$

$$\int_2^5 -x+2-x^2+8x-12\ dx$$

$$= \frac{9}{2}$$

12) $x = -\frac{y^2}{2} - y + \frac{1}{2},\ x = -y - 4,$
$y = -5,\ y = 0$

$$\int_{-5}^{-3} -y-4+\frac{y^2}{2}+y-\frac{1}{2}\ dy\ +$$

$$\int_{-3}^{0} -\frac{y^2}{2}-y+\frac{1}{2}+y+4\ dy$$

$$= \frac{49}{3}$$

13) $x = y^3 + y^2 - 5y,\ x = y$

$$\int_{-3}^{0} y^3+y^2-5y-y\ dy\ +$$

$$\int_0^2 y-y^3-y^2+5y\,dy$$

$$= \frac{253}{12}$$

14) $x = 2y^2 + 4y - 2,\ x = -2$

$$\int_{-2}^{0} -2-2y^2-4y+2\ dy$$

$$= \frac{8}{3}$$

15) $y = -\frac{x^2}{2} - 4x - 9,\ y = -x - 4,$
$x = -5,\ x = -2$

$$\int_{-5}^{-2} -x-4+\frac{x^2}{2}+4x+9\ dx$$

$$= 3$$

16) $x = -y^2 - 4y - 3,\ x = y^2 + 2y - 3,$
$y = -4,\ y = -2$

$$\int_{-4}^{-3} y^2+2y-3+y^2+4y+3\ dy$$

$$+\int_{-3}^{-2} -y^2-4y-3-y^2-2y+3\ dy$$

$$= 6$$

17) $y = -\frac{x^2}{2} - 4x - 6,\ y = \frac{x^2}{2} + 2x - 1$

$$\int_{-5}^{-1} -\frac{x^2}{2}-4x-6-\frac{x^2}{2}-2x+1\ dx$$

$$= \frac{32}{3}$$

18) $x = 2\sqrt[3]{y^2},\ x = y$

$$\int_0^8 2\sqrt[3]{y^2}-y\ dy$$

$$= \frac{32}{5}$$

19) $x = y^2 - 4y + 3,\ x = \frac{y^2}{2} - 2y + 3$

$$\int_0^4 \frac{y^2}{2}-2y+3-y^2+4y-3\,dy$$

$$= \frac{16}{3}$$

20) $x = 2y^2 - 8y + 5,\ x = -\frac{y^2}{2} + 4y - 10,$
$y = 1,\ y = 4$

$$\int_1^4 2y^2-8y+5+\frac{y^2}{2}-4y+10\ dy$$

$$= \frac{15}{2}$$

Honors Calculus — Name ______________________________

Average Function Value/MVT — Date ___________________ Block _______

For each problem, find the average value of the function over the given interval.

1. $f(x) = -x^2 - 2x + 5;\ [-4,0]$

$$\frac{1}{0+4}\int_{-4}^{0} -x^2-2x+5\,dx$$

$$= \frac{11}{3}$$

2. $f(x) = -x^4 + 2x^2 + 4;\ [-2,1]$

$$\frac{1}{3}\int_{-2}^{1} -x^4+2x^2+4\,dx$$

$$= \frac{19}{3}$$

3. $f(x) = 4 - x^2;\ [-2,2]$

$$\frac{1}{4}\int_{-2}^{2} 4-x^2\,dx$$

$$= \frac{8}{3}$$

4. $f(x) = \frac{4(x^2+1)}{x^2};\ [1,3]$

$$\frac{1}{2}\int_{1}^{3} \frac{4(x^2+1)}{x^2}\,dx$$

$$= \frac{16}{3}$$

5. $f(x) = \frac{x^2+5}{x};[1,2]$

$$\int_{1}^{2} \frac{x^2+5}{x}\,dx$$

$$= 4.966$$

6. $f(x) = 4xe^{-2x^2};\ [-1,1]$

$$\frac{1}{2}\int_{-1}^{1} 4xe^{-2x^2}\,dx$$

$$= 0$$

7. $f(x) = \sin x;\ [0,\pi]$

$$\frac{1}{\pi}\int_{0}^{\pi} \sin x\,dx$$

$$= .6366$$

8. $f(x) = \cos x;\ \left[0,\frac{\pi}{2}\right]$

$$\frac{2}{\pi}\int_{0}^{\pi/2} \cos x\,dx$$

9. $f(x) = 4\cos(2x);\ \left[\frac{\pi}{6},\frac{\pi}{2}\right]$

$$\frac{3}{\pi}\int_{\pi/6}^{\pi/2} 4\cos(2x)\,dx$$

$$= -1.654$$

10. $f(x) = x^5 - 2x^3 - 2;\ [-1,1]$

$$\frac{1}{2}\int_{-1}^{1} x^5-2x^3-2\,dx$$

$$= -2$$

For each problem, find the values of c that satisfy the Mean Value Theorem for Integrals.

11. $f(x) = -\frac{x^2}{2} + x + \frac{3}{2}$; $[-3,1]$

$4\left(-\frac{c^2}{2} + c + \frac{3}{2}\right) = \int_{-3}^{1} -\frac{x^2}{2} + x + \frac{3}{2}\,dx$

$-2c^2 + 4c + 6 = -\frac{8}{3}$

$c^2 - 2c - 3 = 4/3$

$c = -1.3094, 3.3094$

12. $f(x) = \frac{4}{x^2}$; $[-4,-2]$

$2\left(\frac{4}{c^2}\right) = \int_{-4}^{-2} \frac{4}{x^2}\,dx$

$2\left(\frac{4}{c^2}\right) = 1$

$\frac{4}{c^2} = \frac{1}{2}$

$c^2 = 8$

$c = \pm 2\sqrt{2}$

13. $f(x) = 4\sqrt{x}$; $[0,3]$

$3(4\sqrt{c}) = \int_0^3 4\sqrt{x}\,dx$

$12\sqrt{c} = 13.8564$

$\sqrt{c} = 1.1547$

$c = 1/3$

14. $f(x) = \frac{1}{x}$; $[2,3]$

$\frac{1}{c} = \int_2^3 \frac{1}{x}\,dx$

$\frac{1}{c} = .4055$

$c = 2.4663$

15. $f(x) = x^5 - 2x^3 + x$; $[-1,0]$

Do Not Do

16. $f(x) = x^5 - 4x^3 + 2x - 1$; $[-2,2]$

Do Not Do

17. $f(x) = -x + 2$; $[-2,2]$

$4(-c+2) = \int_{-2}^{2} -x + 2\,dx$

$4(-c+2) = 8$

$-c + 2 = 2$

$-c = 0$

$c = 0$

18. $f(x) = \frac{4}{(2x+6)^2}$; $[-6,-5]$

$\frac{4}{(2c+6)^2} = \int_{-6}^{-5} \frac{4}{(2x+6)^2}\,dx$

$\frac{4}{(2c+6)^2} = \frac{1}{6}$

$24 = (2c+6)^2$

$\pm 2\sqrt{6} = 2c + 6$

$c = \frac{-6 \pm 2\sqrt{6}}{2}$

$c = -3 \pm \sqrt{6}$

19. $f(x) = -x^2 - 8x - 17$; $[-6,3]$

$9(-c^2 - 8c - 17) = \int_{-6}^{3} -x^2 - 8x - 17\,dx$

$9(-c^2 - 8c - 17) = -126$

$c^2 + 8c + 17 = 14$

$c^2 + 8c + 3 = 0$

$c = -.3944, -7.6056$

20. $f(x) = -3(2x-6)^{\frac{1}{2}}$; $[3,5]$

$2\left(-3(2c-6)^{1/2}\right) = \int_3^5 -3(2x-6)^{1/2}\,dx$

$2\left(-3(2c-6)^{1/2}\right) = -8$

$(2c-6)^{1/2} = 4/3$

$2c - 6 = \frac{16}{9}$

$c = \frac{35}{9}$

Honors Calculus Name ___________________

Solve the separable Differential Equations: Use your own paper to work the problems.

1. $\frac{dy}{dx} = \frac{y}{x}$

$x\,dy = y\,dx$

$\frac{1}{y}dy = \frac{1}{x}dx$

$\ln y = \ln x + c$

$y = e^{\ln x + c}$

$y = Ce^{\ln x}$

$\boxed{y = Cx}$

2. $\frac{dy}{dx} = -xy$

$\frac{1}{y}dy = -x\,dx$

$\ln y = -\frac{x^2}{2} + c$

$\boxed{y = Ce^{-\frac{x^2}{2}}}$

3. $\frac{\sqrt{1+x^2}}{1+y}\frac{dy}{dx} = x$

$\frac{1}{1+y}dy = \frac{x}{\sqrt{1+x^2}}dx$

$\ln|1+y| = \sqrt{1+x^2} + c$

$1+y = Ce^{\sqrt{1+x^2}}$

$\boxed{y = Ce^{\sqrt{1+x^2}} - 1}$

4. $(1+y^2)\frac{dy}{dx} = e^x y$

Do Not Do

5. $e^y \sin x + \frac{dy}{dx}\cos^2 x = 0$

$\cos^2 x\,dy = -e^y \sin x\,dx$

$\frac{1}{-e^y}dy = \tan x \sec x\,dx$

$e^{-y} = \sec x + c$

$-y = \ln(\sec x + c)$

$\boxed{y = -\ln(\sec x + c)}$

6. $\frac{dy}{dx} - \frac{y^2 - y}{\sin x} = 0$

7. $(1+x^4)\frac{dy}{dx} = \frac{x^3}{y}$

$y\,dy = \frac{x^3\,dx}{1+x^4}$

$\frac{y^2}{2} = \frac{1}{4}\ln|1+x^4| + c$

$y^2 = \frac{1}{2}\ln(1+x^4) + c$

$\boxed{y = \sqrt{\frac{1}{2}\ln(1+x^4) + c}}$

8. $\frac{dy}{dx} = (1+y^2)x^2$

Solve the differential equation. Use the value given to find C.

9. $\frac{dy}{dx} - xe^y = 2e^y \quad y(0) = 0$

$\frac{dy}{dx} = 2e^y + xe^y$

$\frac{dy}{dx} = e^y(2+x)$

$e^{-y}dy = 2+x\,dx$

$-e^{-y} = 2x + \frac{x^2}{2} + c$

$e^{-y} = -2x - \frac{x^2}{2} + c$

$-y = \ln(-2x - \frac{x^2}{2} + c)$

$y = -\ln(-2x - \frac{x^2}{2} + c)$

$0 = -\ln(-c)$

$0 = \ln(-c)$

$1 = c$

$y = -\ln(-2x - \frac{x^2}{2} + 1)$

10. $\frac{dy}{dx} = \cancel{xy + x}\ x(y+1) \quad y(0) = 3$

$\frac{1}{y+1}dy = x\,dx$

$\ln(y+1) = \frac{x^2}{2} + c$

$\ln 4 = c$

$\ln(y+1) - \ln 4 = \frac{x^2}{2}$

$\ln\frac{y+1}{4} = \frac{x^2}{2}$

$\frac{y+1}{4} = e^{\frac{x^2}{2}}$

$y+1 = 4e^{\frac{x^2}{2}}$

$\boxed{y = 4e^{x^2/2} - 1}$

11. $\frac{dy}{dx} + y = 2 \quad y(0) = 1$

$\frac{1}{2-y}dy = dx$

$-\ln(2-y) = x + c$

$-\ln(2-1) = 0 + c$

$c = 0$

$-\ln(2-y) = x$

$\ln(2-y) = -x$

$2 - y = e^{-x}$

$\boxed{y = 2 - e^{-x}}$

12. $\frac{dy}{dx} = \frac{4x^2}{y + \cos y} \quad y(0) = \pi$

Do Not Do

Honors Calculus Name________________________________

Volumes of Known Cross Sections

Date________________ Period____

For each problem, find the volume of the specified solid.

1) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$. Cross-sections perpendicular to the x-axis are squares.

$$\int_{-3}^{3} \left(\sqrt{16 - \frac{16x^2}{9}} + \sqrt{16 - \frac{16x^2}{9}} \right)^2 dx$$

$$= 256$$

2) The base of a solid is the region enclosed by $y = -\frac{x^2}{4} + 1$ and $y = 0$. Cross-sections perpendicular to the x-axis are squares.

$$\int_{-2}^{2} \left(-\frac{x^2}{4} + 1 \right)^2 dx$$

$$= \frac{32}{15}$$

3) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 25$. Cross-sections perpendicular to the y-axis are semicircles.

$$\frac{\pi}{8} \int_{-5}^{5} \left(\sqrt{25 - y^2} + \sqrt{25 - y^2} \right)^2 dy$$

$$= \frac{250\pi}{3}$$

4) The base of a solid is the region enclosed by $y = 4$ and $y = \frac{x^2}{4}$. Cross-sections perpendicular to the x-axis are equilateral triangles.

$$\frac{\sqrt{3}}{4} \int_{-4}^{4} \left(4 - \frac{x^2}{4} \right)^2 dx$$

$$= \frac{256\sqrt{3}}{15} \approx 29.56$$

5) The base of a solid is the region enclosed by the semicircle $y = \sqrt{16 - x^2}$ and the x-axis. Cross-sections perpendicular to the y-axis are equilateral triangles.

$$\frac{\sqrt{3}}{4} \int_{0}^{4} \left(\sqrt{16 - y^2} + \sqrt{16 - y^2} \right)^2 dy$$

$$\frac{128\sqrt{3}}{3} \approx 73.901$$

6) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. Cross-sections perpendicular to the y-axis are squares.

$$\int_{-4}^{4} \left(\sqrt{25 - \frac{25y^2}{16}} + \sqrt{25 - \frac{25y^2}{16}} \right)^2$$

$$= \frac{1600}{3}$$

7) The base of a solid is the region enclosed by the semicircle $y = \sqrt{9 - x^2}$ and the x-axis. Cross-sections perpendicular to the y-axis are equilateral triangles.

$$\frac{\sqrt{3}}{4} \int_0^3 \left(\sqrt{9 - y^2} + \sqrt{9 - y^2} \right)^2 dy$$

$$= 18\sqrt{3} \approx 31.177$$

8) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 16$. Cross-sections perpendicular to the y-axis are semicircles.

$$\frac{\pi}{8} \int_{-4}^{4} \left(\sqrt{16 - y^2} + \sqrt{16 - y^2} \right)^2 dy$$

$$\frac{128}{3}\pi \approx 134.041$$

9) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 36$. Cross-sections perpendicular to the x-axis are equilateral triangles.

$$\frac{\sqrt{3}}{4} \int_{-5}^{5} \left(\sqrt{36 - x^2} + \sqrt{36 - x^2} \right)^2 dx$$

$$= 288\sqrt{3} \approx 498.831$$

10) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$. Cross-sections perpendicular to the y-axis are semicircles.

$$\frac{\pi}{8} \int_{-5}^{5} \left(\sqrt{9 - \frac{9y^2}{25}} + \sqrt{9 - \frac{9y^2}{25}} \right)^2 dy$$

$$= 30\pi \approx 94.248$$

11) The base of a solid is the region enclosed by the semicircle $y = \sqrt{36 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are squares.

$$\int_{-6}^{6} \left(\sqrt{36 x 2} \right)^2 dx$$

$$= 288$$

12) The base of a solid is the region enclosed by $y = -x^2 + 1$ and $y = 0$. Cross-sections perpendicular to the x-axis are equilateral triangles.

$$\frac{\sqrt{3}}{4} \int_{-1}^{1} (-x^2 + 1)^2 dx = \frac{4\sqrt{3}}{15} \approx .462$$

Honors Calculus
Volumes of Revolution: Disk Method Worksheet

Name ____________________
Date ______________ Block ________

Find the Volumes of Revolution:

1. $y = \sqrt{x}, x = 1, x = 4, y = 0$ about the x-axis

$$\pi \int_1^4 x \, dx = \frac{15\pi}{2}$$

2. $y = -x + 1, y = 0, x = 0$ about the x-axis

$$\pi \int_0^1 (-x+1)^2 \, dx = \frac{\pi}{3}$$

3. $y = 4 - x^2, y = 0, x = 0$, (in the 1st quadrant) about the x-axis

$$\pi \int_0^2 (4-x^2)^2 \, dx = \frac{256\pi}{15}$$

4. $y = x^2, x = 0, y = 4$, (in the 1st quadrant) about the y-axis

$$\pi \int_0^4 \sqrt{y}^{\,2} \, dy = 8\pi$$

5. $y = \sqrt{4 - x^2}, y = 0, x = 0$, (in the 1st quadrant) about the x-axis

$$\pi \int_0^2 4 - x^2 \, dx$$

$$= \frac{16\pi}{3}$$

6. $x = 4y - y^2, y = 1, x = 0$, about the y-axis

$$\pi \int_0^1 (4y - y^2)^2$$

$$= \frac{53\pi}{15}$$

7. $y = x^{\frac{2}{3}}, y = 1, x = 0$, about the y-axis

$$\pi \int_0^1 (y^{3/2})^2 \, dy$$

$$= \frac{\pi}{4}$$

8. $y = 5x - x^2, y = 0$, about the x-axis

$$\pi \int_0^5 (5x - x^2)^2 \, dx$$

$$\frac{625\pi}{6}$$

9. $y = \frac{x^2}{2}, y = 8$, about the line $y = 8$

$$\pi \int_0^4 \left(8 - \frac{x^2}{2}\right)^2 dx$$

$$\frac{2048\pi}{15}$$

10. $x = \sqrt{y}, x = 9, y = 0$, about $x = 9$

$$\pi \int_0^{81} (9 - \sqrt{y})^2 \, dy$$

$$\frac{2187\pi}{2}$$

Honors Calculus
Volumes of Revolution: Washer Method Worksheet

Name ____________________
Date ______________ Block ________

Find the Volumes of Revolution:

1. $f(x) = 2\sqrt{x}$ *and* $g(x) = x^2$ about the x-axis

$\pi \int_0^{\sqrt[3]{4}} (2\sqrt{x})^2 - (x^2)^2 \, dx$

9.5

2. $f(x) = 2\sqrt{x}$ *and* $g(x) = x^2$ about the y-axis

$\pi \int_0^{4^{2/3}} (\sqrt{y})^2 - \left(\frac{y}{2}\right)^2 dy$

$= 5.78574$

3. $f(x) = 2\sqrt{x}$ *and* $g(x) = x^2$ about the line $y = 3$

$\pi \int_0^{3\sqrt{4}} (3-x^2)^2 - (3-2\sqrt{x})^2 \, dx$

$= 15.633$

4. $f(x) = 2\sqrt{x}$ *and* $g(x) = x^2$ about the line $y = -1$

$\pi \int_0^{3\sqrt{4}} (2\sqrt{x}+1)^2 - (x^2+1)^2 \, dx$

17.8772

5. $y = x^2 + 1, y = 0, x = 1,\ x = 0$ about the y-axis

$\pi \int_0^1 1^2 \, dy + \int_1^2 1^2 - \sqrt{y-1}^2 \, dy$

$\pi + \pi/2 = \frac{3\pi}{2}$

6. $y = \frac{1}{x}, y = 2,$ *and* $x = 2$ about the y-axis

$\int_0^{1/2} 2^2 + \int_{1/2}^{2} \left(\frac{1}{y}\right)^2 dx$

$2 + 3/2 = 7/2$

7. $y = x, y = 2 - x^2,$ *and* $x = 0$ about the x-axis

$\pi \int_0^1 (2-x^2)^2 - (x)^2 \, dx$

7.9587 or $\frac{38}{15}\pi$

8. $y = x^2$ *and* $y = 2x$, about the y-axis

$\pi \int_0^4 y - \left(\frac{y}{2}\right)^2 dy$

$\frac{8\pi}{3}$

9. $y = x^2$, *and* $y = x + 2$, about the $x - axis$

$\pi \int_1^2 (x+2)^2 - (x^2)^2 \, dx$

38.54
or
$\frac{184\pi}{15}$

10. $y = 2x + 2$ *and* $y = x^2 + 2$ about the x-axis

$\pi \int_0^2 (2x+2)^2 - (x^2+2)^2$

$\frac{48}{5}\pi$

Integral Applications Test Review Name ____________________

Determine the area of the bounded region:

1. $y = \frac{1}{x^2},\ y = 0,\ x = 1,\ x = 5$

$\int_1^5 \frac{1}{x^2}\,dx = \boxed{\frac{4}{5}}$

2. $x = y^2 - 2y, x = -1, y = 0$

$y^2 - 2y = -1$
$y^2 - 2y + 1 = 0$
$(y-1)(y-1)$
$y = 1$

$\int_0^1 y^2 - 2y + 1\,dy$
$= \boxed{\frac{1}{3}}$

3. $y = x,\ y = x^3$

Intersection
$x = x^3$
$x^3 - x = 0$
$x(x^2-1) = 0$
$0, +1, -1$

$\left|\int_{-1}^0 x - x^3\,dx\right| + \left|\int_0^1 x - x^3\,dx\right| = +\frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2}}$

4. $x = y^2 + 1,\ x = y + 3$

$y^2 + 1 = y + 3$
$y^2 - y - 2 = 0$
$(y-2)(y+1) = 0$
$2, -1$

$\int_{-1}^2 y^2 + 1 - y - 3\,dy$
$= \boxed{\frac{9}{2}}$

5. $y = \sin x, y = \cos x, \frac{\pi}{4} \le x \le \frac{5\pi}{4}$

$\sin x = \cos x$
$\frac{\pi}{4}, \frac{5\pi}{4}$

$\int_{\pi/4}^{5\pi/4} \sin x - \cos x\,dx$
$= \boxed{2.8284}$

Find the average value of the function over the interval:

6. $f(x) = \frac{1}{\sqrt{x-1}}$ [5,10]

$\frac{1}{5}\int_5^{10} \frac{1}{\sqrt{x-1}}\,dx = \frac{2}{5}$

7. $f(x) = x^3$ [0,2]

$\frac{1}{2}\int_0^2 x^3\,dx = 2$

Find the of c guaranteed by the mean value theorem:

8. $f(x) = x$ [0,4]

$4(c) = \int_0^4 x\,dx = 8$
$\boxed{c = 2}$

9. $f(x) = x^2 - \frac{1}{x^2}$ [1,2]

$c^2 - \frac{1}{c^2} = \int_1^2 x^2 - \frac{1}{x^2}\,dx$
$c^4 - 1 = \frac{11}{6}c^2$
$6c^4 - 11c^2 - 6 = 0$
$c = -.44,\ 2.27$

Solve the differential equation:

10. $\frac{dy}{dx} = \frac{x^2+3}{x}$

$dy = x + \frac{3}{x}\,dx$
$y = \frac{x^2}{2} + 3\ln|x| + C$

11. $\frac{dy}{dx} = \frac{x}{y},\ y(0) = -3$

$\frac{y^2}{2} = \frac{x^2}{2} + c$
$\frac{9}{2} = c$
$\frac{y^2}{2} = \frac{x^2}{2} + \frac{9}{2}$
$y^2 = x^2 + 9$

$y = -\sqrt{x^2 + 9}$

12. $y' - e^y \sin x = 0$

$-y = \ln(-\cos x + c)$

$\boxed{y = -\ln(-\cos x + c)}$

$\frac{dy}{dx} = e^y \sin x$

$e^{-y}\, dy = \sin x\, dx$

$-e^{-y} = \cos x + C$

$e^{-y} = -\cos x + c$

13. $\frac{dy}{dx} = \frac{\ln x}{xy}$, $y(1) = 2$

$y\, dy = \frac{\ln x}{x}\, dx$

$\frac{y^2}{2} = \frac{(\ln x)^2}{2}$

$y = \ln x + c$

$2 = c$

$\boxed{y = \ln x + 2}$

14. $\frac{dy}{dx} = xy^2$

$\frac{1}{y^2}\, dy = x\, dx$

$-\frac{1}{y} = \frac{x^2}{2} + c$

$-y = \frac{2}{x^2} + c$

$\boxed{y = -\frac{2}{x^2} + c}$

15. $\frac{du}{dt} = \frac{2t+\sec^2 t}{2u}$, $u(0) = -5$

$2u\, du = 2t + \sec^2 t\, dt$

$u^2 = t^2 + \tan t + c$

$25 = c$

$u^2 = t^2 + \tan t + 25$

$u = \sqrt{t^2 + \tan t + 25}$

16. $xy^2 y' = x + 1$

$y^2\, dy = 1 + \frac{1}{x}\, dx$

$\frac{y^3}{3} = x + \ln x + c$

$y^3 = 3x + 3\ln x + c$

$y = \sqrt[3]{3x + 3\ln x + c}$

17. $\frac{dP}{dt} = \sqrt{Pt}$, $P(1) = 2$

$\frac{1}{\sqrt{P}}\, dP = \sqrt{t}\, dt$

$P^{-1/2}\, dP = t^{1/2}\, dt$

$2\sqrt{P} = \frac{2}{3} t^{3/2} + C$

$2\sqrt{2} = \frac{2}{3} c$

$C = 2\sqrt{2} - \frac{2}{3}$

$2\sqrt{P} = \frac{2}{3} t^{3/2} + 2\sqrt{2} - \frac{2}{3}$

$\sqrt{P} = \frac{1}{3} t^{3/2} + \sqrt{2} - \frac{1}{3}$

$P = \left(\frac{1}{3} t^{3/2} + \sqrt{2} - \frac{1}{3}\right)^2$

18. Find the volume of the region generated by $y = \sqrt{25 - x^2}$ and the x-axis. The cross sections are perpendicular to the x-axis:

a. Squares

$\int_{-5}^{5} \left(\sqrt{25 - x^2}\right)^2 dx = \frac{500}{3}$

b. Isosceles triangles

$\frac{1}{2}\int_{-5}^{5} \sqrt{25 - x^2}^{\,2}\, dx = \frac{1}{2}\left(\frac{500}{3}\right) = \frac{250}{3}$

c. Semi Circles

$\frac{\pi}{8}\int 25 - x^2\, dx = \frac{125\pi}{6}$

19. Find the volume of the region generated by $y = \frac{1}{\sqrt{x}}$, $x = 1$, and $x = 4$. The cross sections are perpendicular to the y-axis:

$\sqrt{x} = \frac{1}{y} \quad x = \frac{1}{y^2}$

a. Squares

$\int_{1}^{1/2} \left(\frac{1}{y^2}\right)^2 dy = \frac{7}{3}$

b. Isosceles triangles

$\frac{1}{2}\left(\frac{7}{3}\right) = \frac{7}{6}$

c. Semi Circles

$\frac{\pi}{8} \cdot \frac{7}{3} = \frac{7\pi}{24}$

20. Find the volume of the region generated by $y = -\frac{x^2}{9} + 4$ and $y = 0$. The cross sections are perpendicular to the x-axis. The cross sections are rectangles with a height twice the base.

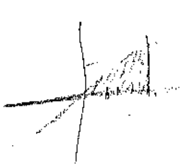
$\int_{-6}^{6} \left(-\frac{x^2}{9} + 4\right)\left(-\frac{2x^2}{9} + 8\right) dx = 204.8$

Find the volume of the solid generated by revolving the plane region bounded by the indicated equations:

21. $y = x, y = 0, x = 4$

a. x-axis

b. y-axis



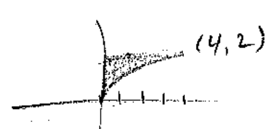
a) Disk $\pi\int_0^4 x^2\, dx = \frac{64\pi}{3}$

b) Washer $\pi\int_0^4 4^2 - y^2\, dy = \frac{128\pi}{3}$

22. $y = \sqrt{x}, y = 2, x = 0$

a. x-axis

b. y-axis



a) Washer $\pi\int_0^4 2^2 - x\, dx = 8$

b) Disk $\pi\int_0^2 \sqrt{x}^{\,2} = 2\pi$

23. $y = \frac{1}{x^4+1}$, $y = 0$, $x = 0, x = 1$

x-axis Disk

$\pi\int_0^1 \left(\frac{1}{y^4+1}\right)^2 dx$

$.7752\pi$