Application of Vectors and Vector Addition

EQ: How can I use vectors to model a physical scenario?

For those of you who have or are taking Physics, we will limit the scope of our application to the same unit. For instance, the applications will not discuss forces and then ask you to talk about the velocity at a certain time. If the vectors discuss forces, the question will ask about forces.
bearing = clockwise from North

angle for unit circle = 450° - bearing
**when bearing is not in Quadrant 1**

90° - bearing in Quad. I

Component Problem - only one vector is represented, and we are asking about the result of one of the components of that vector.

Example: A ball is thrown into the air at a speed of 42 mph with a trajectory of 40° to the horizon. What is the ground speed of the ball?

\[
\begin{align*}
\langle 42 \cos 40°, 42 \sin 40° \rangle \\
ground\ speed &= \text{horizontal component} \\
&= 32.17 \text{ mph}
\end{align*}
\]
Vector Addition Problem - Two or more vectors are represented that present different events or forces happening to the same object. We must add the vectors and then use the resultant vector to answer the question.

Example: A plane is flying across the U.S. at 550 mph with a bearing of 290°. If there is a tailwind blowing due west at 25 mph, what is the bearing and speed of the resultant vector?

\[ \Theta = 450° - 290° = 160° \]
\[ \overrightarrow{V}_p = \langle 550 \cos 160°, 550 \sin 160° \rangle \]
\[ \overrightarrow{V}_c = \langle 25 \cos 180°, 25 \sin 180° \rangle \]
\[ \overrightarrow{V}_{pw} = \langle 550 \cos 160° + 25 \cos 180°, 550 \sin 160° + 25 \sin 180° \rangle \]
\[ ||\overrightarrow{V}_{pw}|| = \sqrt{(541.83)^2 + (188.11)^2} = 573.55 \text{ mph} \]
\[ \Theta = \tan^{-1}\left(\frac{188.11}{541.83}\right) = -9.15° + 180° = 160.85° \text{ bearing} \]

Suppose that you swim across a stream that has a 5-km/hr current.

a. Find your actual velocity vector if you swim perpendicular to the current at 3 km/h.

\[ \overrightarrow{V}_s = \langle 3, 0 \rangle \quad \overrightarrow{V}_c = \langle 0, -5 \rangle \]
\[ \overrightarrow{V}_s + \overrightarrow{V}_c = \langle 3, -5 \rangle \]
\[ \Theta = \tan^{-1}\left(\frac{-5}{3}\right) = 59.04° \]

b. Find your speed through the water if you swim perpendicular to the current but your resultant velocity makes an angle of 34° with the direction you are heading.

\[ \overrightarrow{V}_s = \langle x, 0 \rangle \quad \overrightarrow{V}_c = \langle 0, -5 \rangle \]
\[ \Theta = \tan^{-1}\left(\frac{-5}{x}\right) \]
\[ 34° = \tan^{-1}\left(\frac{-5}{x}\right) \]
\[ \tan 34° = \frac{5}{x} \]
\[ x = \frac{5}{\tan 34°} = 7.41 \text{ km/hr} \]
A ship near the coast is going 9 knots at an angle of $130^\circ$ to a current of 4 knots. What is the ship's resultant velocity with respect to the shore?

\[
\mathbf{v}_s = \langle 9 \cos 130^\circ, 9 \sin 130^\circ \rangle \\
\mathbf{v}_c = \langle 4, 0 \rangle \\
\mathbf{v}_{cs} = \mathbf{v}_s + \mathbf{v}_c = \langle -5.79, 6.89 \rangle \\
\|
\mathbf{v}_{cs}\| = \sqrt{(-5.79)^2 + (6.89)^2} \\
= 7.12 \text{ knots}
\]

Victoria walks 90 m due south (bearing $180^\circ$), then turns and walks 40 m more along a bearing of $250^\circ$.

a. Find Victoria's resultant displacement vector from the starting point.

\[
\mathbf{w}_1 = \langle 0, -90 \rangle \\
\mathbf{w}_2 = \langle 40 \cos 200^\circ, 40 \sin 200^\circ \rangle \\
\mathbf{w}_{1+2} = \langle 37.59, -103.68 \rangle
\]

b. What is the starting point's bearing from the place where Victoria stops?

\[
\begin{align*}
\Theta & = \tan^{-1}\left(\frac{103.68}{37.59}\right) \\
& = 70.07^\circ \\
\text{bearing} & = 90 - 70.07^\circ \\
& = 19.93^\circ
\end{align*}
\]
Abe and Bill cooperate to pull a tree stump out of the ground. They think it will take a force of 350 lb to do the job. They tie ropes around the stump. Abe pulls his rope with a force of 200 lb, and Bill pulls his rope with a force of 150 lb. The force vectors make an angle of 40°.

a. Find the magnitude of the resultant force vector and the angle the resultant vector makes with Abe’s vector.

\[
\begin{align*}
\mathbf{V}_A &= \langle 200, 0 \rangle \\
\mathbf{V}_B &= \langle 150 \cos 40°, 150 \sin 40° \rangle \\
\mathbf{V}_{AB} &= \langle 314.91, -96.42 \rangle \\
|\mathbf{V}_{AB}| &= \sqrt{314.91^2 + (-96.42)^2} = 329.34 \text{ lb} \\
\theta &= \tan^{-1}\left(\frac{-96.42}{314.91}\right) = -17.02°
\end{align*}
\]

b. What false assumption about vectors did Abe and Bill make?

It didn’t matter the direction of the force.