

# Toward chaotic electron transport in bismuth nanocluster wires

M. S. Fairbanks<sup>\*,a</sup>, B. C. Scannell<sup>a</sup>, T. P. Martin<sup>a,b</sup>, R. D. Montgomery<sup>a</sup>, S. A. Brown<sup>c</sup>, R. P. Taylor<sup>a,c</sup>

<sup>a</sup>Department of Physics, University of Oregon, Eugene, OR 97403-1274, USA

<sup>b</sup>School of Physics, University of New South Wales, Sydney NSW 2052, Australia

<sup>c</sup>Department of Physics and Astronomy, University of Canterbury, Christchurch, Private Bag 4800 New Zealand

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## Abstract

We present a generic model for electron transport through nanoscale 2-D devices based on chaotic scattering statistics. Here we focus on wires assembled from bismuth nanoclusters and propose that our scattering model extends to this novel class of devices. The wires are thin enough to confine electrons to a 2-D plane, with widths as narrow as the cluster diameter of  $\approx 50$  nm. We discuss the consequences of our model for both classical and quantum conduction, which includes the prediction of room temperature quantum interference effects for 50 nm diameter cavities.

*Key words:* bismuth, electron transport, scattering

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## Introduction

Future generations of nanoscale electronic devices will require both new approaches to fabrication and new models of transport physics at the nanometer scale. “Self-assembly” fabrication techniques represent a highly practical and versatile approach to constructing electronic devices and circuits from the ‘bottom-up’ in contrast to traditional ‘top-down’ lithography [1]. In this paper, we focus on the self-assembly of nanowires via the deposition of nanoscale metallic or semiconducting clusters formed by inert gas aggregation [2]. In particular, bismuth nanoclusters are used to fabricate wires with heights small enough to confine electrons to a 2-D plane and widths that are as narrow as the cluster diameter. These wires have significant variation of the wire width along the wires length. This variation is a generic feature of this self-assembled system and stands in sharp contrast to the smooth, straight walls of nanowires defined by traditional lithography techniques such as etching and electrostatic gate confinement.

We present a generic electron transport model for self-assembled wires of this type, in which this intrinsic variation in width produces an array of connected quantum cavities. Based on previous experiments performed on micron-scale 2-D cavities and wires [3], scattering sites in the material are expected to act as Sinai diffusers that induce chaotic dynamics in the electron trajectories. The role of the cavity walls is simply to reflect the trajectories repeatedly through these scatterers. We will discuss the consequences of this chaotic model for classical and quantum transport through self-assembled wires. In particular, we expect the chaotic trajectories to generate fractal behavior in the electron wave interference. This manifests itself as a power-law spectral density in the

magnetoconductance fluctuations. Whereas observation of these fractal fluctuations has been limited to temperatures below 100 K for micron-sized systems, we will show that the material parameters of small (30 – 50 nm) bismuth cavities lead to the prediction that fluctuations will be observed at significantly higher temperatures for the self-assembled bismuth nanocluster wires.

Two examples of self-assembled bismuth nanowires are shown in the SEM images of Figure 1. Details of the fabrication process can be found elsewhere [2]. Briefly, bismuth is evaporated from a crucible in high vacuum, and flow of inert gas (typically Ar) causes the bismuth vapor to aggregate into clusters. A series of nozzles and pumping stages removes the Ar gas and forms a beam of clusters that can be deposited on substrates of  $\text{SiO}_2$  or  $\text{Si}_3\text{N}_4$ . The cluster size can be tuned by changing the gas flow rate. The clusters shown in Figure 1 are roughly 30 nm in diameter. The clusters flatten as they impinge on the substrate (velocity =  $45 \pm 5$  m/s), which results in a thickness ( $\approx 5$  nm) of the clusters that is smaller than the Fermi wavelength ( $\approx 10$  nm), based on measurements of the carrier density in a bismuth nanocavity [4]. Thus, carrier transport is expected to be confined to a 2-D plane. The clusters can be further confined to wires with widths as narrow as the cluster diameter. For the wires in Figure 1, this confinement is achieved by depositing a layer of PMMA resist on the substrate and using electron beam lithography to create a window such that deposited clusters adhere to the exposed substrate and self-assemble to form a wire. Note that the clusters stick only to the exposed substrate ( $\text{SiO}_2$  or  $\text{Si}_3\text{N}_4$ ), not the surrounding PMMA film [2].

The parameters that characterize electronic transport of bismuth, particularly in low-dimensional devices, are still the subject of current research. Low dimensional bismuth systems

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\*Corresponding author

Email address: mfairban@uoregon.edu (M. S. Fairbanks)

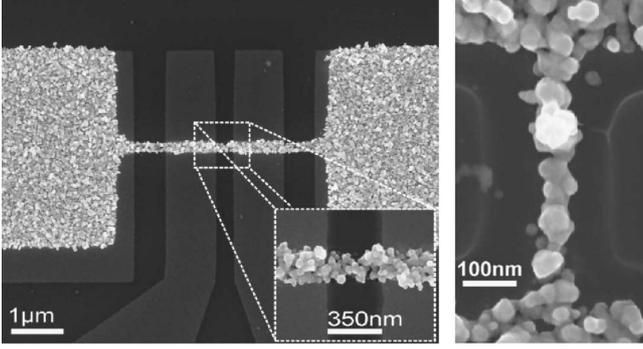


Figure 1: Examples of bismuth nanocluster wires with approximate widths of 350 nm (a) and 100 nm (b).

have shown a variety of behaviors, including a semimetal-semiconductor transition on the 50 nm size scale (e.g. in thin films [5] and nanowires [6]). The electron effective mass of bismuth ( $m^* = 0.002 - 0.1m_e$ ) is highly anisotropic, but can in principle be engineered to be very low. The associated increase in the spacing between quantum energy levels therefore makes this material highly attractive for studies of high temperature quantum effects.

To model classical and quantum conduction through self-assembled bismuth nanowires, we picture transport as a continuous flow of electrons through a series of connected cavities defined by the variation in wire width shown in Figure 2. These cavities feature material-induced scatterers, each of which provide an electrostatic scattering profile to the passing electrons that is approximately circular. Circular scatterers have been the subject of much research because they can act as Sinai diffusers [7], inducing chaotic dynamics in the electron trajectories.

The principle of Sinai diffusers is shown in Figure 3(a). Whereas an empty cavity generates stable electron dynamics, insertion of the circular scatterer leads to an exponential sensitivity to initial conditions. For example, electrons entering the cavity with slightly different launch conditions (shown by the green and blue trajectories) evolve along distinct trajectories. In Figure 3(a), the role of the cavity walls is to force the electron trajectories to undergo multiple scattering events off the circular scatterer. This amplifies the diverging effect of the scatterers curved walls, generating chaotic dynamics. The array of circular scatterers shown in Figure 3(b) generates precisely the same electron dynamics. In this second case, divergence occurs via multiple Sinai scatterers rather than repeated reflections from one scatterer. Finally, Figure 3(c) shows an array of scatterers where significant spatial disorder has been introduced, more closely approximating the situation of defects in a device.

A Gaussian random distribution of scatterers results trivially in diffusive transport. A sample path for a single particle is shown in Figure 4(a). However, the introduction of some correlation in the scatterers' positions can result in superdiffusive chaotic transport (i.e. 'enhanced' diffusion), nearly identical to that in the ordered systems. Specifically, if for instance in Fig-

ure 3(c) we consider the case where each scatterer is displaced from its ordered position (shown by the dotted circles) under the constraint  $\Delta_0 < (L - 2r)/2$ , the disordered Sinai array is known to exhibit superdiffusive behavior [8].

This behavior results from an increased prevalence of long trajectories without scattering. Compared to the Gaussian distribution of trajectory lengths in diffusive transport, this increased prevalence results in a distribution with a 'long tail'. In the instance of a power law fall-off in the distribution, the superdiffusive behavior is referred to as a Lévy flight [9]. A single particle path following Lévy statistics is shown in Figure 4(b) over the same time interval as the diffusive path in (a). Interestingly, the power law distribution of trajectory lengths for a Lévy flight results in a fractal set [10], which is crucial in linking the classical dynamics in this system to the observed quantum behavior.

The statistics of classical trajectories followed by carriers will impact classical conduction properties such as electron mobility, resistivity and mean free path. Quantum conduction properties will instead be sensitive to the way in which electron waves flowing along the classical trajectories undergo interference. One way to interpret quantum electron interference in 2-D systems is via the Aharonov-Bohm effect, in which a time-varying magnetic field modulates the phase difference between a pair of waves interfering in a scattering loop [11]. The oscillation period ( $\Delta B$ ) is inversely proportional to the area ( $A$ ) of the loop,  $\Delta B \sim h/(eA)$ . The distribution of loop areas will there-

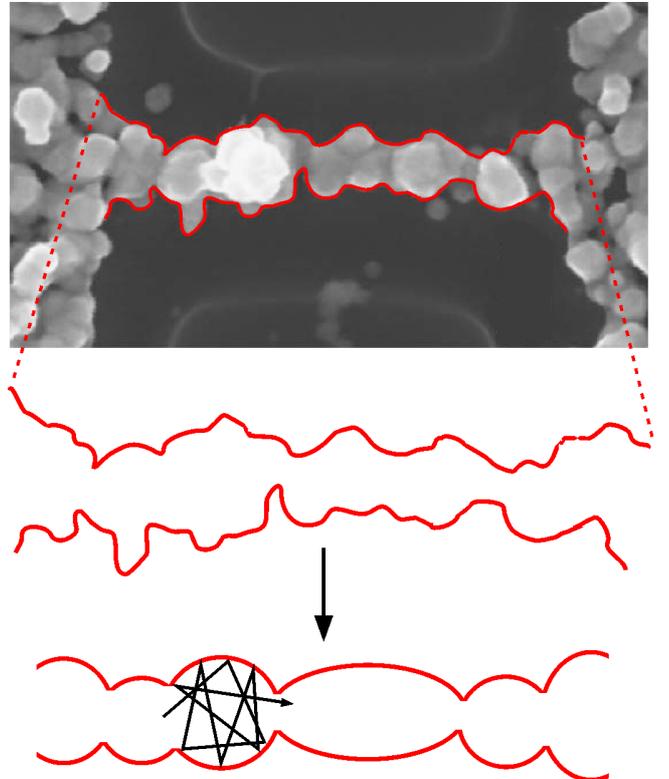


Figure 2: Transport through a self-assembled nanowire is modeled as a series of irregular, connected cavities defined by the widening and narrowing of the wire width.

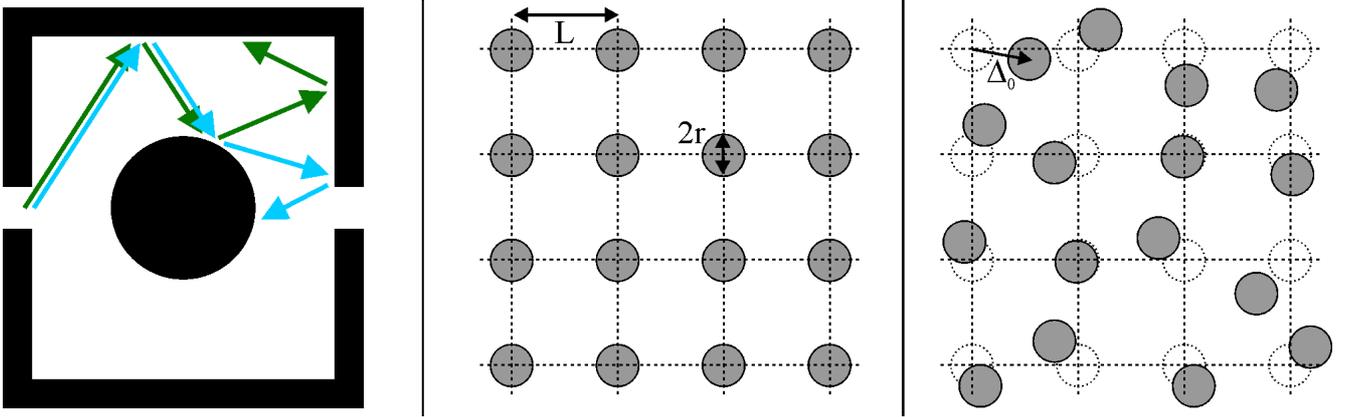


Figure 3: Three different Sinai billiard configurations: a single Sinai diffuser in an open billiard (a), an ordered Sinai scattering array with spacing  $L$  and diffusers with radius  $r$  (b), and a Sinai scattering array where the ordered diffusers have each been randomly displaced by  $\leq \Delta_0$  (c).

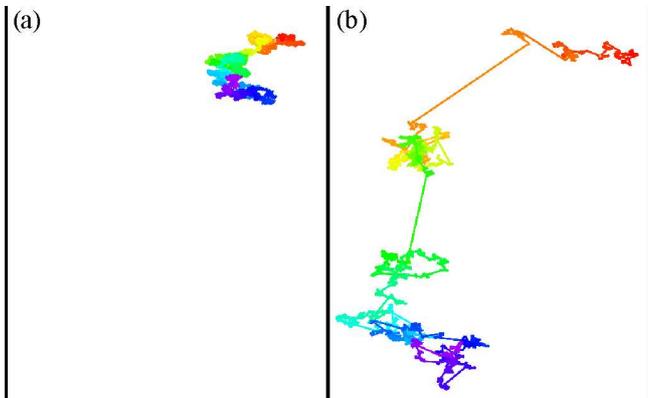


Figure 4: Simulations showing a path with a Gaussian distribution of trajectory lengths (a) and a corresponding path drawn from a Lévy (power law) distribution (b). The origin for each path is fixed. Time elapsed is shown as a color gradient along the paths from  $t = 0$  (red) to the finish (violet).

fore determine the distribution of fluctuation periods in traces of conductance versus magnetic field,  $B$ . Previous studies of quantum interference in 2-D devices have established a universal behavior for the statistics of these interfering loops, in which material-induced scatterers act as Sinai diffusers to generate *fractal* conductance fluctuations (FCF) [3]. The FCF are universal in the sense that they are generated across diverse material systems, including both large angle (e.g., in GaAs wires with in-plane donor ions) and small angle scatterers (e.g. in semiconductor electron billiards with remote donor ions). This suggests that the underlying classical fractal trajectory distribution is manifesting itself in the observed quantum interference statistics, *and* that the scatterers are sufficiently correlated to generate the requisite Lévy (fractal) distribution of trajectory lengths.

Given this universality and the expected presence of a similar distribution of scattering sites in the bismuth clusters, we anticipate that self-assembled nanowires will also generate FCF in their quantum transport properties. A potential measurement configuration for confirming this prediction is shown in Figure 5 (inset), where nanoclusters are assembled between a source

and drain and the FCF are generated using an electrostatic gate to vary the phase of the electron waves. This technique has been shown to generate FCF with statistics similar to those generated by a time-varying magnetic field. Figure 5 shows an example of fluctuations generated by a changing top gate voltage in a high-mobility semiconductor (InGaAs/InP) billiard.

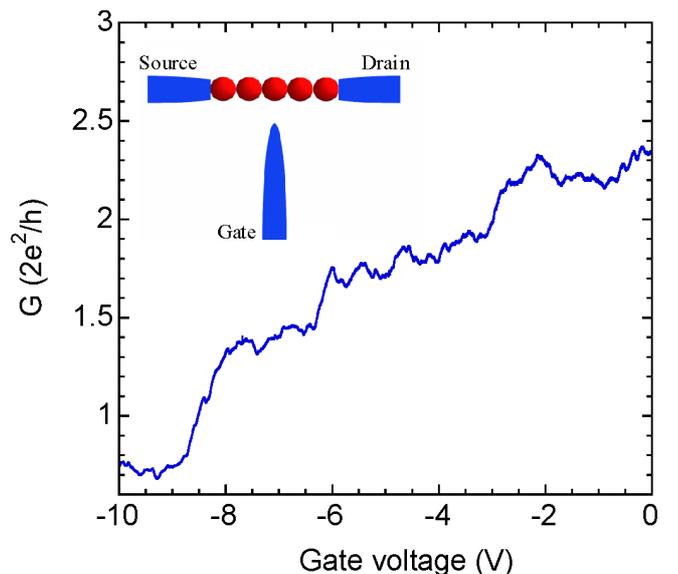


Figure 5: Fractal conductance fluctuations of an electron billiard in an InGaAs/InP heterostructure, which are induced by varying the billiard's electrostatic environment via a top gate. The inset shows an analogous measurement configuration proposal for the bismuth nanocluster wires.

Of particular interest for the bismuth nanowires is the possibility of observing FCF at much higher temperatures due to bismuth's low effective mass and relatively high phase coherence lengths. In previous work, the complexity of the FCF were found empirically to depend on the parameter  $Q$  [3]:

$$Q = \frac{\Delta E_s}{\Delta E_b} = \frac{2\pi\hbar/m^*A}{\sqrt{(\hbar/\tau_\phi)^2 + (k_B T)^2}} \quad (1)$$

where  $\Delta E_s$  is the average spacing of the systems quantum en-

ergy levels,  $\Delta E_b$  is their average broadening,  $A$  is the enclosed area of a cavity, and  $\tau_\phi$  is the phase coherence time of the electron states. The FCF were found to be most complex for the condition  $Q = 1$ . Even at ambient temperatures ( $T = 300$  K), the bismuth nanowires should be able to exhibit this peak fractal character for effective masses at the low end of the known range for bismuth ( $< 0.01m_e$ ), sufficiently high phase coherence times ( $> 5$  fs)<sup>1</sup>, and cavity areas equal to those experimentally demonstrated ( $\approx 50$  nm). Even in the absence of these conditions, the phase coherence times at 4 K are large enough ( $\approx 1$  ps)[4] that the ‘universal’ fractal character of the conductance fluctuations should be easily observable.

## Conclusions

In this paper, we have described the self-assembly of nanowires fabricated by deposition of bismuth clusters. The varying width of these wires is a generic property of self-assembled wires, and forms the geometry of our proposed model for conduction through these systems, that of connected series of ballistic cavities. We propose that previously observed ‘universal’ quantum interference statistics based on the underlying fractal classical dynamics should lead to similar behavior in these nanowires since they share key material and device parameters. Additionally, there is a potential for the observation of ambient temperature quantum interference effects in the bismuth nanowires due to their low effective mass and high phase coherence times.

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## References

- [1] G. M. Whitesides, B. Grzybowski, Self-assembly at all scales, *Science* 295 (2002) 2418 – 2421.
- [2] R. Reichel, J. G. Partridge, F. Natali, T. Matthewson, S. A. Brown, A. Lassesson, D. M. A. Mackenzie, A. I. Ayesh, K. C. Tee, A. Awasthi, S. C. Hendy, From the adhesion of atomic clusters to the fabrication of nanodevices, *Applied Physics Letters* 89 (2006) 213105.
- [3] C. A. Marlow, R. P. Taylor, T. P. Martin, B. C. Scannell, H. Linke, M. S. Fairbanks, G. D. R. Hall, I. Shorubalko, L. Samuelson, T. M. Fromhold, C. V. Brown, B. Hackens, S. Faniel, C. Gustin, V. Bayot, X. Wallart, S. Bollaert, A. Cappy, Unified model for fractal conductance fluctuations for diffusive and ballistic semiconductor devices, *Physics Review B* 73 (2006) 195318.
- [4] B. Hackens, J. P. Minet, S. Faniel, G. Farhi, C. Gustin, J. P. Issi, J. P. Heremans, V. Bayot, Quantum transport, anomalous dephasing, and spin-orbit coupling in an open ballistic bismuth nanocavity, *Physical Review B* 67 (2003) 121403(R).

- [5] C. A. Hoffman, J. R. Meyer, F. J. Bartoli, A. Di Venere, X. J. Yi, C. L. Hou, H. C. Wang, J. B. Ketterson, G. K. Wong, Semimetal-to-semiconductor transition in bismuth thin films, *Physical Review B* 48 (1993) 11431 – 11434.
- [6] J. Heremans, C. M. Thrush, Y.-M. Lin, C. S., Z. Zhang, M. S. Dresselhaus, J. F. Mansfield, Bismuth nanowire arrays: Synthesis and galvanomagnetic properties, *Physical Review B* 61 (2000) 2921 – 2930.
- [7] Y. G. Sinai, Dynamical systems with elastic reflections, *Russian Mathematical Surveys* 25 (1970) 137 – 189.
- [8] D. N. Armstead, B. R. Hunt, E. Ott, Anomalous diffusion in infinite horizon billiards, *Physical Review E* 67 (2003) 021110.
- [9] M. F. Schlesinger, G. M. Zaslavsky, U. Frisch (Eds.), *Lévy Flights and Related Topics in Physics*, Springer-Verlag, 1995.
- [10] B. B. Mandelbrot, *The Fractal Geometry of Nature*, W. H. Freeman, 1982, Ch. 30 - 32.
- [11] S. Washburn, R. A. Webb, Quantum transport in small disordered samples from the diffusive to the ballistic regime, *Reports on Progress in Physics* 55 (1992) 1311 – 1383.
- [12] J. C. G. de Sande, J. M. Guerra, Nernst-ettingshausen effect in polycrystalline bismuth at high temperature, *Physical Review B* 45 (2000) 11469 – 11473.
- [13] V. M. Grabov, K. G. Ivanov, A. A. Za'itsev, Magneto-optical study of bismuth at 80 - 280 k, *Semiconductors* 34 (2000) 1287 – 1289.
- [14] V. Y. Kashirin, Y. F. Komnik, Electron-electron interaction in thin bismuth films, *Physical Review B* 50 (1994) 16845 – 16850.

<sup>1</sup>Ambient temperature estimates of phase coherence times vary widely with device geometry and crystal quality ( $\sim 1 - 100$  fs)[12, 13, 14] but our  $> 5$  fs constraint appears easily achievable.