**NL2481** **Lévy Flights**

In 1937, the French mathematician Paul Lévy (1886-1971) introduced statistical descriptions of motion that extend beyond the more traditional Brownian motion discovered over one hundred years earlier. A diverse range of both natural and artificial phenomena are now being described in terms of Lévy statistics, from the flights of an albatross across the Antarctic skies to the trajectories followed by the abstract painter Jackson Pollock as he constructed his famous drip paintings.

In 1828, Robert Brown published his studies of the random motion followed by soot particles as they were buffeted around by the thermal motion of water molecules. In 1905, Albert Einstein provided a theoretical basis for this diffusion process. A particle's "Brownian motion" is pictured as a sequence of jumps. For a single jump, the probability dependence on jump size x produces a Gaussian distribution. A consequence of Gaussian statistics is that the size distribution for N jumps is also described by a Gaussian.

Paul Lévy generalised beyond Brownian motion by considering other distributions for which one jump and N jumps share the same mathematical form. These Lévy distributions decrease according to the power law $1/x^{1+\gamma}$ for large x values, where $\gamma$ lies between 0 and 2. Since Gaussians correspond to $\gamma = 2$, Brownian motion can be regarded as an extreme case of Lévy motion. Compared to Gaussian distributions, Lévy distributions do not fall off as rapidly at long distances. For Brownian motion, each jump is usually small and the variance of the distribution, $\langle x^2 \rangle$, is finite. For Lévy motion, however, the small jumps are interspersed with longer jumps, or "flights", causing the variance of the distribution to diverge. As a consequence, Lévy jumps do not have a characteristic length scale!

This scale invariance is a signature of fractal patterns. Indeed, Lévy's initial question of when does the whole look like its parts addresses the fractal property of self-similarity. An important parameter for assessing the scaling relationship of fractal patterns is dimension. What, then, is the dimension of the pattern traced out by Lévy motion? The short jumps making up Brownian motion build a clustered pattern that is so dense that area is a more appropriate measure than length - the pattern is actually two dimensional. In contrast, whereas the short jumps of Lévy motion produce a clustering, the longer, less frequent jumps initiate new clusters. These clusters form a self-similar pattern with a dimension of less than two. Fractional dimensions are an exotic property of fractals.

Today, Lévy motion is as widely explored in non-linear, chaotic, turbulent and fractal systems as Brownian motion is in simpler systems. Following Mandelbrot's research in the 1970s demonstrating the prevalence of fractal patterns in nature, an increasing number of natural phenomena have been described using Lévy statistics (Mandelbrot, 1982). Lévy distributions are also having an impact on artificial systems. A recent example concerns nano-scale electronic devices in which chaotic electron trajectories produce Lévy statistics in the electrical conduction properties (Micolich *et al*., 2001). Other examples include diffusion in Josephson junctions (Geisel *et al.,* 1985) and at liquid-solid interfaces (Stapf *et al.,* 1995).

It is even possible to picture relatively simple systems in which both Brownian and Lévy motion appear and a transition between the two can be induced. Consider, for example, dropping tracer particles into a container of liquid. This, of course, is Brown's original experiment. In 1993, Harry Swinney extended this experiment by considering a rotating container of liquid shaped like a washer. As turbulence set in, vortices appeared in the liquid and Swinney's group showed that the tracer particles followed Lévy flights between the vortices with $\gamma = 1.3$ (Solomon *et al*., 1993).

In addition to spatial distributions, Lévy statistics can also be applied to distributions measured as a function of time. A famous example is the dripping faucet. In 1995 Thadeu Penna's group showed that the fluctuations in the time intervals between drips follow a Lévy distribution with $\gamma = 1.66 - 1.85$. A significant appeal of this result lies in a comparison with earlier medical work by Ary Goldberger's group showing that fluctuations in the human heart beat follow $\gamma = 1.7$ (Goldberger, 1996). This prompted Penna to ask, "Is the heart a dripping faucet?"

Goldberger suggested that the Lévy statistics describing the human heart arise from non-linear processes that regulate the human nervous system. He has since extended his research of the fractal dynamics of physiology to other examples of involuntary behaviour. This includes studies of the human gait that show that fluctuations in stride intervals display fractal variations (Hausdorff *et al*., 1996). Fractal variations might therefore be a general signature of healthy human behaviour, exhibited whenever conscious control is not involved.

It is interesting to consider this speculation within the context of the results of the British Antarctic Survey in 1996, which showed that albatrosses follow Lévy flights. Other species of animals, such as ants and bees, also follow Lévy flights when foraging for food. Due to the diverging variance of the flight distribution, Lévy trajectories represent an efficient way of covering large regions of space, especially when compared to Brownian motion. Significantly, these animal behavioural patterns represent yet another example of Lévy behaviour generated by actions that are devoid of intellectual deliberation.

This relationship between "unconscious" actions and Lévy statistics has even touched on human creativity. In particular, the Surrealist art movement developed a technique called "automatic" painting, in which artists painted with such speed that any conscious involvement was thought to be eliminated. Jackson Pollock adopted this approach during the 1940s-50s when he dripped paint onto large horizontal canvases (see *Autumn Rhythm* below). Remarkably, his paintings are fractal and his motions have been described in terms of Lévy flights (Taylor *et al.,* 1999). This work triggered visual perception tests that identified an aesthetic preference for fractal patterns with dimensions between 1.3 and 1.5 (Taylor, 2001).

Lévy distributions represent a truly inter-disciplinary phenomenon that will continue to be useful as novel artificial and natural systems are explored.

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*See also* Brownian Motion; Dimensions; Fractals

**Further reading**

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