

## Double Angle Identities

p. 60

\*\*\*Write these in graphic organizer on p.51

$$\sin 2u = 2 \sin u \cos u \quad \tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\text{or } 2\cos^2 u - 1$$

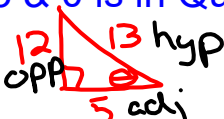
$$\text{or } 1 - 2\sin^2 u$$

Write on this page (p.60)

\*Use identities on p. 51!

If  $\cos \theta = 5/13$  &  $\theta$  is in Quadrant I, find each:

$\uparrow$   
 Same as  $u$   
 $\downarrow$



opp 12 hyp 13  
 5 adj  
 $5^2 + b^2 = 13^2$   
 $25 + b^2 = 169$   
 $b^2 = 144$   
 $b = 12$

1.  $\sin 2\theta = 2 \sin u \cos u$

$$\frac{2}{1} \left( \frac{12}{13} \right) \left( \frac{5}{13} \right) = \frac{120}{169}$$

2.  $\cos 2\theta = \cos^2 u - \sin^2 u$

$$\left( \frac{5}{13} \right)^2 - \left( \frac{12}{13} \right)^2$$

$$\frac{25}{169} - \frac{144}{169} = \frac{-119}{169}$$

3.  $\tan 2\theta = \frac{2 \tan u}{1 - \tan^2 u}$

$$\frac{\frac{2}{1} \left( \frac{12}{5} \right)}{1 - \left( \frac{12}{5} \right)^2} = \frac{\frac{24}{5}}{\frac{25}{25} - \frac{144}{25}} = \frac{\frac{24}{5}}{\frac{-119}{25}}$$

$$\frac{24}{5} \div \frac{-119}{25} = \frac{24}{5} \cdot \frac{-25}{119} = \frac{-120}{119}$$

## Half Angle Identities

p. 61

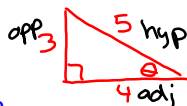
$$\bullet \sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\bullet \cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\bullet \tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} \text{ or } \frac{\sin u}{1 + \cos u}$$

Write on this page (p.61)

\* Use identities on p. 61

If  $\tan \theta = 3/4$  &  $\theta$  is in Quadrant I, find the exact value of each:

only need positive ans.

1.  $\sin \theta/2$ 

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\sqrt{\frac{1 - 4/5}{2}} = \sqrt{\frac{5/5 - 4/5}{2}} = \sqrt{\frac{1/5}{2}} = \sqrt{\frac{1}{5} \cdot \frac{1}{2}}$$

$$= \sqrt{\frac{1}{10}} = \frac{\sqrt{1}}{\sqrt{10}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

2.  $\cos \theta/2$ 

$$\sqrt{\frac{1 + \cos \theta}{2}} = \sqrt{\frac{1 + 4/5}{2}} = \sqrt{\frac{5/5 + 4/5}{2}} = \sqrt{\frac{9/5}{2}}$$

$$\sqrt{\frac{9}{5} \cdot \frac{1}{2}} = \sqrt{\frac{9}{10}} = \frac{\sqrt{9}}{\sqrt{10}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

3.  $\tan \theta/2$ 

$$\frac{1 - \cos \theta}{\sin \theta} = \frac{1 - 4/5}{3/5} = \frac{5/5 - 4/5}{3/5} = \frac{1/5}{3/5} = \frac{1}{5} \cdot \frac{5}{3} = \frac{1}{3}$$

Find the exact value of  $\sin 105^\circ$  using the half-angle identity\* Look on the unit circle for an angle that  $1/2$  of it is  $105^\circ$ 

$$\sin \left( \frac{210^\circ}{2} \right) = \pm \sqrt{\frac{1 - \cos u}{2}} \quad \left( -\frac{\sqrt{3}}{2}, -\frac{1}{2} \right)$$

↑ use - because sin is - in Q3 ( $210^\circ$ )

$$= -\sqrt{\frac{1 - (-\frac{\sqrt{3}}{2})}{2}} = -\sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2 + \sqrt{3}}{2}}$$

$$= -\sqrt{\frac{2 + \sqrt{3}}{2} \cdot \frac{1}{2}} = -\sqrt{\frac{2 + \sqrt{3}}{4}}$$

$$= -\frac{\sqrt{2 + \sqrt{3}}}{\sqrt{4}} = -\frac{\sqrt{2 + \sqrt{3}}}{2}$$