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Dependence of fractal conductance fluctuations on soft-wall profile in a double-layer semiconductor billiard

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We present a semiconductor system featuring two billiards located one on top of the other. We use this system to study the dependence of fractal conductance fluctuations on soft-wall potential profile and show the fluctuations to be surprisingly robust to changes in profile. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1485103]

A consequence of scaling semiconductor devices down to submicron sizes is that the sidewalls have a profound effect on device conductance. In particular, the use of surface gates to define submicron geometries in the two-dimensional electron gas (2DEG) of AlGaAs/GaAs heterostructures has led to a wide variety of fundamental and applied physics phenomena.[1] This electrostatic technique typically produces a two-dimensional potential well with a flat bottom and ‘‘soft’’ walls that have approximately parabolic energy profiles.[2] The well forms a billiard in the ballistic scattering regime and electron trajectories are determined predominantly by the billiard geometry and profile. For phase-coherent electrons, an applied magnetic field varies the electron-wave interference, generating magnetoconductance fluctuations that serve as ‘‘magnetofingerprints’’ of the *precise* trajectory distribution.[1] Recent low-temperature experiments showed that these fluctuations have a generic property—the fluctuations are fractal, exhibiting a recurrence of structure at increasingly fine field scales.[3,4] A semiclassical theory[5] proposed that fractal conductance fluctuations (FCF) are a consequence of the ‘‘mixed’’ stable–chaotic classical dynamics generated by soft walls and predicted that the scaling relationship of the fluctuations, quantified by the fractal dimension D_F,[6] has a critical sensitivity to the *precise* profile.[4,5] To investigate this FCF dependence on profile, we designed a ‘‘double-layer’’ billiard system in which a common set of surface gates define billiards in two 2DEGs located at different depths beneath the heterostructure surface. The two billiards have nominally identical geometries but different soft-wall profiles. We find that the change in profile induces noticeable differences in the individual fluctuations of the two magnetoconductance traces. However, in contrast to predictions, the fractal properties are robust to these individual changes and D_F remains constant.

The soft-wall profile of billiards is known[7,8] to be sensitive to both surface-gate bias V_g and 2DEG depth z. Previously, FCF were investigated[4] as a function of V_g due to the simplicity of adjusting V_g compared to z. However, adjustments to V_g also modify the billiard area A and the number of modes n in the quantum point contact leads, both of which are known[3] to induce independent changes in the FCF. This obscures the relationship between the profile and the measured FCF. The double-layer billiard is designed to overcome this problem. Figure 1(a) shows the device, which uses a common set of surface gates to define billiards in ‘‘shallow’’ ($z=90$ nm) and ‘‘deep’’ ($z=140$ nm) 2DEGs. With this 50 nm separation, electron tunneling and interaction effects are expected to be negligible.[9,10] The shallow billiard (which can be measured independently of the deep billiard, see below) exhibits *identical* D_F behavior to shallow billiards in single-layer systems,[3] confirming that the FCF are not affected by coupling effects between the shallow and deep billiards. We now describe the procedure for measuring the FCF shown in Fig. 2(b). First, the deep billiard is electrically isolated (see later) to measure the shallow billiard. The biases of the three gates forming the billiard (Fig. 1) are then individually tuned to give $n=2$ in the two leads and the FCF are measured (upper trace). The shallow billiard is then isolated and the three gate biases are retuned to achieve $n=2$ for the deep billiard and the FCF are measured (lower trace). Using this technique, the two billiards are independently measured for identical n values and have A values (measured using the

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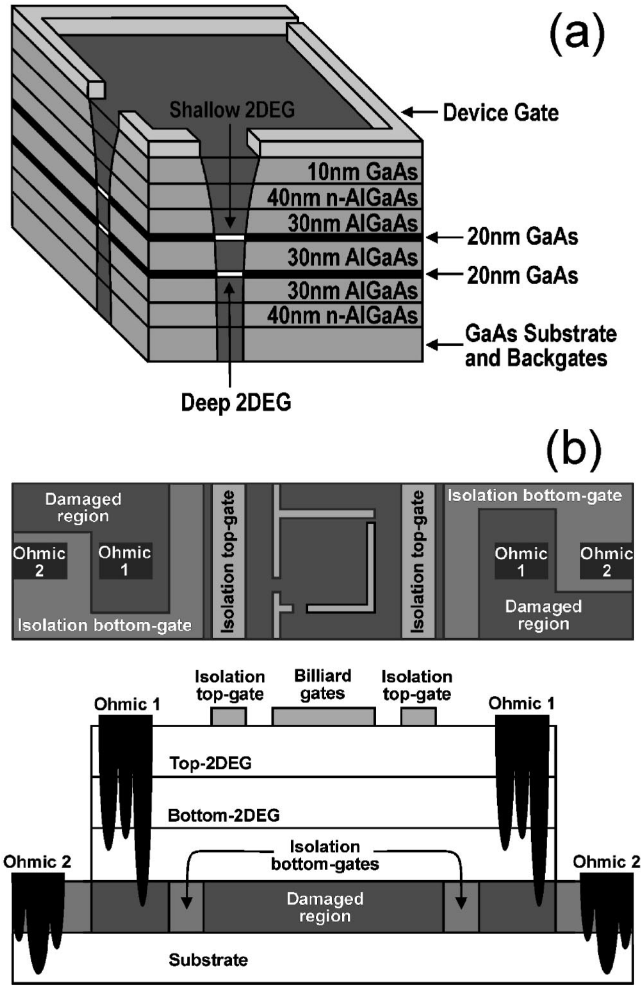

FIG. 1. (a) Schematic representation of the double-layer billiard. The gate widths are 100 nm. (b) Schematic representation of the technique used to achieve independent electrical contact to the two billiards (after Ref. 13).

high-field Aharonov–Bohm effect)[1] matched to within 14%, a difference that previous studies[3] indicate will produce less than a 1% change in D_F. The shallow billiard profile is softer than that of the deep billiard due to the smaller V_g required to define it.[11]

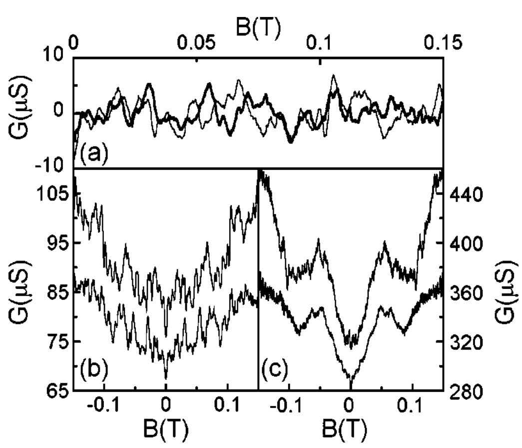

FIG. 2. (a) An overlay of FCF traces (with classical backgrounds removed and for $B>0$ only) from the deep (thin line) and shallow (thick line) billiards for $n=2$. $G(B)$ traces for the shallow (upper traces) and deep (lower traces) billiards at (b) $n=2$ and (c) $n=8$. Shallow billiard traces in (b) and (c) are offset by 20 μS and 5 μS, respectively, for clarity.

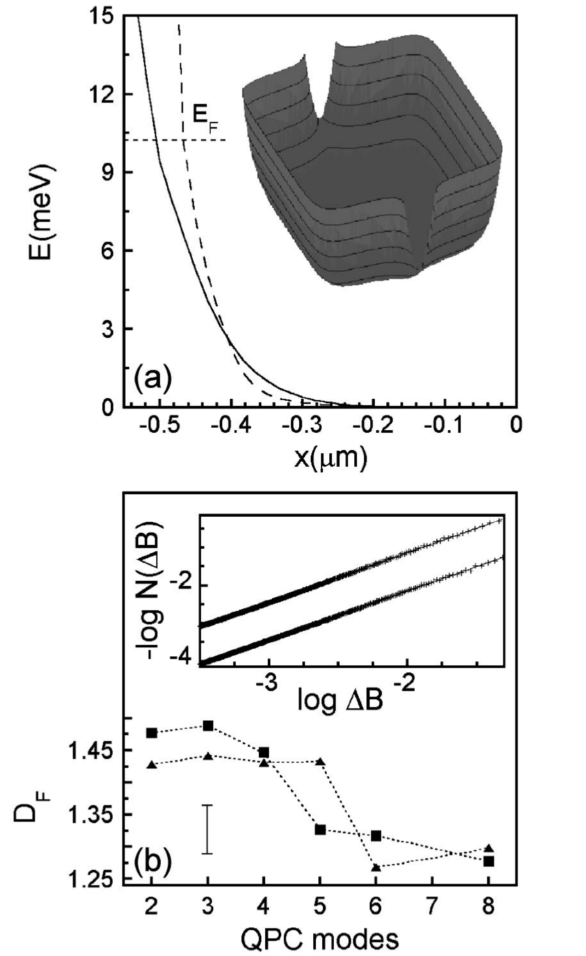

FIG. 3. (a) Self-consistent profile models for the shallow (solid line) and deep (dashed line) billiards with $n=6$. A 16% change in A is predicted from the differences in the profiles. The arrow on the y axis indicates the Fermi energy in the two billiards (Inset) The three-dimensional soft-wall profile. (b) Fractal dimension D_F vs n for the deep (▲) and shallow (■) billiards. (Inset) Fractal scaling plot ($-\log N(\Delta B)$ vs $\log \Delta B$) for the $G(B)$ data obtained at $n=6$ from the shallow (upper data and fit) and deep (lower data and fit) billiards.

The profiles, calculated using self-consistent numerical simulations, are shown in Fig. 3(a). The profile gradient S at the Fermi energy E_F differs by a factor of three between the two billiards. Note also that the difference in profile is not limited to the edge of the billiard. Figure 3(a) shows that the two profiles differ by at least 0.5 meV (corresponding to 5% of E_F) across more than a quarter of the width of the billiard. Given the predicted critical sensitivity to profile,[4,5] this is expected to significantly redistribute the classical trajectories. Our double-layer billiard strategy of using a common set of surface gates offers significant advantages over other possible approaches to this experiment. One alternative approach, adopted in Ref. 4, simply uses the changes in V_g applied to a single-layer billiard to induce variations in S. However, this does not allow independent control of n and S: for example, our simulations show that a change in S of only a factor of 2 is accompanied by a change in n from 2 to 6. Another approach would be to use two single-layer billiards of different lithographic sizes and to apply different V_g to the two billiards to induce the same A and n but different S. This approach introduces problems associated with lithographic variations in the two sets of gate patterns defining the two billiards and, since the mixed electron dynamics are extremely sensitive to the precise billiard geometry,[4,5] this is not desirable.

Electrical contact is achieved via NiGeAu ohmic contacts patterned at the ends of a Hall bar and annealed to penetrate *both* 2DEGs. The billiard gates are deposited using electron-beam lithography on the Hall bar surface. Independent measurement of the two billiards is achieved using a set of "isolation" gates above and below the two 2DEGs at either end of the Hall bar. Eisenstein *et al.*[12] first demonstrated this technique using gates on both surfaces of a thinned wafer. We employ an alternative technique [outlined in Fig. 1(b)] that avoids using thin wafers.[13] To measure the shallow billiard, the electrical path through the deep 2DEG is severed by negatively biasing the isolation bottom gates. These bottom gates are patterned in a conductive n^+ GaAs layer located 350 nm beneath the deep 2DEG by damaging selected regions using *in situ* focused ion-beam lithography (FIBL) during heterostructure growth.[13] Contact to the bottom gates is attained using separate ohmic contacts [marked Ohmic 2 in Fig. 1(b)] that are formed in the same processing steps as the 2DEG ohmic contacts (marked Ohmic 1). Hence, it is necessary to ensure that FIBL-damaged, nonconductive n^+ GaAs regions lay below all of the 2DEG ohmic contacts [see Fig. 1(b)], to prevent shortcircuiting between the bottom gates and the 2DEGs. To measure the deep billiard, the electrical path through the shallow 2DEG is severed by negatively biasing the isolation top gates, which are fabricated using photolithography. Independent contact is verified using Shubnikov–de Haas oscillations. When the two 2DEGs are measured in parallel, the oscillations exhibit beating related to the difference in electron density between the two 2DEGs. In the experiments, we match the two electron densities by adjusting the bias applied to a back gate patterned in the same n^+ GaAs layer as the isolation bottom gates [not shown in Fig. 1(b)] until the beating in the parallel signal is eliminated. We obtain matched electron densities of $2.85\times10^{15}\ \mathrm{m}^{-2}$ (and hence matched Fermi energies of 10.2 meV) in both 2DEGs and electron mobilities of 130 and 110 m^2/Vs in the shallow and deep 2DEGs, respectively.

Magnetoconductance $G(B)$ traces measured at 20 mK are shown in Figs. 2(b) and 2(c) and reveal FCF superimposed on a classical background. For each of the two n settings, the similarity between the classical structure of the deep and shallow billiards confirms that both have the same nominal geometry (i.e., size and shape). To better compare the actual FCF, in Fig. 2(a), we present an overlay of the traces in Fig. 2(b) with their classical backgrounds removed. It is evident that the differing wall profile induces a significant change in the precise details of the FCF, and intuitively, one might expect that the fractal scaling has changed, as predicted by theory.[4,5] D_F is obtained using a variant of the box-counting technique,[3] which calculates the minimum number $N(\Delta B)$ of nonoverlapping squares required to completely cover the trace as a function of square size ΔB. A $G(B)$ trace is fractal[6] if $N(\Delta B)$ scales according to $N(\Delta B) \sim \Delta B^{-D_F}$, where $1<D_F<2$. Therefore, by constructing a scaling plot of $-\log N(\Delta B)$ versus $\log \Delta B$, FCF are detected as a straight line and D_F is obtained from the gradient. We investigate the effect of the change in profile on two FCF parameters—the ΔB range over which fractal scaling is observed and D_F. Figure 3(b) (inset) shows scaling plots for the $G(B)$ traces obtained for $n=6$, corresponding to the profiles shown in Fig. 3(a). The locations of the upper and lower limits of fractal scaling match for the two billiards, and this is expected since neither limit is related to the soft-wall profile.[3] The difference in D_F for the two traces lies within experimental uncertainties. In other words, although the difference in billiard profile is sufficient to induce changes in the individual features of the FCF, the statistical characteristics of the FCF have not changed. This is further demonstrated in Fig. 3(b) where the D_F dependence on n is plotted for the two billiards. For each billiard, D_F falls as n increases due to the reduction of electron phase coherence.[3] In contrast to this marked fall off, any profile-induced changes between the two traces are minimal and lay within experimental scatter.

In summary, we have developed a double-layer billiard system and used it to investigate the role of soft-wall profile on FCF. Given the 50 nm vertical separation between billiards, we expect the influence of coupling effects on the FCF to be small. Having demonstrated the feasibility of double-layer billiards, we note the potential for studying coupled transport phenomena in billiards with reduced vertical separation.

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