Advanced Algebra/Trig
Chapter 6 Review

Name: Answer Key

Find the following using trigonometric identities.

1. If $\sin\theta = \frac{2}{3}$, find $\cos\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{2}{3}\right)^2 + \cos^2\theta = 1$

$\frac{4}{9} + \cos^2\theta = 1 - \frac{4}{9}$

$\cos^2\theta = \frac{5}{9}$

$\cos\theta = \frac{\sqrt{5}}{3}$

$\cos\theta = \frac{\sqrt{5}}{3}$

2. If $\sin\theta = \frac{7}{10}$, find $\cot\theta$.

$\csc\theta = \frac{10}{7}$

$1 + \cot^2\theta = \csc^2\theta$

$1 + \cot^2\theta = \left(\frac{10}{7}\right)^2$

$-1 + 1 + \cot^2\theta = \frac{100}{49} - 1$

$\sqrt{\cot^2\theta} = \sqrt{\frac{51}{49}}$

$\cot\theta = \frac{\sqrt{51}}{7}$

3. If $\tan\theta = \frac{12}{5}$, find $\sin\theta$.

$\cot\theta = \frac{5}{12}$

$1 + \cot^2\theta = \csc^2\theta$

$1 + \left(\frac{5}{12}\right)^2 = \csc^2\theta$

$1 + \frac{25}{144} = \csc^2\theta$

$\sqrt{\frac{169}{144}} = \sqrt{\csc^2\theta}$

$\csc\theta = \frac{13}{12}$

$\sin\theta = \frac{12}{13}$

4. If $\cot\theta = \frac{5}{9}$, find $\tan\theta$.

$\tan\theta = \frac{9}{5}$

Verify the following identities.

5. $\tan\beta\csc\beta = \sec\beta$

$\frac{\sin\beta}{\cos\beta}\cdot\frac{1}{\sin\beta}$

$\frac{1}{\cos\beta}$

$\sec\beta$

6. $\frac{1}{\sec^2\beta} + \frac{1}{\csc^2\beta} = 1$

$\cos^2\beta + \sin^2\beta$

1

7. $\cos x(\csc x - \sec x) = \cot x - 1$

$\cos x\cdot\csc x - \cos x\cdot\sec x$

$\cos x\cdot\frac{1}{\sin x} - \cos x\cdot\frac{1}{\cos x}$

$\frac{\cos x}{\sin x} - \frac{\cos x}{\cos x}$

$\cot x - 1$

8. $\frac{\sin x\cdot\cot x + \cos x}{\sin x} = 2\cot x$

$\frac{\sin x\cdot\frac{\cos x}{\sin x} + \cos x}{\sin x}$

$\frac{2\cos x}{\sin x}$

$2\cot x$

9. $(1 - \cos x)(1 + \cos x) = \frac{1}{\csc^2 x}$

FOIL

$1 + \cos x - \cos x - \cos^2 x$

$1 - \cos^2 x$

$\sin^2 x$

$\frac{1}{\csc^2 x}$

10. $\sin x\tan x = \sec x - \cos x$

$\frac{1}{\cos x} - \frac{\cos x}{1}\left(\frac{\cos x}{\cos x}\right)$

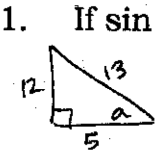
$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$

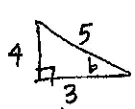
$\frac{1 - \cos^2 x}{\cos x} \to \frac{\sin^2 x}{\cos x} = \sin x\cdot\frac{\sin x}{\cos x}$

$= \sin x\tan x$

If a and b are measures of two first quadrant angles, find the exact value of each function.

11. If $\sin a = \frac{12}{13}$ and $\cos b = \frac{3}{5}$, find $\cos(a + b)$.

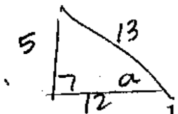


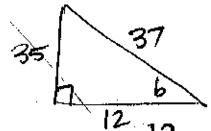


$\cos(a+b) = \cos a\cdot\cos b - \sin a\cdot\sin b$

$= \left(\frac{5}{13}\right)\left(\frac{3}{5}\right) - \left(\frac{12}{13}\right)\left(\frac{4}{5}\right)$

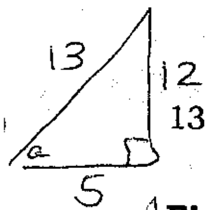
$= \frac{15}{65} - \frac{48}{65} = -\frac{33}{65}$





12. If $\cos a = \frac{12}{13}$ and $\cos b = \frac{12}{37}$, find tan (a – b).

$\tan(a-b) = \frac{\tan a - \tan b}{1+\tan a \cdot \tan b} = \frac{\frac{5}{12} - \frac{35}{12}}{1+\frac{5}{12}\cdot\frac{35}{12}} = \frac{-\frac{30}{12}}{1+\frac{175}{144}} = \frac{-\frac{30}{12}}{\frac{319}{144}} = -\frac{30}{12}\cdot\frac{144}{319} = -\frac{360}{319}$



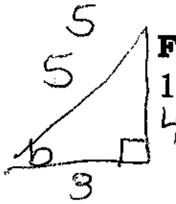
13. If $\csc a = \frac{13}{12}$ and $\sec b = \frac{5}{3}$, find sin (a – b).

$\sin a = \frac{12}{13}$ $\cos b = \frac{3}{5}$

$= \sin a \cdot \cos b - \cos a \sin b$

$\left(\frac{12}{13}\right)\left(\frac{3}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$

$= \frac{36}{65} - \frac{20}{65} = \frac{16}{65}$



Find the exact value.

14. tan 75° $= \tan\frac{150°}{2} = \sqrt{}$

$= \frac{1-\cos 150°}{\sin 150°} = \frac{1-\frac{-\sqrt{3}}{2}}{\frac{1}{2}}$

$= \frac{\left(1+\frac{\sqrt{3}}{2}\right)^2}{\left(\frac{1}{2}\right)^2} = 2+\sqrt{3}$

15. sin (-15°)

$= \sin(30° - 45°)$

$\sin 30° \cos 45° - \cos 30° \sin 45°$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

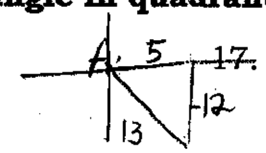
$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$

If $\sin A = -\frac{12}{13}$ and ∠A is an angle in quadrant IV, find the exact value.

16. cos 2A $= 1 - 2\sin^2 A$

$= 1 - 2\left(-\frac{12}{13}\right)^2$

$= \frac{-119}{169}$



~~17.~~ tan 2A $\frac{2\tan A}{1-\tan^2 A} = \frac{2\left(-\frac{12}{5}\right)}{1-\left(-\frac{12}{5}\right)^2} = \frac{-\frac{24}{5}}{\frac{-119}{25}}$

$= -\frac{24}{5}\cdot\frac{25}{-119} = +\frac{120}{119}$

(½ of A is in quad II)

18. $\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}} = \pm\sqrt{\frac{1-5/13}{2}}$

$= \sqrt{\frac{4}{13}} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$

19. $\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}} = \pm\sqrt{\frac{1+5/13}{2}}$

$= \pm\sqrt{\frac{9}{13}} = -\frac{3\sqrt{13}}{13}$

Solve for values of θ such that 0 ≤ θ < 2π.

20. $2\sin^2\theta - 1 = 0$

$\frac{2\sin^2\theta}{2} = \frac{1}{2}$

$\sqrt{\sin^2\theta} = \frac{1}{\sqrt{2}}$

$\sin\theta = \pm\frac{\sqrt{2}}{2}$

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

21. $(2\sin\theta - 1)(2\cos\theta + \sqrt{3}) = 0$

$2\sin\theta - 1 = 0$ $2\cos\theta + \sqrt{3} = 0$

$\frac{2\sin\theta}{2} = \frac{1}{2}$ $\cos\theta = -\frac{\sqrt{3}}{2}$

$\sin\theta = \frac{1}{2}$

$\frac{\pi}{6}, \frac{5\pi}{6}, \pi$

22. $2\sin^2 x - 5\sin x = -2$

$2\sin^2 x - 5\sin x + 2 = 0$

$(2\sin x - 1)(\sin x - 2) = 0$

$2\sin x - 1 = 0$ $\sin x = 2$

$\sin x = \frac{1}{2}$

$\frac{\pi}{6}, \frac{5\pi}{6}$

23. $\tan^2 x - \sqrt{3}\tan x = 0$

$\tan x(\tan x - \sqrt{3}) = 0$

$\tan x = 0$ $\tan x = \sqrt{3}$

$0, \pi, \frac{\pi}{3}, \frac{4\pi}{3}$

Write the expression as the sine, cosine, or tangent of an angle.

24. cos 25° cos 15° – sin 25° sin 15°

$\cos(u \pm v) = \cos u \cdot \cos v \mp \sin u \cdot \sin v$

$\cos(25° + 15°) = \cos 40°$

25. $\frac{\tan 68° - \tan 115°}{1 + \tan 68° \tan 115°}$

$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \cdot \tan v}$

$\tan(68° - 115°) = \tan(-47°)$