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The Abstract Expressionists and Les Automatistes: A shared multi-fractal depth?

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ABSTRACT

Statistical analysis of abstract paintings is becoming an increasingly important tool for understanding the creative process of visual artists. We present a multifractal analysis of 'poured' paintings from the Abstract Expressionism and Les Automatistes movements. The box-counting dimension (D_0) is measured for the analyzed paintings, as is the associated multifractal depth $\Delta D = D_0 - D_\infty$, where D_∞ is the asymptotic dimension. We investigate the role of depth by plotting a 'phase space' diagram that examines the relationship between D_0 and D_∞. We show that, although the D_0 and D_∞ values vary between individual paintings, the collection of paintings exhibit a similar depth, suggesting a shared visual characteristic for this genre. We discuss the visual implications of this result.

1. Introduction

In 1945, Jackson Pollock started to perfect a radically new approach to painting. Abandoning physical contact with the canvas, he dipped his brush in and out of a can and poured the fluid paint onto horizontal canvases. The uniquely continuous paint trajectories served as artistic 'fingerprints' of his motions through the air. Over the next decade, he generated vast abstract works featuring complex patterns formed across many scales—from the width of the canvas down to finest speck of paint.

In doing so, he became one of the leaders of the Abstract Expressionist movement, which shifted the focus of the art world from its traditional base of Paris to New York. Pollock's form of Abstract Expressionism inspired the Quebec-based Les Automatistes who also adopted the pouring style of painting. Art theorists now recognize the "drip and pour" style as a revolutionary approach to esthetics. However, despite the millions of words written about this body of work through the years, the artistic significance behind these complex swirls of paint remained the source of fierce debate in the art world. One of the central questions within this debate concerned the degree of variability between paintings by different artists: for example, are there shared visual characteristics between Pollock's paintings and those of Les Automatistes? Examples of their respective works are shown in Figs. 1 and 2.

Fractal analysis techniques hold great promise for both the academic and artistic communities, since these techniques serve to identify underlying visual signatures in an artwork. The fractal dimension D_F of an artwork can be regarded as a preliminary indicator of complexity in a pattern: lower fractal dimensions are a measure of shallow complexity, while higher fractal dimensions (*i.e.* those which approach the dimension of the embedding space) demonstrate high complexity. That is, a line has fractal dimension $D_F=1$, while a wrapping-curve that densely fills the plane has dimension $D_F \to 2$. Within this scheme, fractal paintings are quantified by D_F values in the range $1 < D_F < 2$, where paintings with D_F values

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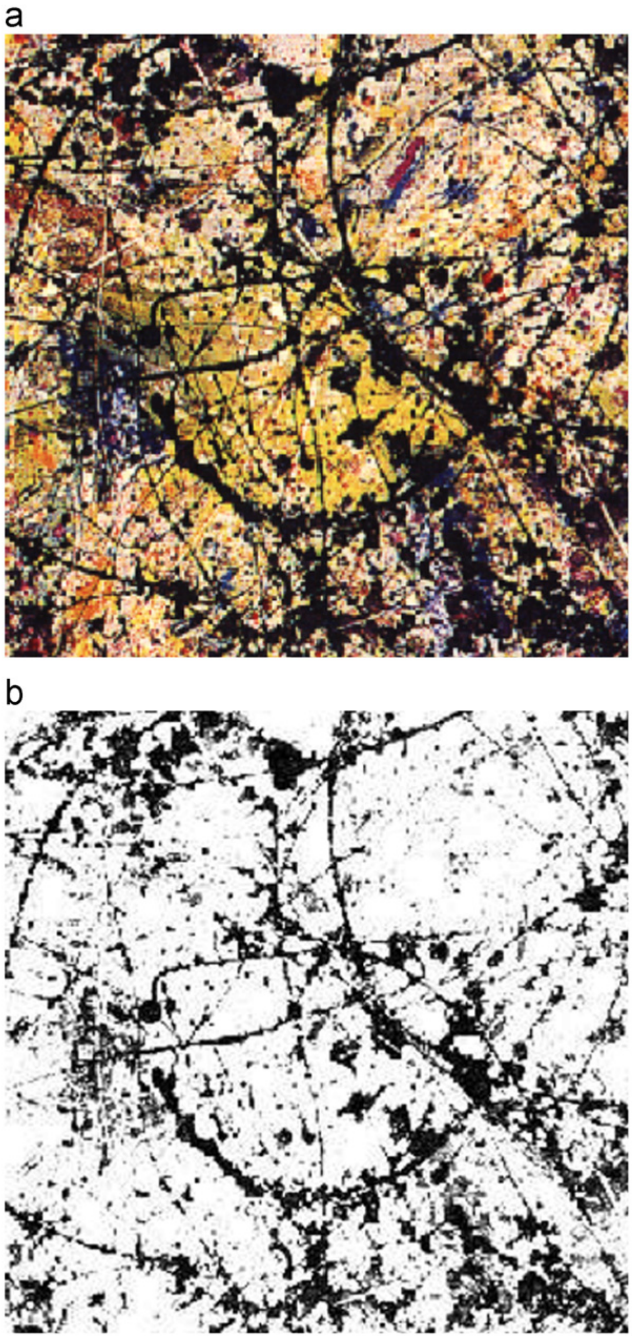

Fig. 1. *Reflections of the Big Dipper* (1947), by Jackson Pollock. Image progression shows section of (a) original painting, and (b) blob structure for the black pigment layer.

closer to 1 appearing simple and sparse, and paintings with D_F values closer to 2 appearing more rich and intricate. The multifractal spectrum takes a closer look at this visual relationship by considering an infinite family of fractal dimensions $\{D_q, q=0, 1, 2 \ldots\}$ that yield key information about the degree to which complexity is manifest in a pattern.

Motivated by these concepts, one of us (R.P.T.) employed an established fractal analysis called the 'box-counting' technique to extract D_0 values for Pollock's poured paintings [1–3]. The fractal character of Pollock's work was confirmed by J.R.M., who also extended the problem to the multifractal arena and extracted the full spectrum of D values for $q=0$ to $q \to \infty$ [4,5]. Based on this initial research, we and a number of other groups have since shown diverse fractal analysis techniques to be useful approaches to quantifying the visual complexity of Pollock's poured patterns [6–24].

Fig. 2. The blob structure of *Tumulte* (1973) by *Les Automatistes*.

It is critical to note a fundamental limitation common to all of these fractal analysis techniques [9,10]. Along with nature's fractals, art works are examples of physical fractals and consequently have inherent upper and lower cut-offs beyond which their fractal geometry either cannot be resolved or does not exist. The magnification range of nature's physical fractals can be surprisingly small – the typical range is only 1.25 orders – and this is an inevitable quality of fractal art also [3]. For this reason, we adopt the term "effective dimension" to highlight this limited-range quality. Nevertheless, the D_q values extracted from the multifractal analysis still quantify the associated visual complexities of the pattern [25,26].

The importance of comparing and contrasting parameters extracted from multiple analysis techniques was emphasized early in this field's development [2]. More recently, the employment of 'phase space' diagrams, generated by plotting different parameters along several axes, was highlighted as an efficient approach to identifying the trademark 'parameter space' for given paintings [22]. Here, we build on earlier multi-fractal comparisons of Pollock's patterns with those of Les Automatistes by constructing a phase space diagram based on the box-counting dimension D_0 and the asymptotic dimension D_∞. This plot is used to investigate the multifractal depth, which is defined as $\Delta D = D_0 - D_\infty$ for the art works. We show that, although the D_0 and D_∞ values vary between individual paintings, the collection of paintings exhibit a similar depth, suggesting a shared visual characteristic for the 'poured' genre of art. The focus of previous fractal studies has been the search for variations between art works by different artists, with the aim of developing novel authenticity techniques. In contrast, the work presented here serves as a reminder that identifying shared qualities is equally important for art history.

2. Multifractal analysis of art

2.1. Fractals and multifractals

The "box-counting" method is a well-established technique for extracting the fractal dimension D_0 for a fractal pattern. In this approach, digitized images of paintings are covered with a computer-generated mesh of identical squares (or "boxes"). The statistical scaling qualities of the pattern are then determined by calculating the proportion of squares occupied by the painted pattern and the proportion that are empty. This process is then repeated for meshes with increasingly small square sizes. Reducing the square size is equivalent to looking at the pattern at finer magnification. In this way, it is possible to compare the pattern's statistical qualities at different magnifications. Specifically, the number of squares, $n(\varepsilon)$, that contained part of the painted pattern can be counted and this is repeated as the size, ε, of the squares in the mesh was reduced. The largest size of square is chosen to match the canvas size (for a Pollock painting this is typically $\varepsilon \sim 2.5$ m) and the smallest is chosen to match the finest paint work ($\varepsilon \sim 1$ mm). For fractal behavior, $n(\varepsilon)$ scales according to the power law relationship $n(\varepsilon) \sim \varepsilon^{-D}$. This power law generates the scale invariant properties that are central to fractal geometry [2,3].

Fractals are a subset of a larger class of objects known as multifractals. While the former is described by a single scaling dimension D_F, the latter is characterized by an infinite number of dimensions D_q, $-\infty < q < +\infty$, where $D_q > D_{q+1}$ [27,28]. Multifractal dimensions with $q>0$ are a measure of the clustering complexity of a set, while those for $q<0$ describe the anti-clustering behavior. The $q=1$ multifractal dimension is equivalent to the information dimension, while the correlation (or mass) dimension corresponds to D_2. The standard fractal dimension is $D_F = D_0$, and if the set is itself pure or "monofractal" then $D_q = D_0$ for all q.

A multifractal is, in essence, an interspersion or overlayering of an infinite number of monofractals characterized by the dimension D_q, whose clustering behavior is dependent on the scale-size at which it is measured. The dimension itself may be understood to be a "microscope," which can be tuned to higher and higher clustering behavior for increasing q. That is, $q>0$ provides a measure of relative (increasing) pattern density, while $q<0$ probes the regions of very low density (*i.e.* the gaps or voids in the pattern). Note the spectra region $q<0$ is similar in spirit to lacunarity analysis [29], in which the size and frequency of void regions within a pattern is measured as a quantification of translational invariance.

As a practical example, the physical distribution of galaxies in the universe is believed to be multifractal (see e.g. [30,31] for a review). Specifically, below a certain cosmological distance scale, it has been shown that galaxies preferentially cluster on surfaces, or sheets. But within those sheets, a finer-tuned scale measurement reveals filamentary clusters of galaxies. In multifractal parlance, these would correspond to scaling dimensions of $D=2$ (small q), and $D=1$ (larger q).

For the paintings considered herein, an equivalent interpretation of the data would suggest scale-dependent construction of the painting. The largest patterns, presumably created by wide-swing arm motions and movement of the artist, are characterized by D_q (small q), while the finer brush motions of the wrist and fingers yield the large q dimensionality. This is a similar interpretation to the bi-fractal behavior discussed in [1–3].

A convenient measure of the complexity depth of a multifractal is defined as

$$\Delta D = D_0 - D_\infty \tag{1}$$

Patterns with a rich multifractal structure will have large ΔD, while those with little or no variation will show the opposite. A monofractal is characterized by $\Delta D = 0$.

The multifractal spectrum of dimensions is readily calculated by a modified box counting algorithm. As described above, the pattern is covered by $N(\varepsilon)$ boxes of scale size ε, of which only $n(\varepsilon)$ actually contain the pattern. These contribute to the multifractal moments [27,28],

$$\Xi(q,\varepsilon) = \sum_{i=1}^{N(\varepsilon)} [p_i(\varepsilon)]^q, \quad p_i(\varepsilon) = \frac{n_i(\varepsilon)}{N(\varepsilon)} \tag{2}$$

where $p_i(\varepsilon)$ is the relative density of the pattern contained in the box. The logarithmic slope $\tau(q)$ of the partition function $\Xi(q,\varepsilon)$ is related to the multifractal dimension D_q by

$$D_q = \frac{\tau(q)}{q-1}, \quad \tau(q) = \lim_{\varepsilon \to 0} \frac{d \log[\Xi(q,\varepsilon)]}{d \log(\varepsilon)} \tag{3}$$

As with the basic box counting technique, practical implementation of the modified box counting algorithm generally involves digitization of the painting image and iteratively binning pixel data into $N(x_i)$ covering boxes of pixel width x_i over a finite range of scales. These generally range from roughly the size of the original image, to boxes a few pixels in width. Numerical estimates of fractal and multifractal dimensions are then obtained by linear regression of the partition function values (2) as a function of scale size.

We note for completeness that there are numerous other analysis methods from which the multifractal spectrum of dimensions may be obtained. These primarily include variations of wavelet transform procedures [32–34], whose advantage over the box-counting technique lies in a more robust estimation of a distribution's multifractal measure. A second adaptation of the wavelet framework is the Wavelet Transform Modulus Maxima (WTMM) method [35–38], a novel approach that associates to the multifractal scaling a thermodynamic description. A more recent technique, the wavelet leader method [39–41], is constructed from discrete wavelet transform coefficients, and has shown particular advantages over other approaches in computing the multifractal spectrum for $q<0$. For consistency with previous relevant results in the literature concerning non-representational art, we opt to use the former (box counting) technique herein.

3. Painting analysis

A comparison was made between two groups of 5 paintings by Pollock and Les Automatistes. The Pollock paintings were: *Blue Poles* (1952), *Autumn Rhythm* (1950), *Lavender Mist* (1950), *Reflections of the Big Dipper* (1947) and *Number One A* 1948. Les Automatistes paintings were: *Au Chatuea d'Argol* (1947), *Feivres* (1976), *Tumulte* (1973), *Voyage Au Bout du Vent* (1978) and *Suite Marocaine no.* 1 (1975). Each painting was scanned at 300 dpi from high-quality prints and stored in the color Portable Network Graphics (png) format, for easy input to custom-written multifractal analysis code. Resulting images were typically between 1000–2500 pixels on the longest side.

Pigment patterns were filtered out according to a specified target color (R_0, G_0, B_0) in RGB space (see Figs. 1 and 2). A variance in the channel values up to a specified "color radius" $r_{RGB} = \sqrt{(R-R_0)^2+(G-G_0)^2+(B-B_0)^2}$ from the target was allowed to account for any small fluctuations in the pigment shade. For a standard RGB color scheme with values ranging between 0–255, a value of 30 units per channel ($r_{RGB} \approx 52$) was found to represent the true color of a typical pigment, based on the relatively normal distribution in each channel's histogram [4,5]. As the patterns are a result of monochromatic pigment deposits we refer to them as "blobs".

Previous research has identified the importance of the 'anchor layer' – the layer that dominates the construction process and determines the D values of the other layers – and so we concentrate on this layer for the current studies [2,3]. For example, Fig. 1 shows the black anchor layer of Pollock's *Reflections of the Big Dipper* (1947), along with the completed painting. For single layer paintings, such as *Tumulte* (1973) by Les Automatistes shown in Fig. 2, this single layer is equivalent to the anchor layer of the multi-layer paintings.

The fractal dimensions of the anchor layer blob patterns were calculated over roughly 3 orders of magnitude of scale (1024 pixels to 4 pixels per side), which corresponds to actual canvas length scales of approximately 1–2 m to just over a few millimeters. A standard least-square fit was performed on the range of box count data thus obtained. Choosing a lower box side of 2 pixels was shown to not appreciably affect the quality or value of the fit. Standard log–log plots of the box-counting data showed linear fits within a 95% confidence level for a wide range of q values, indicating the patterns definitely exhibit scale-invariant fractal structure (the interested reader is directed to [5] for specific details).

This scaling range is dominated by patterns generated by the artists' physical motions, and so we refer to the extracted D values as "motion" dimensions, to differentiate them from "drip" dimensions which concentrate on the finest scale patterns generated by the drip process itself [2,3]. The results are shown in Fig. 3, where a phase space plot of D_0 versus D_∞ allows the observation of any specific 'parameter spaces' for the paintings.

We begin by focusing on the D_0 values alone. A one-way analysis of variance (ANOVA) comparing the Pollock (mean $D_0 = 1.79$) with the Automatistes ($D_0 = 1.73$) paintings indicated that the D_0 indices are not significantly different, $F(1, 14) = 1.18$, $p < 30$. This result suggests the Hausdorff or monofractal dimension of poured paintings (D_0), when

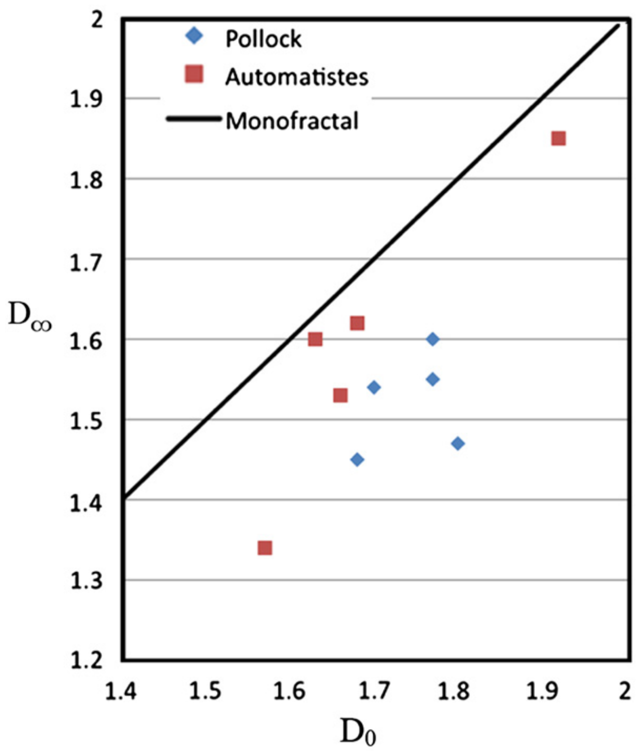

Fig. 3. A phase space plot of D_0 along the horizontal axis and D_∞ along the vertical axis (see text for details).

taken in isolation, is not unique to any one artist. Considering now D_∞, we find as expected the D values converge to an asymptotic value as q increases, such that a reasonable approximation $D_{50} \sim D_\infty$ can be made. A subsequent ANOVA confirms the D_∞ values do not present a clearer means of differentiation than the base dimension, $F(1, 14) = 0.06$, $p < 80$. Significantly, using the phase space diagram to examine any emerging relationship between D_0 and D_∞ fails to identify any distinguishing between the two groups of painters. Thus, the phase space plot adds to previous observations that the blob dimensions associated with the artists painting motions cannot, when taken in isolation, be used as an authenticity tool [2,4,5].

The phase space plot, however, succeeds in highlighting the common elements of these two movements. In each case, the artists generated paintings with a range of D values, with the phase space plot suggesting that perhaps Pollock's work gravitated to a narrower range than that of the Les Automatistes. Inclusion of the "monofractal" black line, representing the condition D_0 and D_∞, reveals a common depth of complexity $\Delta D = D_0 - D_\infty$ to the multi-fractal character of their paintings. It might be argued that Pollock's paintings are slightly deeper but, if so, it is clear that the difference in depth between the two art movements is of the same order as variations between individual paintings. Depth of the multi-fractal spectrum is clearly an important visual characteristic for both movements.

Consider the visual consequence of depth. Whereas the fractal dimension is an intrinsic measure of the scale-invariant complexity of a pattern, the set of multifractal of dimensions gauges the "moments" of such complexity. This effectively translates to the scaling properties of the pattern density: low moment ($q = 0$, 1, etc…) multifractal dimension describe the entire pattern, while high moment (q approching infinity) focus on the densest regions. The multifractal depth therefore is a descriptor of richness of local and global moment-complexity. Visually, this corresponds to the prevalence of small-scale intricacies in design. An image with low depth is globally complex, but possesses no new small-scale characteristics different from the whole (*i.e.* a regular fractal). High depth, on the other hand, possesses an additional local pattern complexity at every point, distinct from the global one.

4. Conclusions

In this paper, we have applied a multifractal analysis to the poured paintings by distinct artists—Jackson Pollock from the Abstract Expressionism movement and Les Automatistes. We have used a phase space plot of D_0 plotted against D_∞ to highlight a common depth for both art movements. This finding is intriguing when taken within the context of previous observations made by art historians. Previously, the uniformity of Pollock's "all-over" style has been identified as key element of Pollock's visual style [42]. Our findings suggest a more subtle effect. Mono-fractals generate a more uniform pattern and yet both Pollock and Les Automatistes generated multi-fractal works, indicating that the associated increased visual complexity and variety is important.

We acknowledge that these results are preliminary and may be somewhat limited by the sample size. A more comprehensive study involving a larger number of paintings may show deviations from the behavior, or may also reveal other interesting aspects including time-dependence of the multifractal depth (a result of e.g. the evolution of a particular artist's style). Such questions call for future research in this area.

In addition to quantifying the common visual impact of these fractal paintings, the depth also provides some intriguing insights into the artistic process that generated them. A recent study performed by the authors applied a multifractal analysis to poured paintings created by adults and children and found that the adult paintings show dimensions with average dimensions of $D_0=1.86$ and $D_{50}=1.82$, while those for the children paintings have lower means ($D_0=1.65$, $D_{50}=1.54$) and a larger depth [24]. It is of interest to compare the corresponding mean fractal depths of $\Delta D=0.04$ for adults and $\Delta D=0.11$ for children with those highlighted by our phase space plot. The connection between a painting's fractal properties and the maturity of the painter will be expanded upon in an upcoming manuscript [43]. We hope that the initial results presented in this paper will serve as motivation for other researchers to adopt pattern analysis techniques and explore the link between the visual characteristics of action art and the physiology of the artist's motion.

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