OIMT 2015 - Open Exam II

Name:____

School: ____

Would you like us to post your score online with your first initial and last name?
(please select one): yes no

Instructions: The exam lasts TWO AND A HALF HOURS. Calculators are NOT allowed. There are three parts to this exam.

- PART I consists of 12 multiple choice questions.
  Scoring: 4 points for each correct answer.
- PART II consists of 8 problems with single number answers.
  Scoring: 6 points for each correct answer.
- PART III consists of 4 longer problems.
  Scoring: 12 points for each correct answer. Partial credit may be given, so you should show your work.

Each part starts with easier problems and ends in harder ones, so consider moving on to the next part if you find you are getting stuck on the later questions. We recommend you spend no more than 50 minutes on each part.

	Score
Part I	
Part II	
Part III	
Total	

PART I.

Write your **letter choice** in the provided answer box.

1. Find the perimeter of a regular hexagon that is inscribed in a circle of radius 6m.

   Answer $\square$

   (a) 24 m
   (b) 36 m
   (c) $24\sqrt{3}$ m
   (d) $36\sqrt{2}$ m
   (e) $36\sqrt{3}$ m
   (f) None of the above

2. Express $\dfrac{1\cdot 2\cdot 3 + 2\cdot 4\cdot 6 + 4\cdot 8\cdot 12 + 7\cdot 14\cdot 21}{1\cdot 3\cdot 5 + 2\cdot 6\cdot 10 + 4\cdot 12\cdot 20 + 7\cdot 21\cdot 35}$ as a simple fraction in lowest terms.

   Answer $\square$

   (a) $\frac{2}{3}$
   (b) $\frac{3}{5}$
   (c) $\frac{1}{2}$
   (d) $\frac{2}{5}$
   (e) None of the above

3. If $f(x) = 3x - 7$ and $g(x) = x^2 - 4$, what is $f(g(f^{-1}(5)))$?

   Answer $\square$

   (a) 27
   (b) 42
   (c) 54
   (d) 173
   (e) None of the above

4. A 180 m long train is traveling at a constant speed. Suppose it takes the train 90 seconds to pass by a fixed signal. Traveling at the same constant speed, the train passes through a tunnel that is 360 m long. How much time passes from the instant the front end of the train enters the tunnel to the instant the back end of the train exits the tunnel?

   Answer $\boxed{}$

   (a) 180 seconds
   (b) 270 seconds
   (c) 360 seconds
   (d) 450 seconds
   (e) None of the above

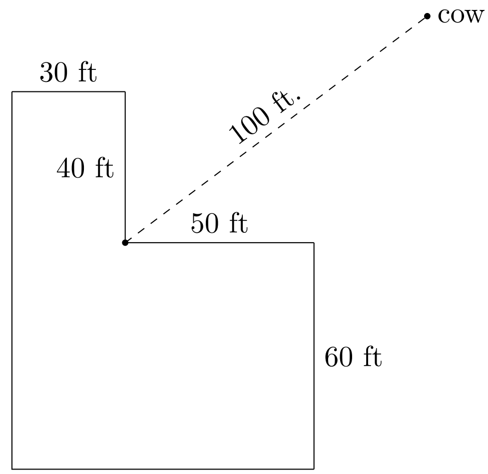
5. Which answer correctly describes all of the solutions to the following equation?

$$\log(x+2) + \log(x-3) = \log(14x) + \log(1/x)$$

   Answer $\boxed{}$

   (a) -4
   (b) 4
   (c) 20
   (d) 5
   (e) Both -4 and 5
   (f) None of the above

6. A cow is tethered on a grass field by a 100 ft rope attached to the inside corner of an L-shaped building, as shown below. Find the area of grass that the cow can graze.



Answer $\boxed{}$

(a) 4250π ft^2
(b) 17000π ft^2
(c) 240π ft^2
(d) 3650π ft^2
(e) 2150π ft^2
(f) None of the above

7. Arrange the numbers 2^{300}, 3^{200} and 5^{120} in increasing order.

Answer $\boxed{}$

(a) $5^{120} < 3^{200} < 2^{300}$
(b) $2^{300} < 5^{120} < 3^{200}$
(c) $5^{120} < 2^{300} < 3^{200}$
(d) $2^{300} < 3^{200} < 5^{120}$
(e) $3^{200} < 5^{120} < 2^{300}$
(f) None of the above

8. Three snails, Alice, Bobby and Cindy, were racing down a road. Whenever one snail passed another, it waved at the snail it passed. During the race, Alice waved 3 times and was waved at twice. Bobby waved 4 times and was waved at 3 times. Cindy waved 5 times. How many times was she waved at?

   Answer $\boxed{}$

   (a) 7
   (b) 3
   (c) 6
   (d) 1
   (e) 2
   (f) None of the above

9. Ben leaves for school 5 minutes after his sister. He follows the same route as her but walks one and a half times faster. If Ben's sister is walking at a constant speed, how long will it take him to catch up to her?

   Answer $\boxed{}$

   (a) 2 minutes and 30 seconds
   (b) 5 minutes
   (c) 7 minutes and 30 seconds
   (d) 10 minutes
   (e) None of the above

10. If $\sqrt{15+\sqrt{216}} = x + \sqrt{y}$, where x and y are integers, what is $x + y$?

   Answer $\boxed{}$

   (a) 7
   (b) 9
   (c) 10
   (d) 12
   (e) 15
   (f) None of the above

11. What are the last four digits of the number 5^{1000} ?

Answer $\boxed{}$

(a) 3125
(b) 5625
(c) 0625
(d) 8125
(e) None of the above

12. Two congruent squares ABCD and EFGH with sides of length 10 cm are placed on the plane so that the vertex E of the second square coincides with the center of square ABCD, and the side EF passes through A. Find the area of the part of the plane covered by these squares.

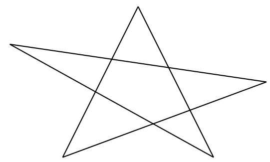
Answer $\boxed{}$

(a) $12\sqrt{2}$ cm^2
(b) 100 cm^2
(c) 150 cm^2
(d) $100 + 50\sqrt{2}$ cm^2
(e) 175 cm^2
(f) None of the above

PART II.

Write your **answer** in the provided answer box.

1. What is sum of the angles (in degrees) at the five points of a pentagonal star?



Answer = $\boxed{}$

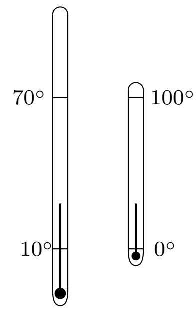
2. For how many integers n between 1 and 200 (inclusive) is n^n a perfect square?

Answer = $\boxed{}$

3. Sam was rowing upstream in a boat and, while going under a bridge, dropped his water bottle. He noticed this 20 minutes later, and immediately turned around and rowed back to fetch it. He reached the bottle 2 miles downstream from the bridge. If Sam rows at a constant rate and the current moves at a constant speed, what is the speed of the current in miles per hour?

Answer = $\boxed{}$

4. Two thermometers are hanging next to each other. As shown in the picture, the $0°$ mark on the small thermometer is aligned with the $10°$ mark on the big thermometer, while the $100°$ mark on the small one is aligned with the $70°$ mark on the big one. What is the temperature when the mercury in the two thermometers is at exactly the same horizontal level?

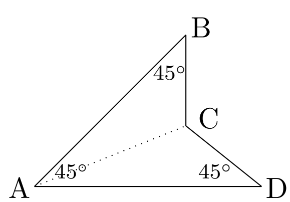


Answer = $\boxed{}$

5. Find an integer n so that $\dfrac{1}{n} < \sqrt{2015} - \sqrt{2014} < \dfrac{1}{n-1}$.

Answer = $\boxed{}$

6. The length AC in the following quadrilateral ABCD is 16 cm.



What is the area of the quadrilateral?

Answer = $\boxed{}$

7. Given that $a + a^{-1} = \frac{7}{3}$, find the value of $a^3 + a^{-3}$. Express your answer as a simple fraction in lowest terms.

Answer = $\boxed{}$

8. Find the smallest positive integer n such that the number $111 \cdots 11$ (the digit 1 repeated n times) is a multiple of 999.

Answer = $\boxed{}$

PART III.

Write your solution in the provided space, and **show your work**.

1. How many 7-digit numbers have the property that each digit (except the units digit) is strictly greater than the digit to its right?

2. Find the smallest possible perimeter of a triangle with one vertex on the x-axis, another on the line $y = x$, and the third at the point $(2, 1)$.

3. Let $f(n)$ be a function that satisfies the following three conditions for all positive integers n:

   - $f(n)$ is a positive integer;
   - $f(n+1) > f(n)$;
   - $f(f(n)) = 3n$.

   Find $f(2015)$.

4. Find integers m and n such that

$$\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right) = \frac{m}{n}.$$