

Sum & Difference Identities

p. 58

Write these on graphic organizer on p. 51:

- $\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$    
 *same*   
  $\pm$  means same sign as prob.
- $\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$    
 *opp sign*   
  $\mp$  means opposite sign as prob.
- $\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$    
 *same sign*   
  $\mp$  means opposite sign as prob.

on this page:

Use identities on p. 51! Find 2 angles on the unit circle that add or subtract to equal your value. Then plug into appropriate identity.

1. Find the exact value of  $\cos 75^\circ$

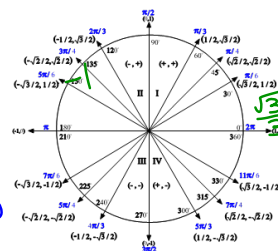
$$\begin{aligned} &\cos(30^\circ + 45^\circ) \\ &\cos(u+v) = \cos u \cos v - \sin u \sin v \\ &\cos 30^\circ \cdot \cos 45^\circ - \sin 30^\circ \sin 45^\circ \\ &\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \\ &\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \\ &\frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

Find the exact value of  $\sin(-135^\circ)$    
 *You can find a coterminal  $\angle$ :  $-135 \rightarrow 180 - 315$   
 $1360 \rightarrow 135 - 270$   
 $225^\circ \rightarrow 45 - 180$*

$$\begin{aligned} &\sin(45^\circ - 180^\circ) \\ &\sin u \cos v - \cos u \sin v \\ &\sin 45^\circ \cos 180^\circ - \cos 45^\circ \sin 180^\circ \\ &\frac{\sqrt{2}}{2}(-1) - \frac{\sqrt{2}}{2}(0) \\ &-\frac{\sqrt{2}}{2} - 0 = -\frac{\sqrt{2}}{2} \end{aligned}$$

3. Find the exact value of  $\tan 165^\circ$

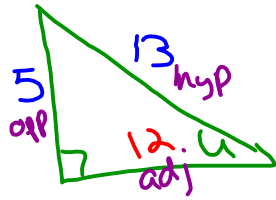
$$\begin{aligned} &\tan(135^\circ + 30^\circ) \\ &\frac{\tan u + \tan v}{1 - \tan u \tan v} \\ &\frac{\tan 135^\circ + \tan 30^\circ}{1 - \tan 135^\circ \tan 30^\circ} \\ &\frac{\left(\frac{\sqrt{3}}{3} + -1\right)}{\left(1 - \frac{\sqrt{3}}{3}(-1)\right)} = \frac{\frac{\sqrt{3}}{3} - \frac{3}{3}}{\frac{3}{3} + \frac{\sqrt{3}}{3}} = \frac{\frac{\sqrt{3}-3}{3}}{\frac{3+\sqrt{3}}{3}} \\ &\frac{\sqrt{3}-3}{3} \cdot \frac{3}{3+\sqrt{3}} = \frac{(\sqrt{3}-3)(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})} \\ &\frac{3\sqrt{3}-3-9+3\sqrt{3}}{9-3\sqrt{3}+3\sqrt{3}-3} = \frac{-12+6\sqrt{3}}{6} \\ &-2 + \sqrt{3} \end{aligned}$$



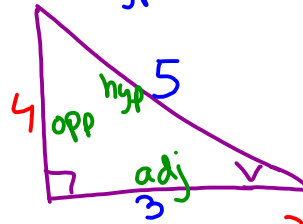
4. Find the exact value of  $\tan 285^\circ$

If 2 angles,  $u$  &  $v$ , are measures in Quadrant I, find <sup>p.59</sup> the exact value of each function:

1. If  $\sin u = \frac{5}{13}$  <sup>opp</sup>/<sub>hyp</sub> &  $\cos v = \frac{3}{5}$  <sup>adj</sup>/<sub>hyp</sub> find  $\sin(u+v)$



$$\begin{aligned} 5^2 + b^2 &= 13^2 \\ 25 + b^2 &= 169 \\ b^2 &= 144 \\ b &= 12 \end{aligned}$$



$$\begin{aligned} 3^2 + b^2 &= 5^2 \\ 9 + b^2 &= 25 \\ b^2 &= 16 \\ b &= 4 \end{aligned}$$

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$= \frac{5}{13} \cdot \frac{3}{5} + \frac{12}{13} \cdot \frac{4}{5}$$

$$= \frac{15}{65} + \frac{48}{65}$$

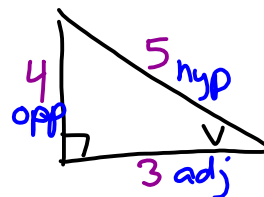
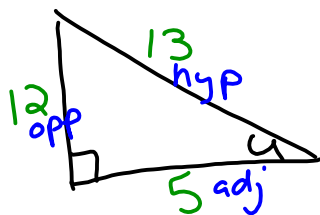
$$= \frac{63}{65}$$

2. If  $\csc u = \frac{13}{12}$  &  $\sec v = \frac{5}{3}$ , find  $\cos(u-v)$

$$\downarrow$$

$$\sin u = \frac{12}{13}$$

$$\cos v = \frac{3}{5}$$



$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$= \frac{5}{13} \cdot \frac{3}{5} + \frac{12}{13} \cdot \frac{4}{5}$$

$$= \frac{15}{65} + \frac{48}{65}$$

$$= \frac{63}{65}$$