Applications of the Derivative

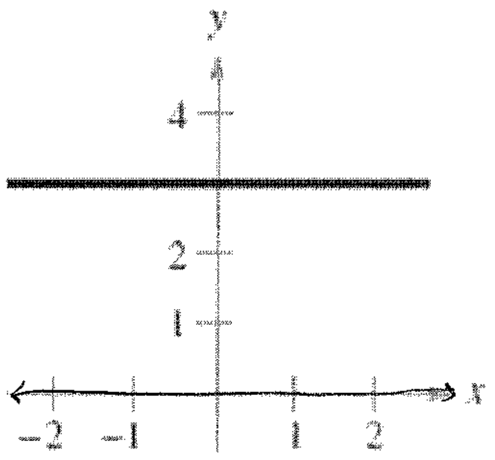
Essential question: ***How can I apply derivatives to the real world?***

Day	Topic	Assignment
Wed - Thurs Feb 28 - 1	Keeper 21 – Curve Sketching - Graphing f' from f -Graphing f'' from f and f'	**Keeper 21 Worksheet – Curve Sketching** **- f from f' Worksheet**
Fri - Mon March 2-5	Keeper 22 – Curve Sketching -Review: Graphing f' from f -Graphing f from f' -Graphing f'' from f'	**Quick Homework Quiz** **Keeper 22 Worksheet – Curve Sketching** **- f' from f Worksheet**
Tues - Wed March 6-7	Keeper 23 – Interpreting Graphs -Graphing unusual functions using characteristics of the first and second derivative.	**Quick Homework Quiz** **Keeper 23 Worksheets –Interpreting Graphs of Derivatives 1 and 2**
Thurs - Fri March 8 - 9	Keeper 24 – Particle Motion -Using derivatives to find velocity, acceleration and other features of particle motion	**Quick Homework Quiz** **Keeper 24 Worksheets – Particle Motion**
Mon - Tues March 12-13	Keeper 25 – Optimization -Finding what maximizes and minimizes lengths, areas, perimeters, volume, surface area, etc. -Finding how to maximize cost, and minimize lost	**Quick Homework Quiz** **Keeper 25 Worksheet – Optimization**
Wed-Fri March 14-16	Keeper 26 – Related Rates - Cubes - Cones -Spheres - Squares	**Quick Homework Quiz** **Keeper 26 Worksheet – Related Rates Packet**
Mon 3/20	Review	**Complete Homework Packet**
Tues 3/20	**Test on Applications of Derivatives**	**Turn in all unit 4 homework assignments (worksheets only) for a unit daily grade!**

Curve Sketching - Graphing f' from f

The graph of f is given below. Sketch a possible graph of f'

1.



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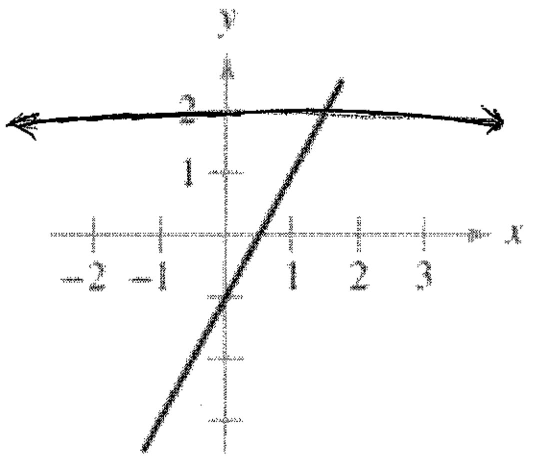

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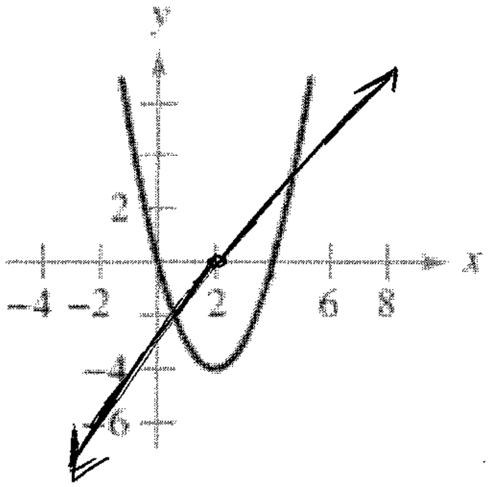

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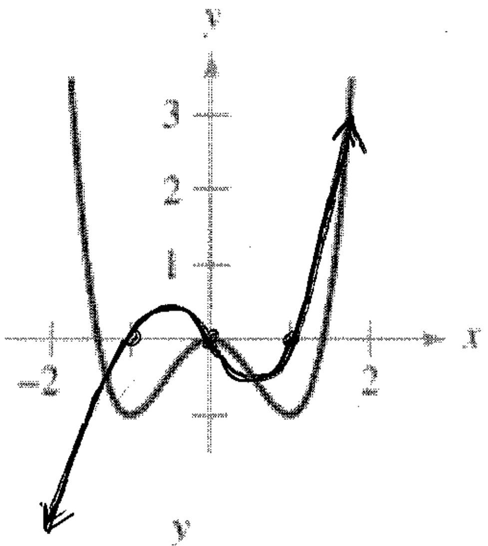

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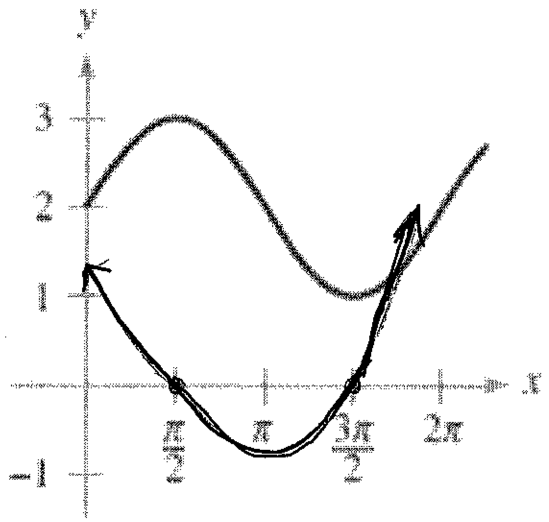

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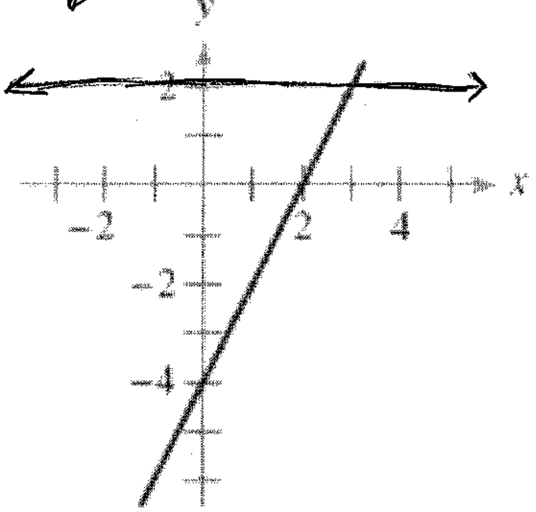

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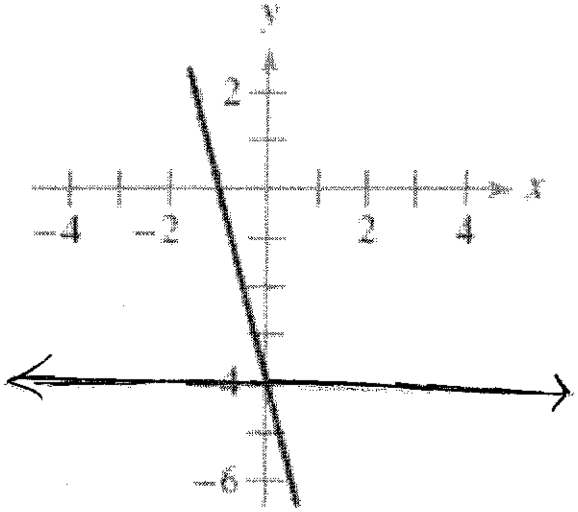

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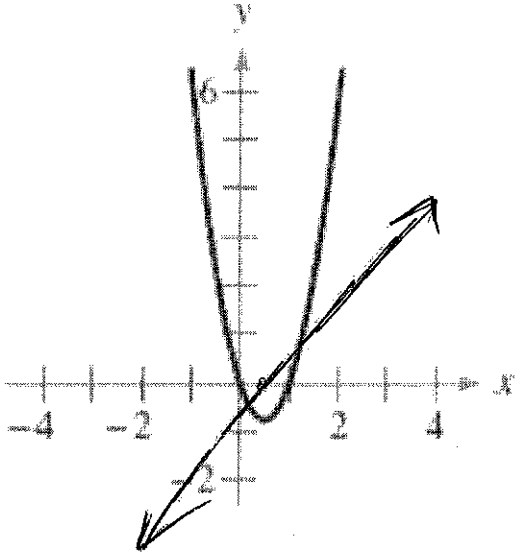

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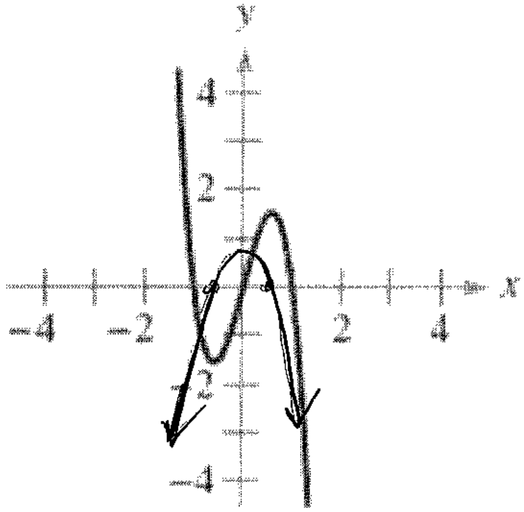

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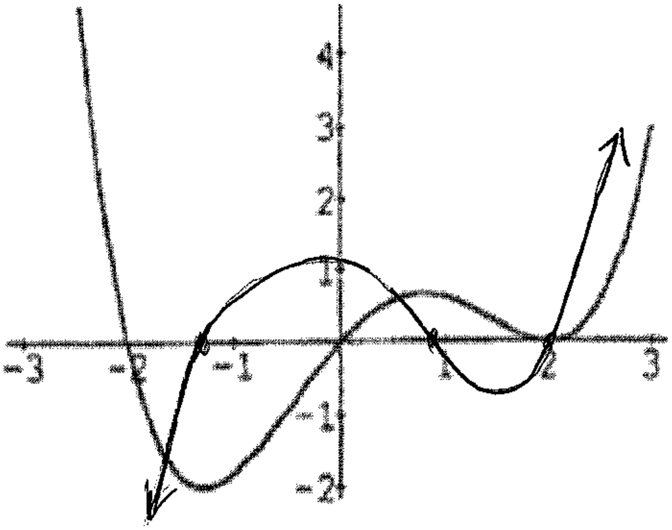
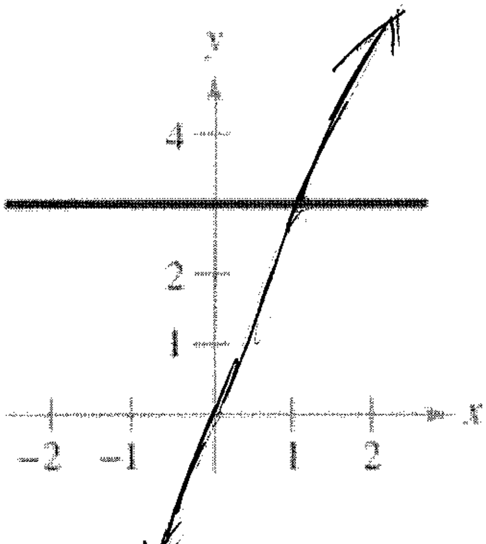


Curve Sketching - Graphing f from f'

The graph of f' is given below. Sketch a possible graph of f

1.



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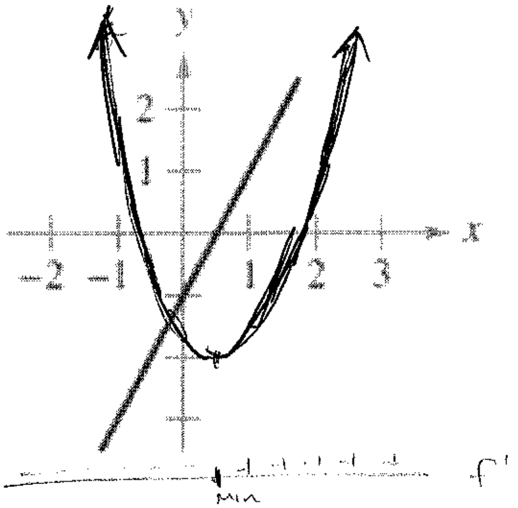

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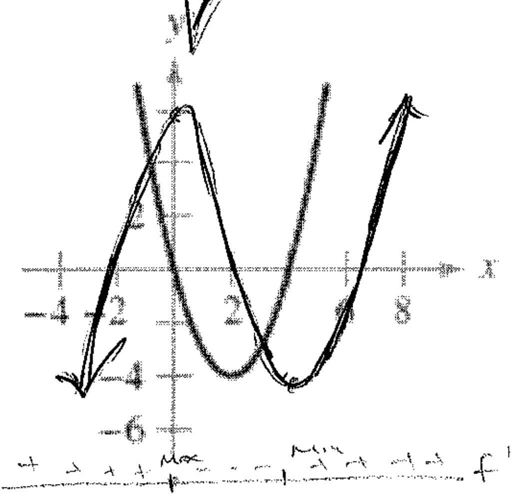

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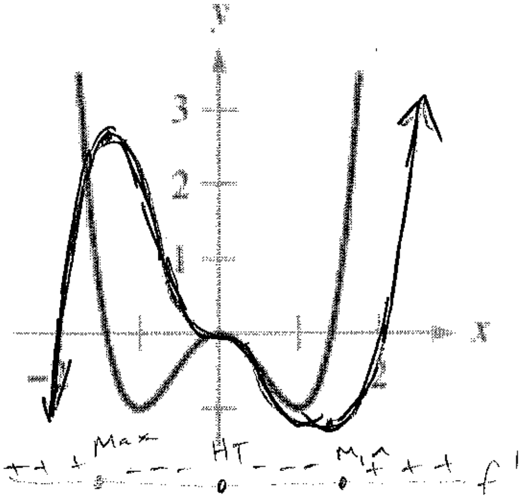

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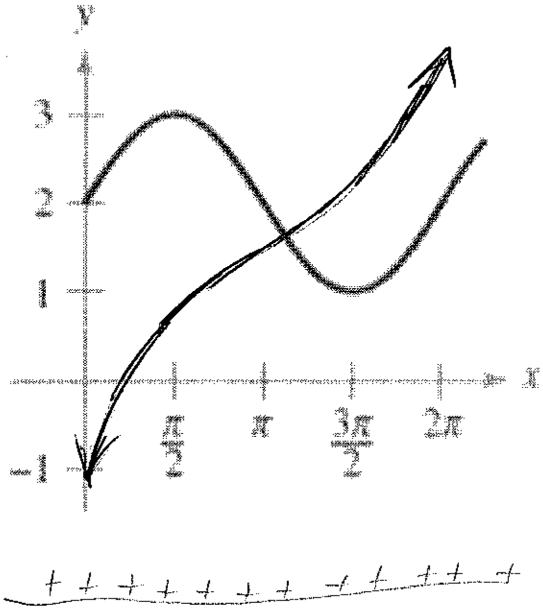

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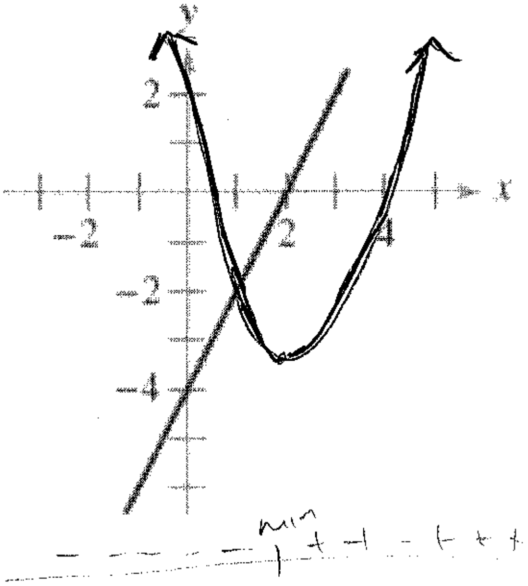

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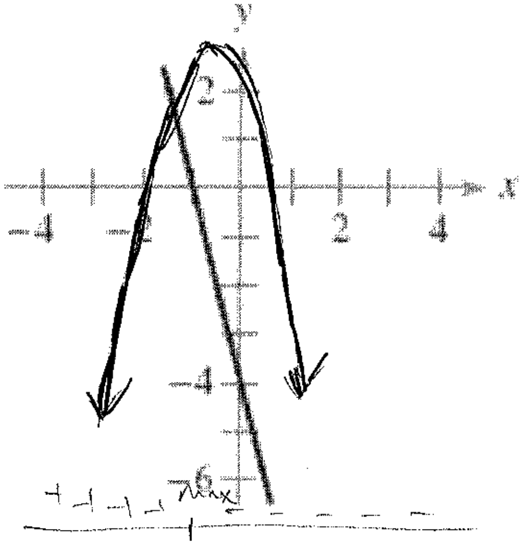
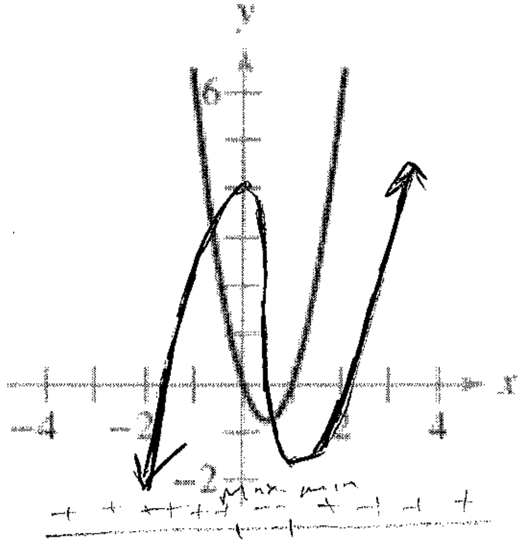


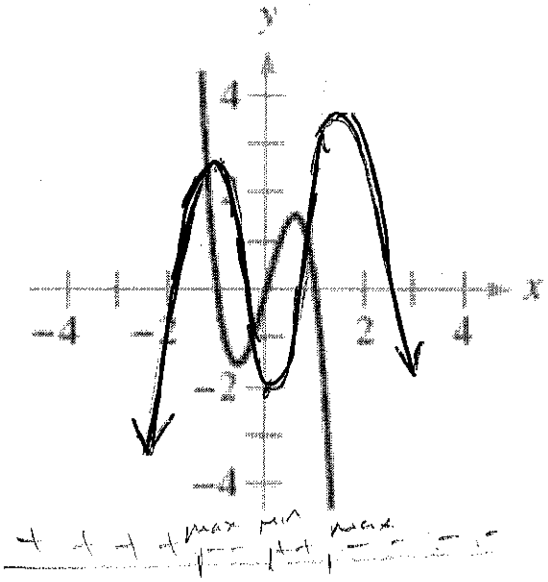
y
x
-4
-2
2
4
2
-4
-6
max

8.



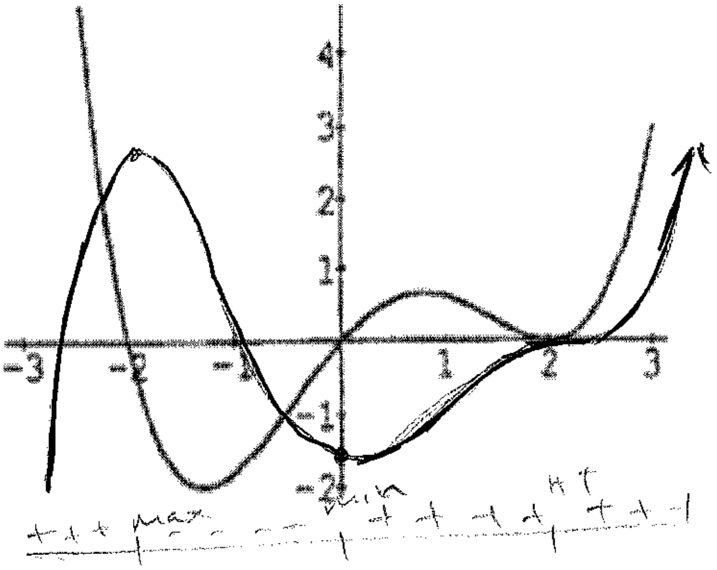
y
x
6
-4
-2
2
4
-2
max
min

9.



y
x
4
-4
-2
2
4
-2
-4
max
min
max

10.



4
3
2
1
-3
-2
-1
1
2
3
-1
-2
max
min
HT

Interpreting Graphs Using Derivatives

For each problem, find the: x and y intercepts, asymptotes, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

1. $y = \frac{x}{x+3}$

$y' = \frac{(x+3)(1) - x}{(x+3)^2} = \frac{3}{(x+3)^2}$

$+ + + + \mid + + + +$; -3

X intercept (0, 0)

Y intercept (0, 0)

Vertical Asymptotes X = -3

Horizontal Asymptotes y = 1

Critical Points None

Interval(s) of Increase (-∞, -3) ∪ (-3, ∞)

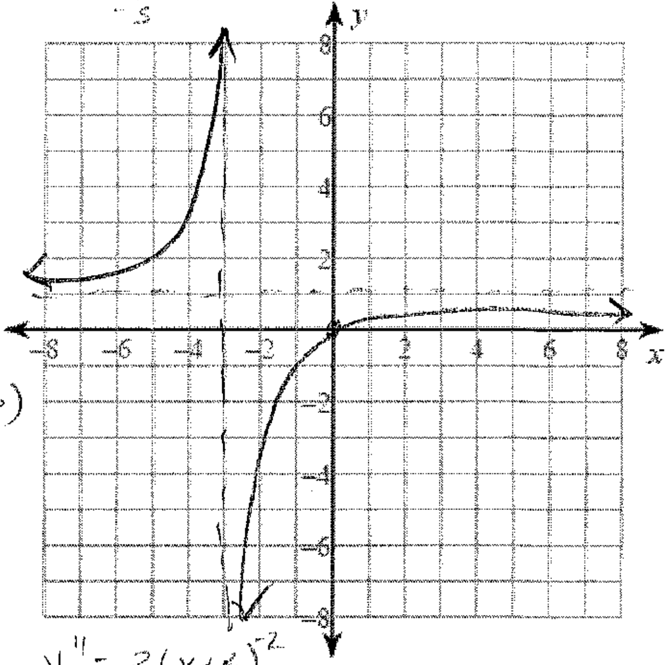
Interval(s) of Decrease None

Concave Up (-∞, -3)

Concave Down (-3, ∞)

Relative Maxima None

Relative Minima None



$y'' = 3(x+3)^{-2}$

$= -6(x+3)^{-3} = \frac{-6}{(x+3)^3}$

$+ + + + \mid - - - - -$; -3

2. $y = \frac{3}{x+3}$

$y' = \frac{-3}{(x+3)^2}$

$- - - \mid - - - -$; -3

X intercept None

Y intercept (0, 1)

Vertical Asymptotes X = -3

Horizontal Asymptotes y = 0

Critical Points X = -3

Interval(s) of Increase None

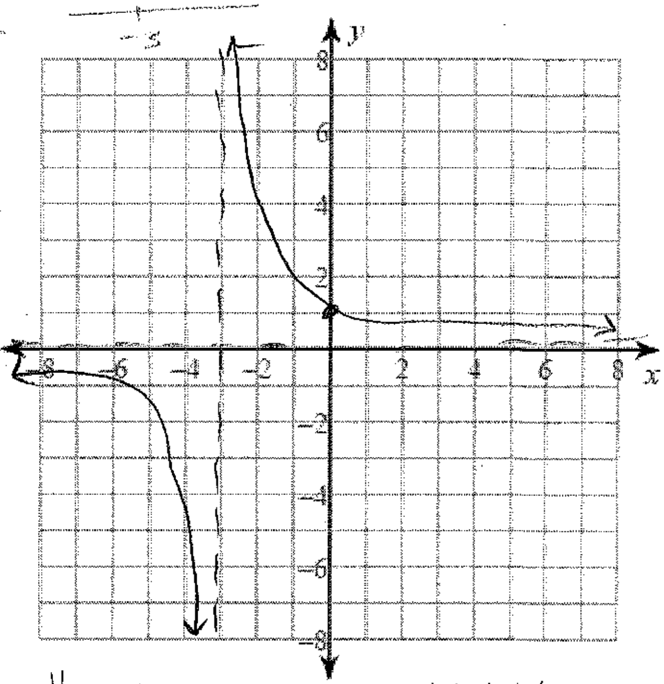
Interval(s) of Decrease (-∞, -3) ∪ (-3, ∞)

Concave Up (-3, ∞)

Concave Down (-∞, -3)

Relative Maxima None

Relative Minima None



$y'' = \frac{6}{(x+3)^3}$

$- - - \mid + + + +$; -3

3. $y = \frac{x^3}{12} + \frac{x^2}{4}$

$12y = x^3 + 3x^2$

$0 = x^2(x+3)$

$y' = \frac{1}{4}x^2 + \frac{1}{2}x = 0$

$x^2 + 2x = 0$

$x = 0, -2$

$\underline{++ \mid -- \mid +++}$

$-2 \quad 0$

X intercept $(0,0) \quad (-3,0)$

Y intercept $(0,0)$

Vertical Asymptotes None

Horizontal Asymptotes None

Critical Points $x = 0, -1/4$

Interval(s) of Increase $(-\infty,-2) \cup (0,\infty)$

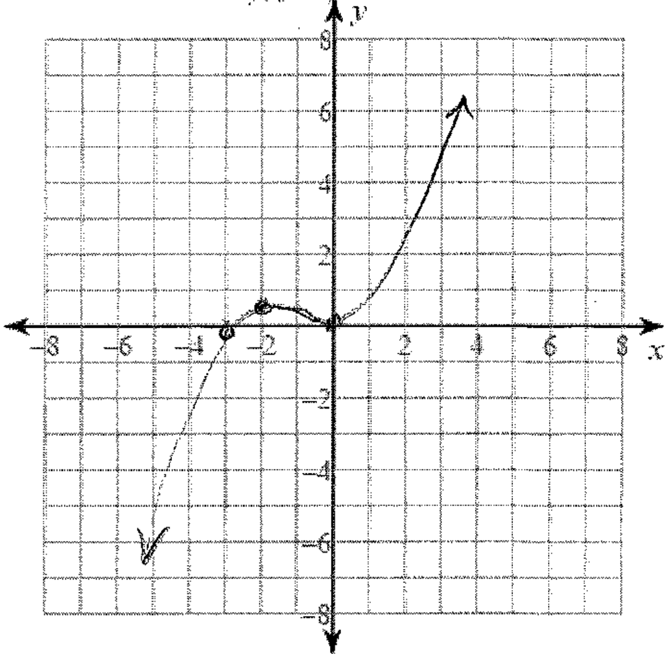
Interval(s) of Decrease $(-2,0)$

Concave Up $(-1,\infty)$

Concave Down $(-\infty,-1)$

Relative Maxima $(-2, 1/2)$

Relative Minima $(0,0)$



$f'' = \frac{1}{2}x + \frac{1}{2} = 0$

$x = -1$

$\underline{--- \mid +++}$

-1

4. $y = -(-2x+12)^{\frac{1}{2}}$

$y' = -\frac{1}{2}(-2x+12)^{-1/2} \cdot -2$

$= \frac{1}{\sqrt{-2x+12}}$

$\underline{++++ \mid \text{DNE}}$

6

X intercept $(6,0)$

Y intercept $(0,-2\sqrt{3})$

Vertical Asymptotes None

Horizontal Asymptotes None

Critical Points $x = 6$

Interval(s) of Increase $(-\infty,6)$

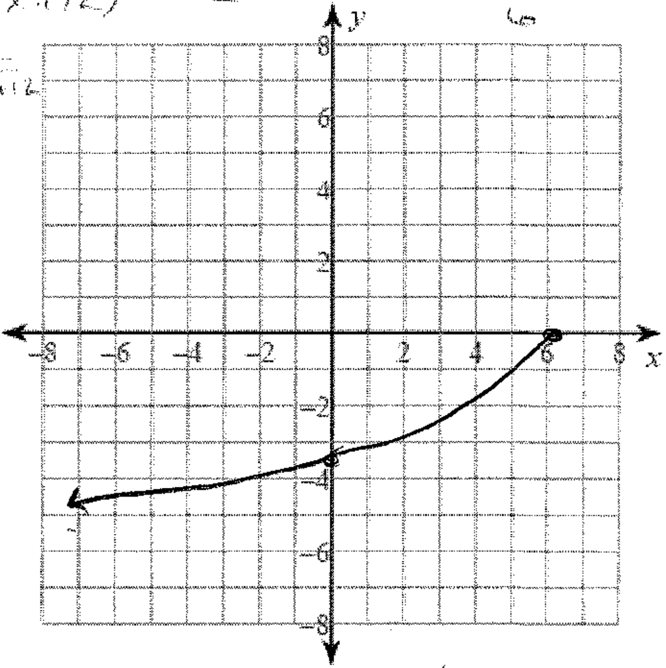
Interval(s) of Decrease None

Concave Up $(-\infty,6)$

Concave Down None

Relative Maxima None

Relative Minima None



$f'' = -\frac{1}{2}(-2x+12)^{-3/2} \cdot 2$

$= -(-2x+12)^{-3/2}$

$\underline{+++ \mid \text{DNE}}$

6

5. $y = -(x-3)^{\frac{2}{3}}$

X intercept (3, 0)

Y intercept (0, $-\sqrt[3]{9}$)

Vertical Asymptotes None

Horizontal Asymptotes None

Critical Points X = 3

Interval(s) of Increase (−∞, 3)

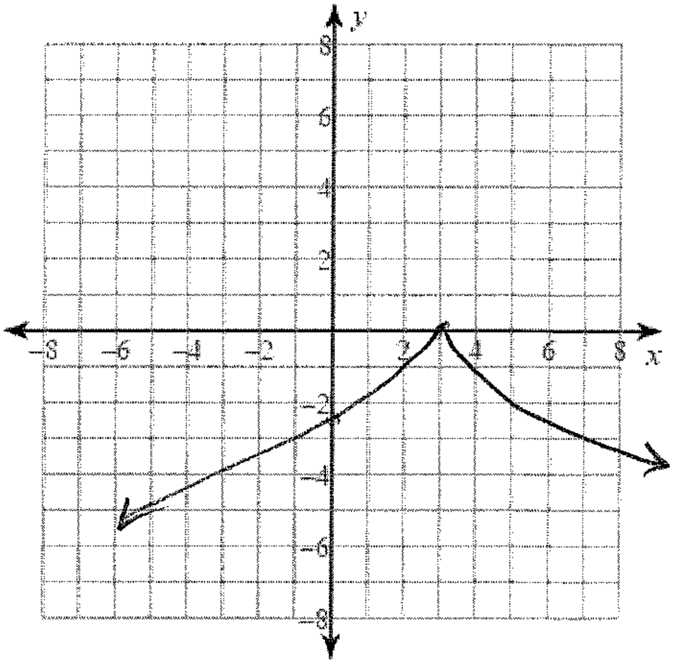
Interval(s) of Decrease (3, ∞)

Concave Up (−∞, 3) (3, ∞)

Concave Down None

Relative Maxima (3, 0)

Relative Minima None



6. $y = \frac{x^2}{4x+8}$ Slant Asymptote

X intercept (0, 0)

Y intercept (0, 0)

Vertical Asymptotes X = −2

Horizontal Asymptotes None

Critical Points X = −4, 0

Interval(s) of Increase (−∞, −4), (0, ∞)

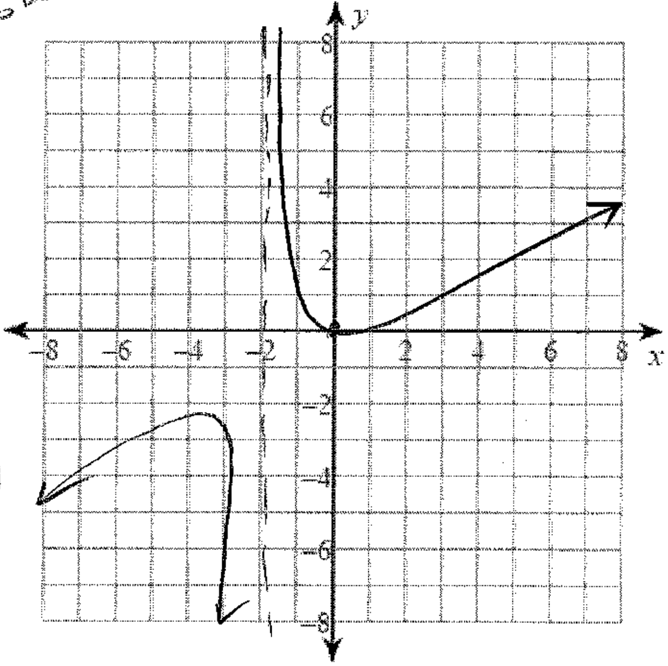
Interval(s) of Decrease (−4, −2), (−2, 0)

Concave Up (−2, ∞)

Concave Down (−∞, −2)

Relative Maxima (−4, −2)

Relative Minima (0, 0)



7. $$y = \frac{-2x}{x-3}$$

X intercept (0, 0)

Y intercept (0, 0)

Vertical Asymptotes X = 3

Horizontal Asymptotes y = -2

Critical Points None

Interval(s) of Increase $(-\infty, 3) \cup (3, \infty)$

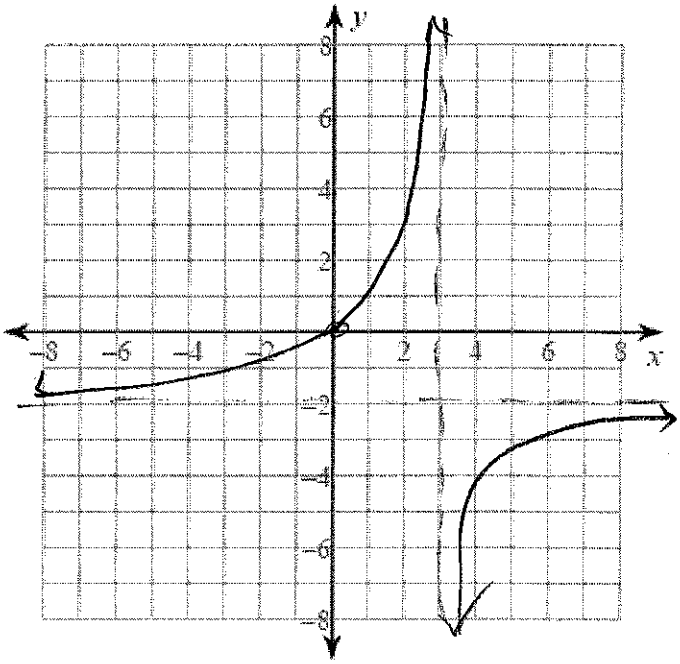
Interval(s) of Decrease None

Concave Up $(-\infty, 3)$

Concave Down $(3, \infty)$

Relative Maxima None

Relative Minima None



8. $$y = x^3 + x^2$$

X intercept (-1, 0) (0, 0)

Y intercept (0, 0)

Vertical Asymptotes None

Horizontal Asymptotes None

Critical Points x = $-2/3$, 0

Interval(s) of Increase $(-\infty, -2/3) \cup (0, \infty)$

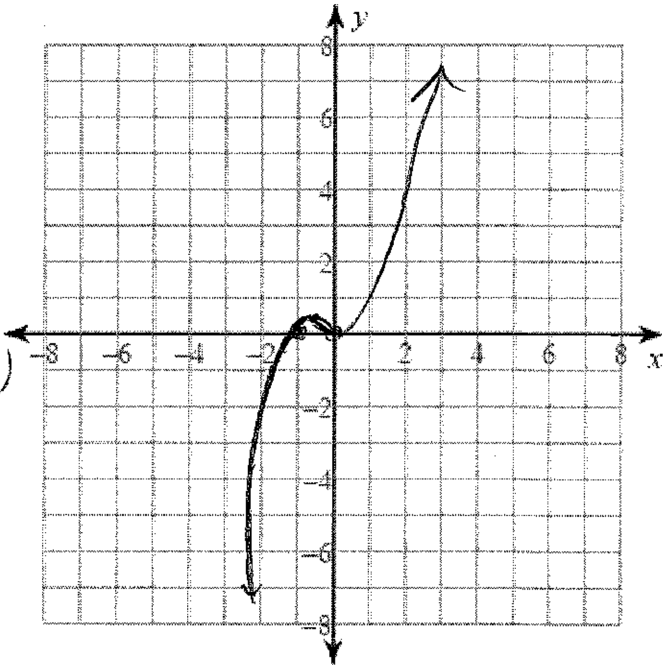
Interval(s) of Decrease $(-2/3, 0)$

Concave Up $(-1/3, \infty)$

Concave Down $(-\infty, -1/3)$

Relative Maxima $(-2/3, -4/27)$

Relative Minima (0, 0)

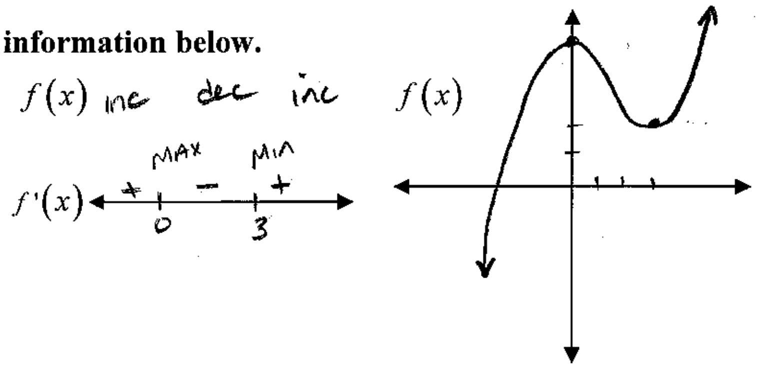


Honors Calculus: 1st & 2nd Derivative
Curve Sketching

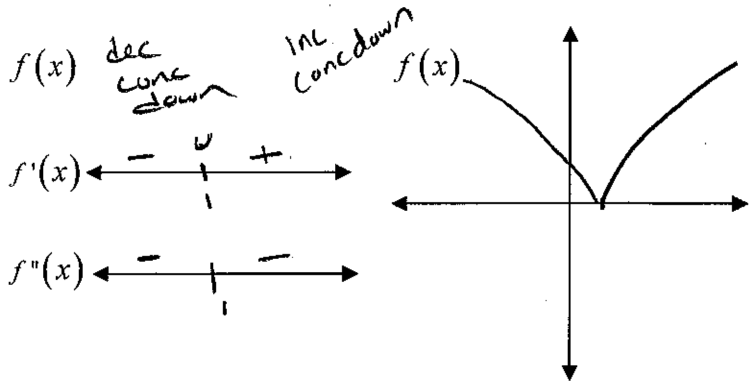
Name ____________________
Date ______________ Period ______

Draw a possible graph of $f(x)$ given the information below.

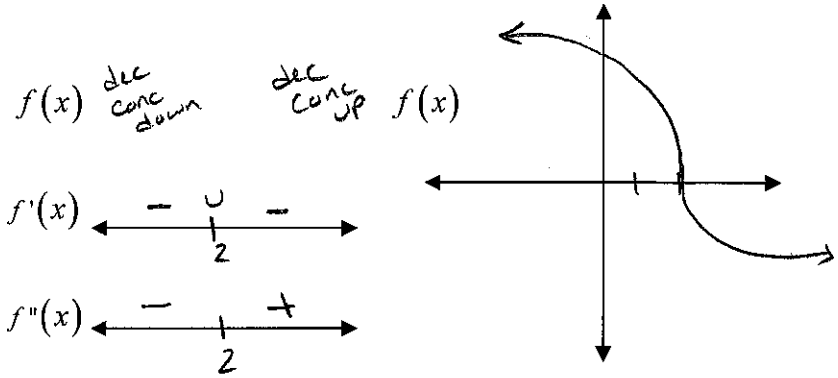
1.
- **a.** $f(x)$ is continuous
- **b.** $f(3)=2$
- **c.** $f'(x)>0$, (-∞,0),(3,∞)
- **d.** $f'(x)<0$, (0,3)
- **e.** $f'(x)=0$ at x=0 ,x=3



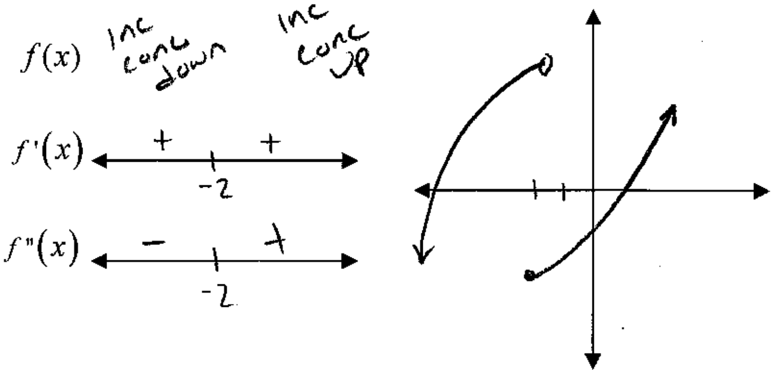
2.
- **a.** $f(x)$ is continuous
- **b.** $f'(x)<0$, (-∞,1)
- **c.** $f'(x)>0$, (1, ∞)
- **d.** $f'(x)=$ undef. at x=1
- **e.** $f''(x)<0$ at $(-\infty,1)\cup(1,\infty)$



3.
- **a.** $f(x)$ is continuous
- **b.** $f'(x)<0;(-\infty,2),(2,\infty)$
- **c.** $f'(x)$ is undefined at $x=2$
- **d.** $f''(x)<0$ when $x<2$
- **e.** $f''(x)>0$ when $x>2$



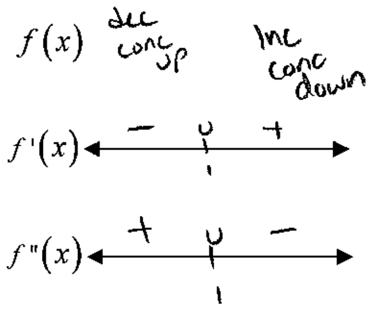
4.
- a. $f(x)$ has jump discont. at $x=$ -2
- **b.** $f'(x)>0;(-\infty,-2),\cup(-2,\infty)$
- **c.** $f''(x)<0;(-\infty,-2)$
- **d.** $f''(x)>0;(-2,\infty)$

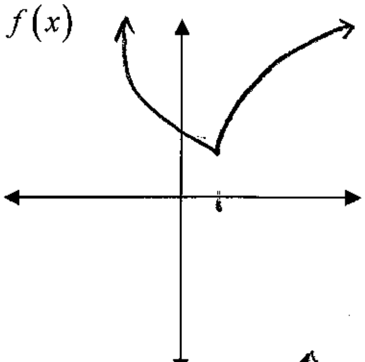


Honors Calculus: 1^{st} & 2^{nd} Derivative Name ______________________________

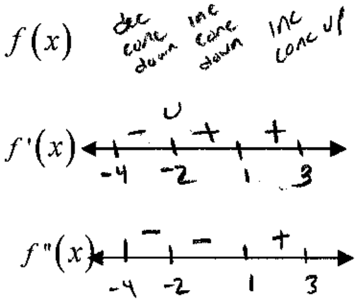
Curve Sketching Date ________________ Period ______

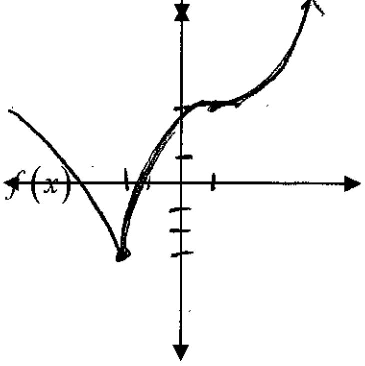
5.
- **a.** $f(x)$ is continuous
- **b.** $f'(x) < 0$ when $x < 1$
- **c.** $f'(x) > 0$ when $x > 1$
- **d.** $f''(x) > 0$ when $x < 1$
- **e.** $f''(x) < 0$ when $x > 1$
- **f.** $f'(x)$ does not exist at $x = 1$
- **g.** $f''(x)$ does not exist at $x = 1$



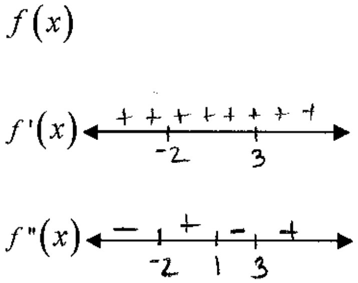


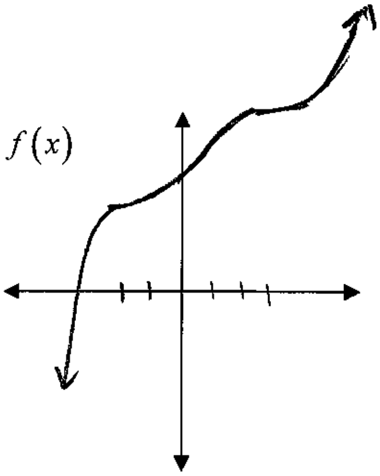
6.
- **a.** $f(x)$ is continuous [-4, 3]
- **b.** $f'(x) < 0$ on $(-4,-2)$
- **c.** $f'(x) > 0$ on $(-2,1)\cup(1,3)$
- **d.** $f'(x) =$ undef. at $x = -2$
- **e.** $f(-2) = -3 \quad f(1) = 3$
- **f.** $f'(x) = 0$ at x=1
- **g.** $f'' < 0$ on $(-4,-2)\cup(-2,1)$
- **h.** $f'' > 0$ on (1, 3)





5.
- **a.** $f(x)$ is continuous
- **b.** $f'(x) > 0$ everywhere
- **c.** $f'(x) = 0$ when $x = -2,\ x = 3$
- **d.** $f''(x) < 0$ on $(-\infty,-2)\cup(1,3)$
- **e.** $f''(x) > 0$ on $(-2,1)\cup(3,\infty)$

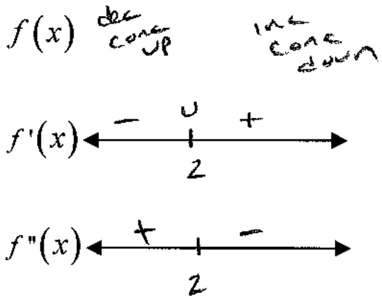


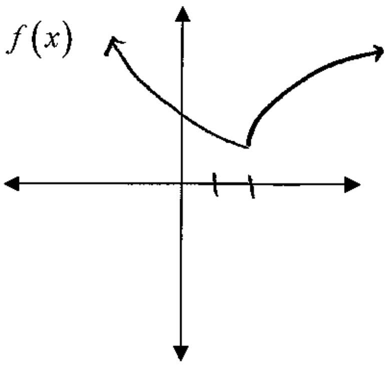


Honors Calculus: 1^{st} & 2^{nd} Derivative
Curve Sketching

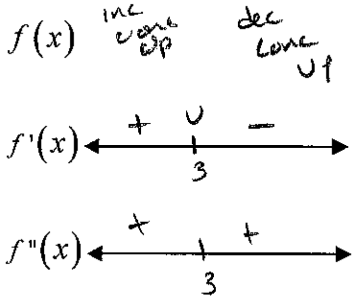
Name ____________________
Date ______________ Period ______

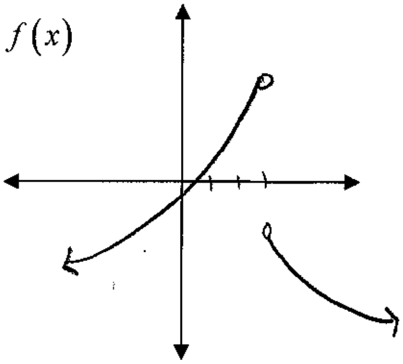
6.
a. $f(x)$ is continuous
b. $f'(x) > 0$ when $x < 2$
c. $f'(x) < 0$ when $x > 2$
d. $f'(x)$ does not exist at $x = 2$
e. $f''(x) > 0$ when $x < 2$
f. $f''(x) < 0$ when $x > 2$



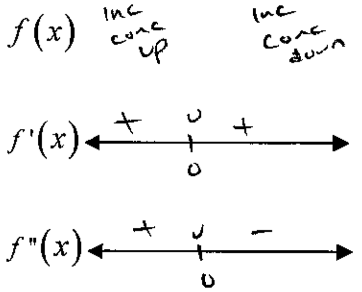


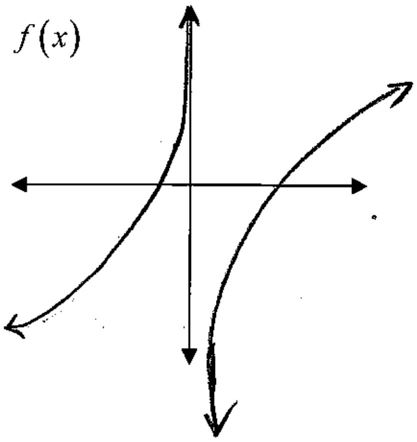
7.
a. $f(x)$ is not continuous at $x = 3$
b. $f'(x) > 0$ when $x < 3$
c. $f'(x) < 0$ when $x > 3$
d. $f''(x) > 0$ when $x < 3$
e. $f''(x) > 0$ when $x > 3$
f. $f'(x)$ does not exist at $x = 3$





8.
a. $f(x)$ is not continuous at $x = 0$
b. $f'(x)$ does not exist at $x = 0$
c. $f'(x) > 0$ when $x < 0$
d. $f'(x) > 0$ when $x > 0$
e. $f''(x)$ does not exist at $x = 0$
f. $f''(x) > 0$ when $x < 0$
g. $f''(x) < 0$ when $x > 0$





Particle Motion

Answer the following questions for each position function $s(t)$ in meters where t is in seconds if a particle is moving along the x-axis.

$s(t) = t^3 - 3t + 3$ [0,6]

a. What is the velocity function?

$V(t) = 3t^2 - 3$

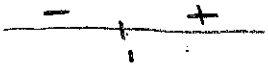
b. What is the velocity at $t = 3$ seconds?

$V(3) = 3(3)^2 - 3 = 27 - 3$
$= 24$ m/s

c. When is the particle at rest?

$V(t) = 3t^2 - 3 = 0$
$t^2 = 1$
$t = 1$ sec

d. When is the particle moving right? Moving left?



Right (1,6) sec
Left (0,1) sec

e. What is the acceleration function?

$a(t) = 6t$

f. What is the acceleration at $t = 1$ second?

$a(1) = 6(1) = 6$ m/s^2

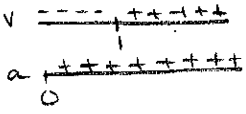
g. What is the displacement?

$S(6) - S(0)$
$201 - 3 = 198$ m

h. What is the total distance traveled?

$(0,1) : S(1) - S(0) = |1 - 3| = 2$
$(1,6) = S(6) - S(1) = 201 - 1 = 200$
$2 + 200 = 202$ m

i. When is the particle speeding up? Slowing Down?



Speed up: (1, 6)
Slow down: (0, 1)

j. Find the velocity when the acceleration is 0.

$a(t) = 6t = 0$ $V(0) = 3(0)^2 - 3$
$t = 0$ $= \boxed{-3 \text{ m/s}}$

$s(t) = t^3 - 6t^2$ [0,7]

a. What is the velocity function?

$V(t) = 3t^2 - 12t$

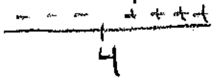
b. What is the velocity at $t = 3$ seconds?

$V(3) = 3(3)^2 - 12(3) = 27 - 36$
$= -9$ m/s

c. When is the particle at rest?

$V(t) = 3t^2 - 12t = 0$
$3t(t - 4) = 0$
$t = 0$ $t = 4$ sec

d. When is the particle moving right? Moving left?



Right (4, 7)
Left (0, 4)

e. What is the acceleration function?

$a(t) = 6t - 12$

f. What is the acceleration at $t = 1$ second?

$a(1) = 6(1) - 12 = -6$ m/s^2

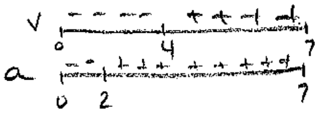
g. What is the displacement?

$S(7) - S(0) = 49 - 0 = 49$ m

h. What is the total distance traveled?

$|S(4) - S(0)| = |-32 - 0| = 32$
$S(7) - S(4) = 49 + 32 = 81$
113 m

i. When is the particle speeding up? Slowing Down?



SU: (0,2), (4,7)
SD: (2,4)

j. Find the velocity when the acceleration is 0.

$a(t) = 6t - 12 = 0$ $V(2) = 3(2)^2 - 12(2)$
$t = 2$
$= 12 - 24$
$= -12$ m/s

$s(t) = 2t^3 - 21t^2 + 60t + 3 \;\; [0,8]$

a. What is the velocity function?

$V(t) = 6t^2 - 42t + 60$

b. What is the velocity at $t = 3$ seconds?

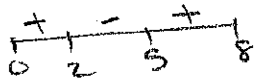
$V(3) = 6(9) - 42(3) + 60 = 240\,m/s$

c. When is the particle at rest?

$V(t) = 6t^2 - 42t + 60 = 0$

$t = 2, 5$ sec

d. When is the particle moving right? Moving left?



Right $(0,2) \cup (5,8)$

left $(2,5)$

e. What is the acceleration function?

$a(t) = 12t - 42$

f. What is the acceleration at $t = 1$ second?

$a(1) = 12(1) - 42$

$= 30\,m/s^2$

g. What is the displacement?

$S(8) - S(0) = 163 - 3 = 160\,m$

h. What is the total distance traveled?

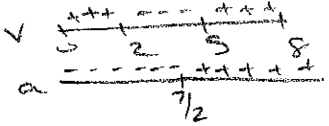
$S(2) - S(0) = 55 - 3 = 52$

$|S(5) - S(2)| = |28 - 55| = 27$

$S(8) - S(5) = 163 - 28 = 135$

214M

i. When is the particle speeding up? Slowing Down?



SU: $(2, 7/2) \cup (5,8)$

SD: $(0,2) \cup (7/2, 5)$

j. Find the velocity when the acceleration is 0.

$a(t) = 12t - 42 = 0$

$t = 7/2$

$V(7/2) = 6(7/2)^2 - 42(7/2) + 60$

$= -13.5\,m/s$

$s(t) = 2t^3 - 14t^2 + 22t - 5 \;\; [0,6]$

a. What is the velocity function?

$V(t) = 6t^2 - 28t + 22$

b. What is the velocity at $t = 3$ seconds?

$V(3) = 6(3)^2 - 28(3) + 22 = -8\,m/s$

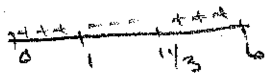
c. When is the particle at rest?

$6t^2 - 28t + 22 = 0$

$(3t - 11)(t - 1) = 0$

$t = 11/3, 1$ sec

d. When is the particle moving right? Moving left?



Right $(0,1) \cup (\frac{11}{3}, 6)$

left $(1, 11/3)$

e. What is the acceleration function?

$a(t) = 12t - 28$

f. What is the acceleration at $t = 1$ second?

$a(1) = 12(1) - 28$

$= -16\,m/s^2$

g. What is the displacement?

$S(6) - S(0) = 55 - {}^-5 = 60\,m$

h. What is the total distance traveled?

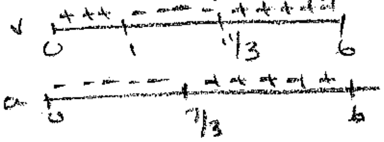
$S(1) - S(0) = 10$

$S(11/3) - S(1) = 18.96$

$S(6) - S(11/3) = 68.96$

$\boxed{97.92\,M}$

i. When is the particle speeding up? Slowing Down?



SU $(1, 7/3) \cup (11/3, 6)$

SD $(0,1) \cup (7/3, 11/3)$

j. Find the velocity when the acceleration is 0.

$V(7/3) = -32/3\,m/s$

Optimization

Solve Each Optimization Problem.

1. A company has started selling a new type of smartphone at the price of $\$110 - 0.1x$ where *x* is the number of smartphones manufactured per day. The parts for each smartphone cost \$60 and the labor and overhead for running the plant cost \$4000 per day. How many smartphones should the company manufacture and sell per day to maximize profit?

revenue
R = quant · cost
C = cost
P = profit

$R: (110 - .1x)x$

$C: 60x + 4000$

$P = R - C$

$= 110x - .1x^2 - 60x - 4000$

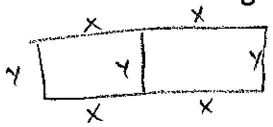
$P = -.1x^2 + 50x - 4000$

$\frac{dP}{dx} = -.2x + 50 = 0$

$-.2x = -50$

$x = \frac{-50}{-.2} = \boxed{250 \text{ Smart phones}}$

2. A rancher wants to construct two identical rectangular corrals using 200 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?



$4x + 3y = 200$

$A = 2xy$

$4x = -3y + 200$

$x = -\frac{3}{4}y + 50$

$A = 2(-\frac{3}{4}y + 50)y$

$A = -\frac{3}{2}y^2 + 100y$

$A' = -3y + 100 = 0$

$y = \frac{100}{3}$

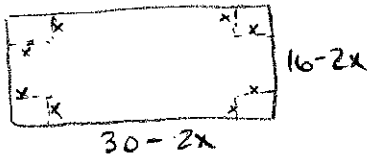
$x = \frac{-3}{4} \cdot \frac{106}{3} + 50$

$= -25 + 50$

$= 25$

$\boxed{25\text{ft} \times \frac{100}{3}\text{ft}}$

3. A supermarket employee wants to construct an open-top box from a 16 by 30 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?



$V = L \cdot w \cdot h$

$= (30-2x)(16-2x)x$

$= 480x - 92x^2 + 4x^3$

$\frac{dV}{dx} = 480 - 184x + 12x^2 = 0$

$3x^2 - 46x + 120 = 0$

$(3x - 10)(x - 12) = 0$

$\frac{10}{3}, 12$

f' + max − min +
10/3 12

$\boxed{\frac{10}{3} \text{in}}$

4. A cryptography expert is deciphering a computer code. To do this, the expert needs to minimize the product of a positive rational number and a negative rational number, given that the positive number is exactly 9 greater than the negative number. What final product is the expert looking for?

$\overset{+}{a} = x + 9$

$\overset{-}{b} = x$

$(x+9)x = x^2 + 9x$

$2x + 9 = 0$

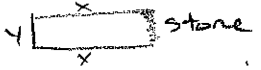
$x = -9/2$

$-9/2 + 9 = 9/2$

$a = 9/2 \quad b = -9/2$

$a \cdot b = \boxed{-81/4}$

5. A farmer wants to construct a rectangular pigpen using 400 ft of fencing. The pen will be built next to an existing stone wall, so only three sides of fencing need to be constructed to enclose the pen. What dimensions should the farmer use to construct the pen with the largest possible area?



$A = x \cdot y$

$= x(400-2x)$

$= 400x - 2y^2$

$A' = 400 - 4x = 0$

$2x + y = 400$

$y = 400 - 2x$

$\boxed{100 \times 200}$

$x = 100$

$y = 400 - 2(100)$

$= 200$

6. Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 864 ft³ of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

$SA = x^2 + 4xh$

$= x^2 + 4x\left(\frac{864}{x^2}\right)$

$= x^2 + \frac{3456}{x}$

$SA' = 2x - \frac{3456}{x^2} = 0$

$2x^3 = 3456$

$x^3 = 1728 \quad x = 12$

$V = x \cdot x \cdot h = 864$

$h = \frac{864}{x^2}$

$h = \frac{864}{144} = 6$

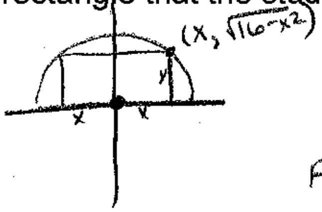
$\boxed{12 \times 12 \times 6}$

7. A geometry student wants to draw a rectangle inscribed in a semicircle of radius 4. If one side must be on the semicircle's diameter, what is the area of the largest rectangle that the student can draw?

$x^2 + y^2 = 16$

$y^2 = 16 - x^2$

$y = \sqrt{16 - x^2}$



$y = \sqrt{16 - 8}$

$y = \sqrt{8}$

$A = 2xy$

$= 2x\sqrt{16 - x^2}$

$A' = 2x \cdot \frac{1}{2}(16 - x^2)^{-1/2} \cdot (-2x)$

$+ 2\sqrt{16 - x^2}$

$= \frac{-2x^2}{\sqrt{16 - x^2}} + 2\sqrt{16 - x^2} = 0$

$2\sqrt{16 - x^2} = \frac{2x^2}{\sqrt{16 - x^2}}$

$2(16 - x^2) = 2x^2$

$16 - x^2 = x^2$

$16 = 2x^2$

$8 = x^2$

$2\sqrt{2} = x$

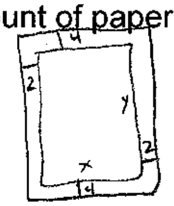
$A = 2(2\sqrt{2})(\sqrt{8})$

$= 4\sqrt{16}$

$= 4 \cdot 4$

$= \boxed{16}$

8. A graphic designer is asked to create a movie poster with a 72 in² photo surrounded by a 4 in border at the top and bottom and a 2 in border on each side. What overall dimensions for the poster should the designer choose to use the least amount of paper?



$A = (x+4)(y+8)$

$A = \left(\frac{72}{y} + 4\right)(y+8)$

$= 72 + \frac{576}{y} + 4y + 32$

$A' = -\frac{576}{y^2} + 4 = 0$

$576 = 4y^2$

$144 = y^2$

$y = 12$

$x \cdot y = 72$

$x = \frac{72}{y}$

$x = 72/12 = 6$

$x + 4 = 6 + 4$

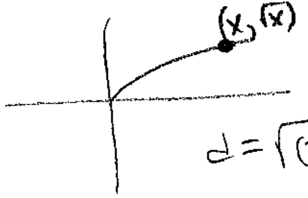
$= 10$

$y + 8 = 12 + 8$

$= 20$

$\boxed{10 \times 20}$

9. Which point on the graph of $y = \sqrt{x}$ is closest to the point (7, 0)?



$(x, \sqrt{x}) \ (7, 0)$

$d = \sqrt{(7-x)^2 + (0-\sqrt{x})^2}$

$= \sqrt{(7-x)^2 + x}$

$= \sqrt{49 - 14x + x^2 + x}$

$= \sqrt{x^2 - 13x + 49}$

$d' = \frac{1}{2}(x^2 - 13x + 49)(2x - 13) = 0$

$x^2 - 13x + 49 = 0 \qquad 2x - 13 = 0$

$13 \pm \sqrt{169 - 4(1)(49} \qquad x = 13/2$

Imaginary

$y = \sqrt{\frac{13}{2}} = \frac{\sqrt{13}}{\sqrt{2}} = \frac{\sqrt{26}}{2}$

$\boxed{\left(\frac{13}{2}, \frac{\sqrt{26}}{2}\right)}$

10. Which points on the graph of $y = 4 - x^2$ are closest to the point (0, 2)?

$(x, 4 - x^2) \quad (0, 2)$

$d = \sqrt{x^2 + (4 - x^2 - 2)^2}$

$= \sqrt{x^2 + (2 - x^2)^2}$

$= \sqrt{x^2 + 4 - 4x^2 + x^4}$

$= \sqrt{4 - 3x^2 + x^4}$

$d' = \frac{1}{2}(4 - 3x^2 + x^4)(4x^3 - 6x) = 0$

$x^4 - 3x^2 + 4 = 0 \qquad 4x^3 - 6x = 0$

$3 \pm \sqrt{9 - 4 \cdot 4} \qquad 4x^2 - 6 = 0$

Imaginary

$x^2 = \frac{6}{4}$

$x = \pm\frac{\sqrt{6}}{2}$

$y = 4 - \left(\frac{\sqrt{6}}{2}\right)^2$

$= 4 - \frac{6}{4}$

$= \frac{10}{4} = \frac{5}{2}$

$\boxed{\left(\pm\frac{\sqrt{6}}{2}, \frac{5}{2}\right)}$

Related Rates – Squares and Circles

Solve each related rate problem.

1. A crowd gathers around a movie star, forming a circle. The radius of the crowd increases at a rate of 3 ft/sec. How fast is the area taken up by the crowd increasing when the radius is 7 ft?

K: $\frac{dr}{dt} = 3$ ft/sec

W: $r = 7$

F: $\frac{dA}{dt}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(7) \cdot 3$

$\frac{dA}{dt} = 42\pi$ ft/sec^2

2. A hypothetical square grows so that the length of its sides are increasing at a rate of 8 m/min. How fast is the area of the square increasing when the sides are 7 m each?

K: $\frac{dL}{dt} = 8$

F $\frac{dA}{dt}$

W $S = 7$

$A = S^2$

$\frac{dA}{dt} = 2s \frac{ds}{dt}$

$\frac{dA}{dt} = 2(7)(8)$

$= 2(56)$

$= 112$ m/min^2

3. Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The radius of the spill increases at a rate of 2 m/min. How fast is the area of the spill increasing when the radius is 14 m?

K $\frac{dr}{dt} = 2$ m/min

F $\frac{dA}{dt}$

W $r = 14$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(14)(2)$

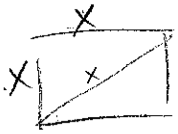
$\frac{dA}{dt} = 56\pi$ m/min^2

4. A hypothetical square grows so that the length of its diagonals are increasing at a rate of 6 m/min. How fast is the area of the square increasing when the diagonals are 15 m each?

K $\frac{dx}{dt} = 6$

F $\frac{dA}{dt}$

W $X = 15$



$A = d^2$

$2d^2 = x^2$

$d = \sqrt{\frac{x^2}{2}}$

$A = \frac{x^2}{2}$

$A = \frac{x^2}{2}$

$\frac{dA}{dt} = x \frac{dx}{dt}$

$\frac{dA}{dt} = 15(6)$

$= \boxed{90 \text{ m/min}^2}$

5. Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 8 cm/min. How fast is the area of the pool increasing when the radius is 13 cm?

K $\frac{dr}{dt} = 8$ cm/min

F $\frac{dA}{dt}$

W $r = 13$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

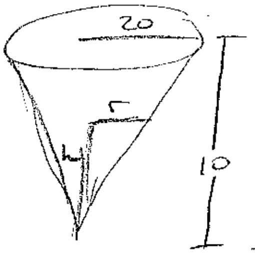
$= 2\pi(13)(8)$

$\frac{dA}{dt} = 208\pi$

Related Rates – Cubes, Spheres, and Cones

Solve each related rate problem.

1. A conical paper cup is 10 cm tall with a radius of 20 cm. The cup is being filled with water so that the water level rises at a rate of 4 cm/sec. At what rate is water being poured into the cup when the water level is 3 cm?



K: $\frac{dh}{dt} = 4$ cm/sec

F $\frac{dV}{dt}$

W: $h = 3$

$\frac{r}{20} = \frac{h}{10}$

$r = \frac{20h}{10} = 2h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (2h)^2 h$

$V = \frac{1}{3}\pi 4h^3$

$V = \frac{4}{3}\pi h^3$

$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$

$= 4\pi (9)(4)$

$\frac{dV}{dt} = 144\pi$ cm/sec^3

2. A spherical balloon is deflated at a rate of $\frac{32\pi}{3}$ cm³/sec. At what rate is the radius of the balloon changing when the radius is 9 cm?

K: $\frac{dV}{dt} = -\frac{32\pi}{3}$ cm^3/sec

F: $\frac{dr}{dt}$

W: $r = 9$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-\frac{32\pi}{3} = 324\pi \frac{dr}{dt}$

$\frac{-8}{243} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{-8}{243}$ cm/sec

3. A spherical balloon is inflated so that its radius increases at a rate of 2 cm/sec. How fast is the volume of the balloon increasing when the radius is 3 cm?

K: $\frac{dr}{dt} = 2$ cm/sec

F: $\frac{dV}{dt}$

W: $r = 3$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi \cdot 9 \cdot 2$

$\frac{dV}{dt} = 72\pi$ cm/sec^3

4. A perfect cube shaped ice cube melts so that the length of its sides are decreasing at a rate of 2 mm/sec. Assume that the block retains its cube shape as it melts. At what rate is the volume of the ice cube changing when the sides are 2 mm each?

K: $\frac{ds}{dt} = -2$ mm/se

F: $\frac{dV}{dt}$

W $s = 2$

$V = s^3$

$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

$= 3(2)^2 \cdot (-2)$

$= 12(-2)$

$= -24$ mm/sec^3

5. A hypothetical cube shrinks so that the length of its sides are decreasing at a rate of 2 m/min. At what rate is the volume of the cube changing when the sides are 4 m each?

K: $\frac{ds}{dt} = -2$ m/min

F $\frac{dV}{dt}$

W: $s = 4$

$V = s^3$

$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

$= 3(4)^2 \cdot -2$

$= 3 \cdot 16 \cdot -2$

$\frac{dV}{dt} = -96$ m^3/min