Chaos and graphics

Universal aesthetic of fractals

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Abstract

Since their discovery by Mandelbrot (The Fractal Geometry of Nature, Freeman, New York, 1977), fractals have experienced considerable success in quantifying the complex structure exhibited by many natural patterns and have captured the imaginations of scientists and artists alike. With ever-widening appeal, they have been referred to both as "fingerprints of nature" (Nature 399 (1999) 422) and "the new aesthetics" (J. Hum. Psychol. 41 (2001) 59). Here, we show that humans display a consistent aesthetic preference across fractal images, regardless of whether these images are generated by nature's processes, by mathematics, or by the human hand.
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1. Introduction

In contrast to the smoothness of many human-made objects, the boundaries of natural forms are often best characterised by irregularity and roughness. Their unique complexity necessitates the use of descriptive elements that are radically different from those of traditional Euclidian geometry. Whereas Euclidian shapes are composed of smooth lines, many natural forms exhibit self-similarity across different spatial scales, a property described by Mandelbrot in the framework of fractal geometry [1]. One such natural fractal object consisting of similar patterns recurring on finer and finer magnifications is the tree shown in Fig. 1. The patterns observed at different magnifications, although not identical, are described by the same statistics.

The fractal character of an image can be quantified by a parameter called the fractal dimension, D. This parameter quantifies the fractal scaling relationship between the patterns observed at different magnifica-

tions. For Euclidean shapes, dimension is a familiar concept described by ordinal integer values of 0, 1, 2, and 3 for points, lines, planes, and solids, respectively. Thus, for a smooth line (containing no fractal structure) D has a value of 1, whereas a completely filled area (again containing no fractal structure) has a value of 2. For the repeating patterns of a fractal line, D lies between 1 and 2. For fractals described by a D value close to 1, the patterns observed at different magnifications repeat in a way that builds a very smooth, sparse shape. However, for fractals described by a D value closer to 2 the repeating patterns build a shape full of intricate, detailed structure [2–4]. Fig. 2 demonstrates how a fractal pattern's D value has a profound effect on its visual appearance. In the three natural scenes shown, the boundaries between different regions form fractal lines with D values of 1.0, 1.3 and 1.9 from top to bottom, respectively. Table 1 shows D values for various classes of natural form.

The ubiquity of fractals in the natural environment has motivated several studies to investigate the relationship between the pattern's fractal character and the corresponding perceived visual qualities [2–6]. Studies by Pentland [3] and Cutting and Garvin [4] have shown a high positive correlation between the dimensional

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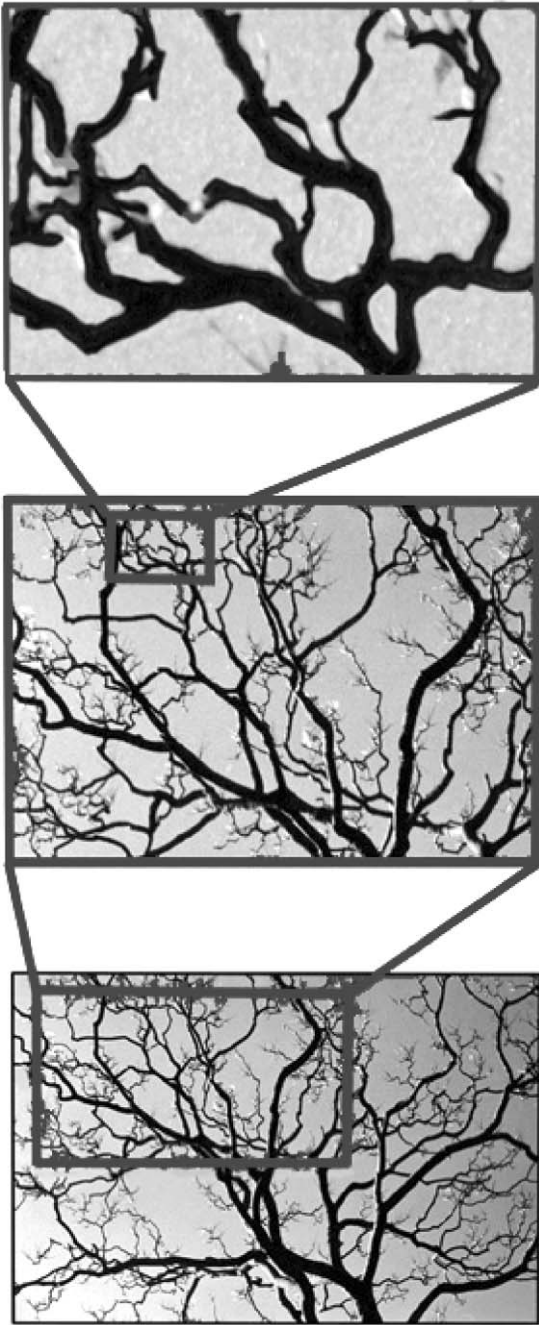*E-mail address:* b.spehar@unsw.edu.au (B. Spehar).

Fig. 1. Trees are an example of a natural fractal object. Although the patterns observed at different magnifications do not repeat exactly, analysis shows them to have the same statistical qualities (photograph by R.P. Taylor).

Fig. 2. Examples of natural forms exhibiting different *D*-values: 1.0 (top: horizon line); 1.3 (middle: clouds); and 1.9 (bottom: tree branches).

value of fractal curves and the pattern's perceived roughness and complexity. Knill et al. [5] reported that observers' ability to discriminate between fractal images based on their fractal dimension varies as a function of

how rough the images are. Interestingly, discrimination performance was maximal for fractal images with dimensions corresponding to those of natural terrain

Table 1
Fractal dimension (*D*) of several typical natural forms

Natural form type	Fractal dimension (*D*)	Source
Coastlines		
South Africa, Australia, Britain	1.05–1.25	Mandelbrot [1]
Norway	1.52	Feder [19]
Galaxies (modelled)	1.23	Mandelbrot [1]
Cracks in ductile materials	1.25	Louis et al. [20]
Geothermal rock patterns	1.25–1.55	Campbel [21]
Woody plants and trees	1.28–1.90	Morse et al. [22]
Waves	1.3	Werner [23]
Clouds	1.30–1.33	Lovejoy [24]
Sea Anemone	1.6	Burrough [25]
Cracks in non-ductile materials	1.68	Skejltorp [26]
Snowflakes	1.7	Nittman and Stanley [27]
Retinal blood vessels	1.7	Family et al. [28]
Bacteria growth pattern	1.7	Matsushita and Fukiwara [29]
Electrical discharges	1.75	Niemyer et al. [30]
Mineral patterns	1.78	Chopard et al. [31]

surfaces, suggesting that the sensitivity of the visual system might be tuned to the statistical distribution of environmental fractal frequency. However, Gilden et al. [6], who investigated the perception of natural contour, cautioned against this notion. They argued that the observed correlation between discrimination sensitivity and environmental fractal frequency might have arisen as a consequence of an alternative principle of perceptual organisation. This principle presumably utilises a smooth–rough decomposition of hierachically integrated structures that is similar to a signal–noise decomposition, and could bear no relationship to the distribution of fractal form.

As well as being rich in structure, fractal images have been widely acknowledged for their instant and considerable aesthetic appeal [7–9]. In Sprott's [10] pioneering empirical study, a collection of about 7500 strange attractors (computer generated fractal images drawn on a plane) was rated by eight observers on a five-point scale for their aesthetic appeal. It was found that images with fractal dimension between about 1.1 and 1.5 were considered to be most aesthetically appealing. More specifically, the 443 images that were rated as the most aesthetically pleasing by his observers had an average fractal dimension of 1.30. A subsequent survey by Aks and Sprott [11] in which 24 observers made direct comparisons among 324 fractal images, agreed with the initial findings and reported that preferred patterns had an average fractal dimension of 1.3. Aks and Sprott noted that the preferred value of 1.3 revealed by their survey corresponds to fractals frequently found in natural environments (for example, clouds have this value) and suggested that perhaps people's preference is actually *set* at 1.3 through continuous visual exposure to nature's patterns. In addition, they explored individual

differences in preferences for these images. Although the observed differences were small in magnitude, they found that individuals who considered themselves creative (self-report measure) had a marginally greater preference for high *D* values, while individuals who actually scored high on objective measures of creativity preferred patterns with lower fractal dimension. Richards [12] and Richards and Kerr [13] also suggested the possibility that high creativity might be related to aesthetic preference for higher fractal dimension but reported preferences for both higher and intermediate *D* values equally among art therapy and psychology students. Pickover [14] reported that among his computer generated fractal images observers expressed a preference for higher fractal dimensions of about 1.8. However, the images used in his survey often exhibited different types of symmetry (bilateral symmetry, inversion symmetry and random-walk symmetry), a highly salient image characteristic that might have interacted with the perceived complexity of the image to affect aesthetic judgements. The discrepancy in the reported fractal dimensions which were judged to be most aesthetically pleasing leaves open the possibility that there is not a universally preferred fractal dimension value. Perhaps the aesthetic qualities of fractals depend specifically on how the fractals are generated, given that the two studies used different mathematical methods for generating the fractal images?

The intriguing issue of the aesthetic appeal of fractal images has recently been reinvigorated in an unexpected way by Taylor's [15] discovery that abstract paintings by Jackson Pollock, a famous 20th Century painter, contain fractal structure. A method used for assessing self-statistical self-similarity over scale of Pollock's paintings has been described in detail elsewhere [15]

and here we present only a brief summary. Referred to as the "box-counting" technique, a digitised image (for example a scanned photograph) of the painting is covered with a computer-generated mesh of identical squares (or "boxes"). The statistical scaling qualities of the pattern are then determined by calculating the proportion of squares occupied by the painted pattern and the proportion that are empty. This process is then repeated for meshes with a range of square sizes. Reducing the square size is equivalent to looking at the pattern at a finer magnification. In this way, we can compare the pattern's statistical qualities at different magnifications. When applied to Pollock's paintings, the analysis extends over scales ranging from the smallest speck of paint (0.8 mm) up to approximately 1 m and we find the patterns to be fractal over the entire size range. The fractal dimension, D, is determined by comparing the number of occupied squares in the mesh, $N(L)$, as function of the width, L, of the squares. For fractal behaviour $N(L)$ scales according to the power law relationship $N(L) \sim L^{-D}$, where D has a fractional value lying between 1 and 2. To detect fractal behaviour we therefore construct a "scaling plot" of $-\log N(L)$ against $\log L$. For a fractal pattern, the data of this scaling plot will lie on a straight line. In contrast, if the pattern is not fractal then the data will fail to lie on a straight line. Furthermore, for a fractal pattern the value of D is simply the gradient of the straight line. In this way, we can use the scaling plot both to detect and quantify fractal behaviour.

Given that systematic research into quantifying people's visual preferences for fractal content has begun only recently, an examination of the methods used by artists to generate aesthetically pleasing images on their canvasses seems an extremely valuable contribution. Pollock dripped paint from a can onto a vast canvasses rolled out across the floor. The analysis of filmed sequences of his painting style reveals that after twenty seconds of the dripping process a fractal pattern with a low-dimensional value would be established on the canvas. Pollock continued to drip paint for a period lasting up to six months, depositing layer upon layer, and gradually creating a highly dense fractal pattern. As a result, the D value of his paintings rose gradually as they neared completion, starting in the range of 1.3–1.5 for the initial springboard layer and reaching a final value as high as 1.9 [15].

Whereas the fractal analysis of Pollock's paintings represents a novel application of the box-counting technique, it is a well-established approach for extracting the D value for natural and computer generated fractals. In particular the D values for many natural objects are well known and have been adopted for the analyses performed here. Here, we examine whether the aesthetic appeal of fractals depends specifically on how the fractals are generated. To determine if there is any systematic difference in the aesthetic quality of fractals of different origin, we carried out a comprehensive study incorporating three categories of fractal pattern:

1. *Natural* fractals—scenery such as trees, mountains, waves, etc.
2. *Mathematical* fractals—computer simulations of coastlines.
3. *Human* fractals—cropped sections of paintings by the artist Jackson Pollock that have recently been shown to be fractal [15].

To our knowledge, a formal investigation of the relationship between fractal dimension and aesthetic appeal for fractal images of natural and human origin has not previously been attempted. Ours is the first direct comparison of aesthetic appeal between fractals of different origin.

2. The present study

2.1. Materials

This study used a range of different fractal images in each category. All stimuli were digitised, scaled to identical geometrical dimensions and presented in achromatic mode. Detailed descriptions of the stimuli in each category are presented below.

2.2. Natural fractals

The natural fractal stimulus set consisted of 11 images of natural scenes with D values ranging from 1.1 to 1.9. The images used, and corresponding estimates of fractal dimension, are shown in Fig. 3.

2.3. Mathematical fractals (computer simulated coastlines)

For the images in this category, we used 15 computer-generated images of simulated coastlines with D values of 1.33, 1.50 and 1.66. There were five exemplars for each of the three different D values, as shown in Fig. 4.

2.4. Human fractals

Cropped images from Jackson Pollock's paintings, with D values of 1.12, 1.50, 1.66 and 1.89 were used as fractals in this category. There were 10 different exemplars for each D value, half of which are shown in Fig. 5.

Whereas mathematical fractals extend from the infinitely large to the infinitesimally small, physical fractals (those generated by nature and humans) are
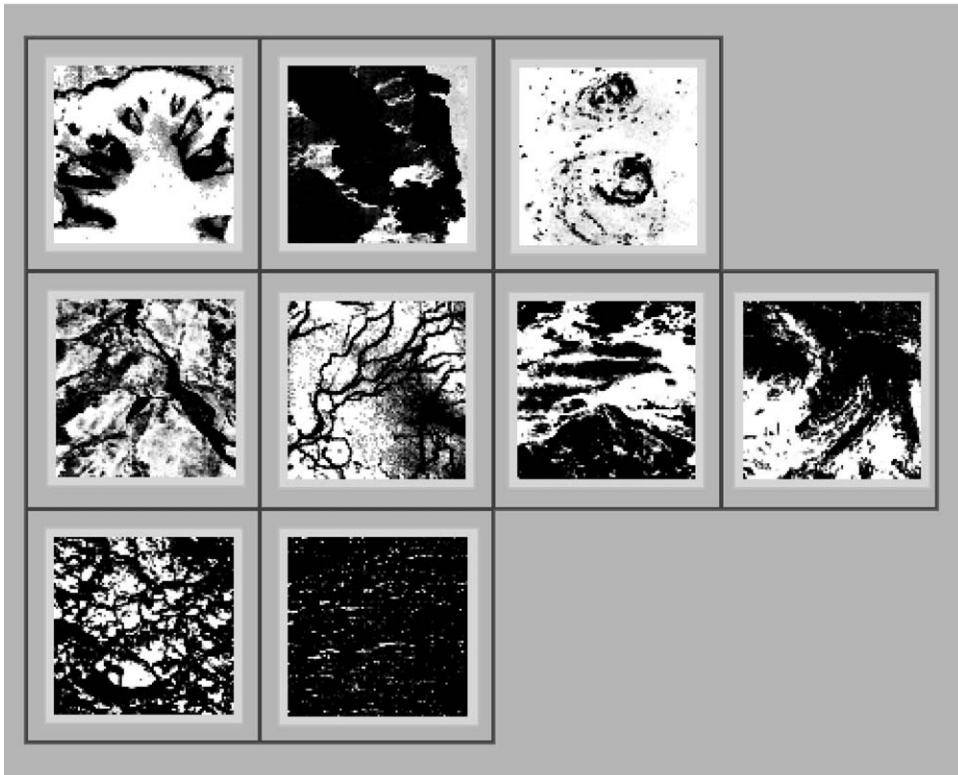
Fig. 3. Natural images and corresponding D-values used in the present study: (top row) cauliflower ($D = 1.1$), mountain ($D = 1.2$), stars ($D = 1.23$); (middle row) river ($D = 1.3$), lightning ($D = 1.3$), waves ($D = 1.3$), clouds (1.33); (bottom row) mud cracks ($D = 1.7$), tree branches (1.9).
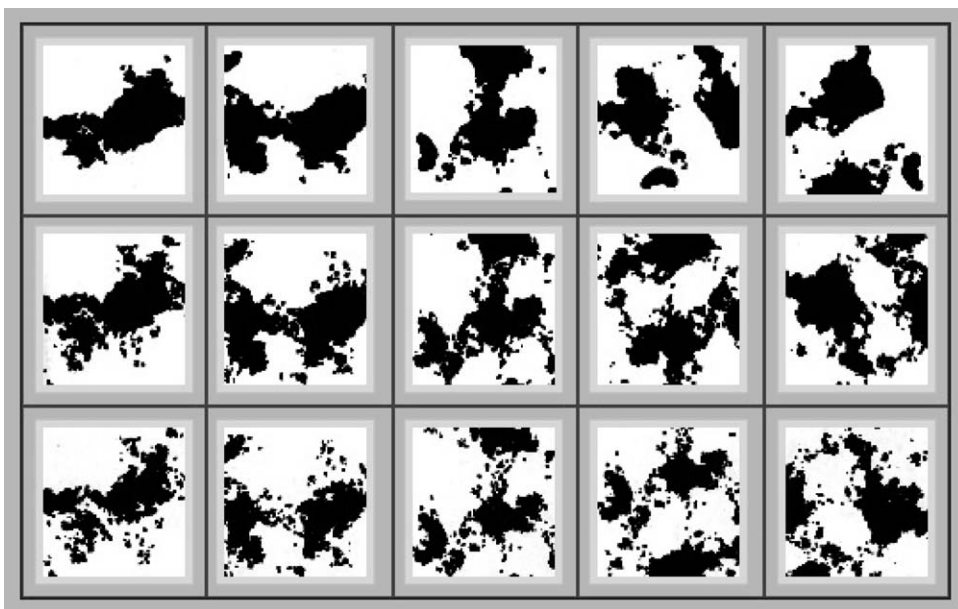

Fig. 4. Mathematical fractal images used in this study: simulated coastline images with D values of 1.33 (top row); 1.50 (middle row); and 1.86 (bottom row).
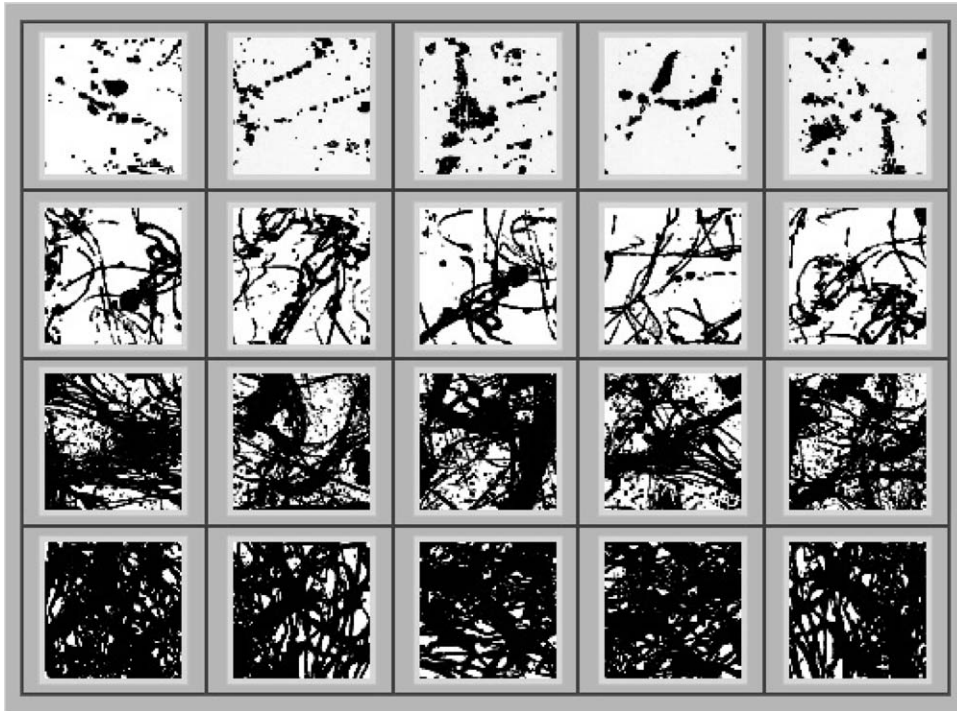
Fig. 5. Selection of cropped images of Pollock's paintings used in this study in the category of human produced fractals: (first row) five cropped images with a fractal dimension of 1.12, extracted from "Untitled", 1945 (private collection); (second row) five cropped images with a fractal dimension of 1.50, extracted from "Number 14", 1948 (Yale University Art Gallery, USA); (third row) five cropped images with a fractal dimension of 1.66, extracted from "Number 32", 1950 (Kunstsammlung Nordhein-Westfalen, Germany); (fourth row) five cropped images with a fractal dimension of 1.89, extracted from an unnamed work from 1950 that is no longer in existence (i.e. Pollock painted over this picture).

limited to a finite range of magnifications. Most physical fractals only occur over a magnification range where the smallest pattern is approximately 25 times smaller than the largest pattern [16]. Although this limited range does not make natural and human images any less fractal than the mathematical variety [17] it necessitates a certain care in the choice of the magnification range over which the images are presented. For reasons of consistency, we present all of our images (mathematical, human and natural) over a range limited by the range over which most physical fractals occur, i.e. in the images shown the smallest resolvable pattern is approximately 25 times smaller than the full image.

2.5. Procedure

Visual preference was determined using a forced-choice method of paired comparison. The method of paired comparison was introduced by Cohn [18] to study colour preferences and it is often regarded as the most adequate way of estimating value judgments. Participants indicated their aesthetic preferences between the two images appearing side-by-side on a monitor. Each

image was paired with every other in the group and each pair of images was presented five times. In different stages of our analysis these comparison groups consisted of fractal images with either identical or different D values. The presentation order was fully randomised and the preference was quantified in terms of the proportion of times each image was chosen.

As a part of the pilot stage, visual preferences for the simulated coastline images were compared separately for each fractal dimension. Each comparison group consisted of patterns with identical D value. This process was repeated for the images from Pollock's paintings. After this initial stage, representative images for each fractal dimension within these two categories were selected for comparison across fractal dimensions. We decided to use three different criteria for this selection: (1) the most preferred image within each fractal dimension; (2) the two images which received ratings closest to the median for each fractal dimension; and (3) the least preferred image within each fractal dimension.

Subsequent to this selection, separate experiments were conducted which compared visual preference for images selected on the basis of these three criteria across

different fractal dimensions within each category of fractal image. For determining the visual preference among the natural images the initial stage of comparing the exemplars with identical D value among themselves was not used and the nine natural images (show in Fig. 3) were directly compared by each image being paired with every other image in the group.

2.6. Participants

A total of 220 University of New South Wales undergraduate volunteers participated in the experiments. Approximately 12–16 observers participated in each condition.

3. Results and discussion

Fig. 6 shows the results obtained in our study. Each panel depicts the proportion of preferences as a function of fractal dimension for images of a particular origin. The top panel shows the pattern of preferences amongst natural images, the middle panel amongst simulated coastlines, and the bottom for a range of representative images from Pollock's paintings. The data shown for the simulated coastlines and Pollock's images compare the images which received the median ratings within each fractal dimension (from the pilot stage). Comparison between the images selected on the basis of the two other criteria, i.e. the most preferred and the least preferred images for each fractal dimension, show the same trend and data are not shown. The three panels reveal a consistent trend for aesthetic preference to peak within the fractal dimension range 1.3–1.5 for the three different origins of fractal image. Taken together, the results indicate that we can establish three ranges with respect to aesthetic preference for fractal dimension: 1.1–1.2 low preference, 1.3–1.5 high preference and 1.6–1.9 low preference.

In order to demonstrate that the aesthetic preference observed with fractal images is indeed a function of fractal dimension and not simply a function of the density (area covered) of a particular image, we performed one additional analysis. We measured aesthetic preference among a set of computer generated random dot patterns with no fractal content but matched in terms of density to the low, medium and high fractal patterns. Fig. 7 shows that there was no systematic preference between these images as a function of their density.

In summary, our analysis extends previous studies that have concentrated on only one category of fractals [15,12] by demonstrating an aesthetic preference for a particular fractal dimension across images of distinctly different origins. Given that fractals define our natural environment, identification of the fractal characteristic
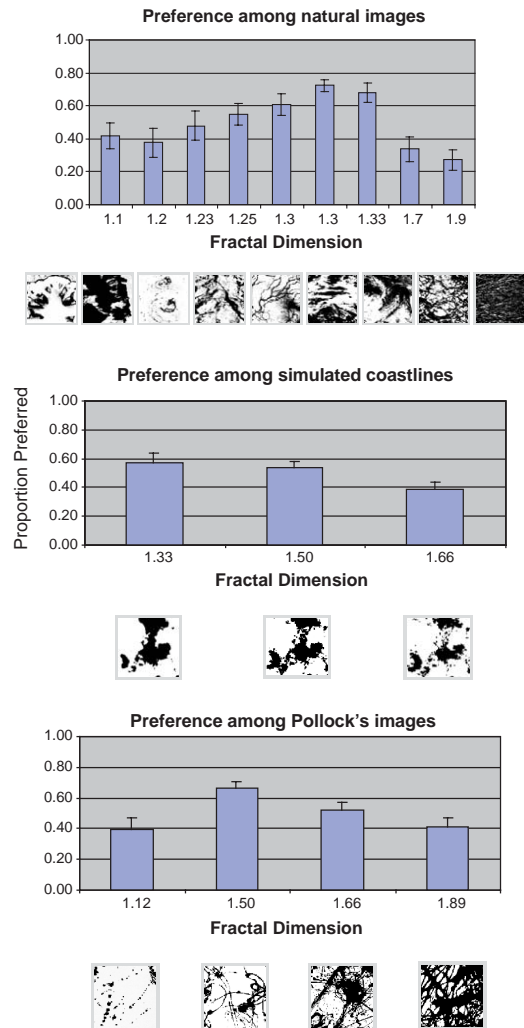

Fig. 6. Aesthetic preference for fractal images of different origin: average proportion by which the image was preferred among others as a function of fractal dimension for natural images (top panel); simulated coastlines (middle panel); and cropped images of Pollock's paintings (bottom panel).

determining aesthetic preference could be of fundamental importance in understanding the way in which our perception in general and our appreciation of art in particular are shaped by the world around us.

Our study is in line with the majority of previous studies of aesthetics of fractals that have chosen to consider the fractal scaling parameter D. However, there are other parameters that can be used in assessing the qualities of a fractal pattern. For example, Aks and Sprott investigated the effect of Lyaponov exponent (quantifying the dynamics that produce fractal patterns) on visual appeal [11]. Another important parameter is the Lacurnarity, which assesses the spatial distribution
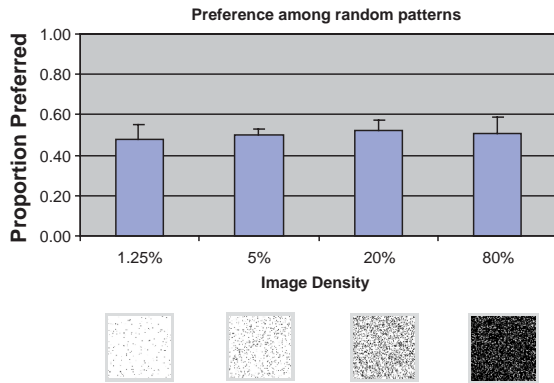
Fig. 7. Aesthetic preference for control random images: average proportion by which the image was preferred among others as a function of image density.

of the fractal pattern at a given magnification. We regard our investigations as preliminary and hope that this work will encourage further work aimed at investigating the impact of various parameters on visual preference.

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