

**Double & Half Identities:**

Double:

- $\sin 2\theta = 2 \cdot \sin \theta \cdot \cos \theta$
- $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$   
 $= 2\cos^2 \theta - 1$   
 $= 1 - 2\sin^2 \theta$
- $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

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Half:

- $\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$
- $\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$
- $\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta}$   
 $= \frac{\sin \theta}{1 + \cos \theta}$

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Ex:

①  $\sin 22.5^\circ = \sin \frac{45^\circ}{2}$

$$= \sqrt{\frac{1 - \cos 45^\circ}{2}}$$

$$= \sqrt{\frac{\frac{1}{2} - \frac{\sqrt{2}}{2}}{2}}$$

$$= \sqrt{\frac{\frac{1 - \sqrt{2}}{2}}{2}}$$

$$= \frac{\sqrt{1 - \sqrt{2}}}{2} = \sqrt{\frac{(1 - \sqrt{2}) \cdot \frac{1}{2}}{2}} = \sqrt{\frac{1 - \sqrt{2}}{4}}$$

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②  $\tan 15^\circ = \tan (45^\circ - 30^\circ)$

or

$$= \tan \frac{30^\circ}{2}$$

$$= \frac{\sin 30^\circ}{1 + \cos 30^\circ}$$

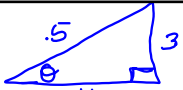
$$= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{\sqrt{3}}{2}} = \frac{\frac{1}{2}}{\frac{1 + \sqrt{3}}{2}}$$

$$= \frac{1}{2} \cdot \frac{2}{1 + \sqrt{3}} = \frac{1}{(1 + \sqrt{3})}$$

$$= \frac{1 - \sqrt{3}}{(1 + \sqrt{3})(1 - \sqrt{3})} = \frac{1 - \sqrt{3}}{1 - 3} = \frac{1 - \sqrt{3}}{-2} = \frac{\sqrt{3} - 1}{2}$$

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③ given  $\sin \theta = \frac{3}{5}$



find  $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

$$\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

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④  $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

$$= \frac{2 \left(\frac{3}{4}\right)}{1 - \left(\frac{3}{4}\right)^2}$$

$$\frac{\frac{16}{16} - \frac{9}{16}}{\frac{7}{16}} = \frac{\frac{7}{16}}{\frac{7}{16}} = \frac{7}{16} \cdot \frac{16}{7} = 1$$

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