Algebraic Vectors p. 70

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Component Form of a Vector Given Initial & Terminal Points

$P_1\ (x_1, y_1)$ initial point

$P_2\ (x_2, y_2)$ terminal point

$\langle x_2 - x_1,\ y_2 - y_1 \rangle$

* can be written as ordered pairs or sum of unit vectors

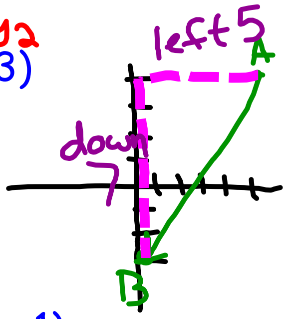
Write the component form of $\overrightarrow{AB}$

1. A (5, 4) & B (0, -3)

x_1, y_1 $x_2\ y_2$

$\langle 0-5, -3-4 \rangle$

$\langle -5, -7 \rangle$



2. A (7, -3) & B (-2, -1)

$\langle -2-7, -1--3 \rangle$

$\langle -9, 2 \rangle$

Magnitude of a Vector Given Component Form

$P_1\ (x_1, y_1)$ initial point

$P_2\ (x_2, y_2)$ terminal point

distance formula:

$\overrightarrow{P_1P_2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

or pythagorean theorem if you know components

$\sqrt{c^2} = \sqrt{a^2 + b^2}$

$|v| = \sqrt{x^2 + y^2}$

Write the magnitude of $\overrightarrow{AB}$

1. A (5, 4) & B (0, -3)

$|v| = \sqrt{(0-5)^2 + (-3-4)^2} =$

$\sqrt{(-5)^2 + (-7)^2} = \sqrt{25 + 49}$

$\sqrt{74}$ or 8.6

2. A (7, -3) & B (-2, -1)

components: $\langle -9, 2 \rangle$

$|v| = \sqrt{(-9)^2 + (2)^2}$

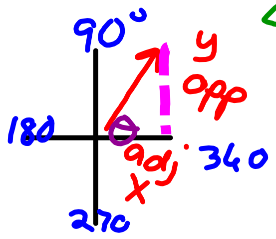
$|v| = \sqrt{81 + 4}$

$|v| = \sqrt{85}$ or 9.2

Find Amplitude (Direction) of a Vector Given Components

Sketch the vector (no protractor) on an xy axis

Create a right triangle with the reference angle



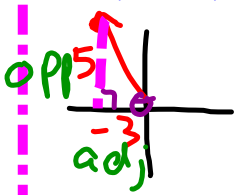
⟨x, y⟩

$\tan\theta = \frac{opp}{adj}$ or $\frac{y}{x}$

★add θ to 180° or 360° if it's not in QI

Find the direction angle of each:

1. (-3, 5)



$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{5}{-3}$

$\theta = \tan^{-1}\left(\frac{5}{-3}\right)$

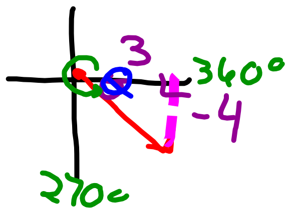
$\theta = -59°$

+180° bc Q2

$\theta = 121°$

2. 3i - 4j

⟨3, -4⟩



$\tan\theta = \frac{-4}{3}$

$\theta = \tan^{-1}(4/3)$

$\theta = -53.1°$

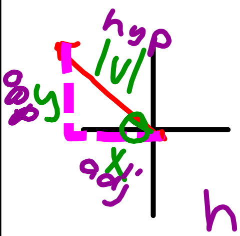
+360

$\theta = 306.9°$

Find Component Form Given Magnitude & Amplitude (Direction)

Sketch the vector (no protractor) on an xy axis

Create a right triangle with the reference angle



|v| = magnitude

θ = amplitude

horiz: $\cos\theta = \frac{x}{|v|}$

$x = |v|\cos\theta$

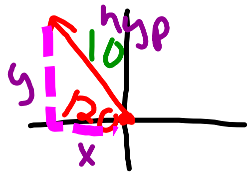
vertical: $\sin\theta = \frac{y}{|v|}$

$y = |v|\sin\theta$

$\langle |v|\cos\theta, |v|\sin\theta \rangle$

Find the component form given the magnitude & direction.

1. |v|=10 θ=120°



$\cos\theta = \frac{x}{10}$

$x = 10\cos 120°$

$x = -5$

$\sin\theta = \frac{y}{hyp}$

$y = 10\sin 120$

$y = 8.7$

⟨-5, 8.7⟩

2. |v|=6 θ=250°

$\langle 6\cos 250°, 6\sin 250° \rangle$

⟨-2.1, -5.6⟩

Add, Subtract, & Scalar Multiplication with Vectors

If $\vec{a} = (x_1, y_1)$ and $\vec{b} = (x_2, y_2)$

Add: $\vec{a} + \vec{b} =$

$\langle x_1 + x_2, y_1 + y_2 \rangle$

Subtract: $\vec{a} - \vec{b} =$

$\langle x_1 - x_2, y_1 - y_2 \rangle$

Scalar Multiplication: $k\vec{a} =$

$\langle k \cdot x_1, k \cdot y_1 \rangle$

Given $\vec{q} = (3, 9)$ and $\vec{r} = (-1, 6)$, find:

1. $\vec{q} + \vec{r} =$

$\langle 3 + -1, 9 + 6 \rangle$

$\langle 2, 15 \rangle$

2. $\vec{q} - \vec{r} =$

$\langle 3 - -1, 9 - 6 \rangle$

$\langle 4, 3 \rangle$

3. $5\vec{q} =$ $5\langle 3, 9 \rangle = \langle 5 \cdot 3, 5 \cdot 9 \rangle$

$\langle 15, 45 \rangle$

4. $3\vec{r} - 2\vec{q} =$

$3\langle -1, 6 \rangle + 2\langle 3, 9 \rangle$

$\langle -3, 18 \rangle + \langle -6, -18 \rangle$

$\langle -9, 0 \rangle$

Unit Vectors

Any vector can be represented by an ordered pair or a sum of vectors.

$If\ \vec{a} = (a_1, a_2)$

$a_1 i + a_2 j$

horiz. vert.

Write as the sum of unit vectors:

1. (2, -5) $2i - 5j$

2. (-1, 2) $-1i + 2j$ or $-i + 2j$

3. (0, -7) $0i - 7j$ or $-7j$

Orthogonal Vectors (Dot Product)

If 2 vectors have a dot product = 0, then they are orthogonal (meaning perpendicular) $\perp$

$\langle x_1, y_1 \rangle + \langle x_2, y_2 \rangle$

dot product:

$x_1 \cdot x_2 + y_1 \cdot y_2$

Are the vectors orthogonal? Show why.

1. (2, -5) and (-2, 5)

$2(-2) + (-5)(5)$

$-4 + -25 = -29$ not $\perp$

2. (4, -2) and (-5, -10)

$(4)(-5) + (-2)(-10)$

$-20 + 20 = 0$

yes $\perp$ (orthogonal)