Oregon Invitational Mathematics Tournament

Algebra 2 level exam
May 16, 2015

Name:____

School: ____

Would you like us to post your score online with your first initial and last name?
(please select one): yes no

Instructions:
There are two parts to this exam.

- Part I consists of 15 problems with numerical answers. Each is worth 4 points. Only your final answer will be graded. There will be NO partial credit given. You must write your answer in the place provided.
- Part II consists of 6 problems. Each is worth 10 points. Partial credit may be given for these questions. Each question requires a fully written-up solution. You should clearly write out each of your steps and provide justification when necessary. A correct answer without explanation will receive no credit.

Calculators are not allowed.

	Score
Part I	
Part II	
Total	

PART I: 4-Point Problems:

(1) To convert from degrees Celsius to Fahrenheit, you multiply by $9/5$ and then add 32. If T degrees Celsius is the same as T degrees Fahrenheit, what is the value of T?

Answer:

(2) The product of the ages of two friends is 1140. If they are both less than 40 years of age, how old are they?

Answer:

(3) The sum of two numbers is 165 and their product is 2015. Find the sum of their squares.

Answer:

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(4) To number pages in a book 1000 digits were used. How many pages are in the book if we start numbering from page three and number it as page 3?

Answer:

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(5) Find all real solutions of the equation $(x^2+3)^2-4x^2-24=0$.

Answer:

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(6) Find the number of integers in $1, 2, \cdots, 300$ that are divisible by 3,5, or 7.

Answer:

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(7) For what value(s) of c will the equation $\dfrac{1}{x(x-c)} = \dfrac{1}{1-c}$ have exactly one solution?

Answer:

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(8) Given $f(x) = \dfrac{x+1}{x+2}$, find the x value in terms of a for which $f(x) = 1 - \dfrac{1}{a}$.

Answer:

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(9) Find the point (a, b) on the line $y = 2x - 1$ which is closest to the point $(0, 1)$.

Answer:

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(10) A polynomial $P(x) = x^3 + ax^2 + bx + c$ satisfies $P(1) = 1^2$, $P(2) = 2^2$ and $P(3) = 3^2$. What is $P(4)$?

Answer:

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(11) A new operation $\otimes$ satisfies the following equations.

- $0 \otimes b = b + 1$
- $(a+1) \otimes b = a \otimes (a \otimes b)$ for all $a \geq 0$.

What is the value of $4 \otimes 5$?

Answer:

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(12) Find the smallest 4 digit number $abcd$ (for example 2375), where a, b, c, d are all distinct integers between 1 to 9 (inclusive) such that the two digit numbers ab, ac, ad, bc and cd are all prime.

Answer:

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(13) It takes Tim 4 hours to get from Seattle to Eugene. It takes Mary 6 hours to get from Seattle to Eugene. Aaron's speed is an average of Tim's and Mary's speeds. How long will it take Aaron to get from Seattle to Eugene?

Answer:

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(14) Simplify:

$z^{\frac{p-3}{p^2+3p}} \div z^{\frac{12}{9-p^2}} \cdot z^{\frac{3}{3p-p^2}}$

Answer:

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(15) Numbers a, b, and c are not equal to each other and $a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}$. Find $a^2b^2c^2$.

Answer:

PART II: 10-Point Problems:

(1) For any numbers x and y such that $x + y \neq 0$, define $x \oplus y$ as

$$x \oplus y = \frac{xy}{x + y}.$$

Show that

$$(x \oplus y) \oplus z = x \oplus (y \oplus z)$$

whenever both the expressions make sense.

(2) We know that for any numbers a and b, $f(a+b)+f(a-b)=2f(a)+2f(b)$.
Is it true that $f(x)$ is even (i.e. $f(x)=f(-x)$ for all values of x from the domain of $f(x)$)?

(3) For which real values of the parameter t does the equation

$$t^2x^2 + 2(t^2 - 2)x + 1 = 0$$

have a unique real solution x?

(4) A function from the non-negative integers to the non-negative integers satisfies

- $f(mn) = mf(n) + nf(m)$
- $f(15) = 60$
- $f(10) = 15$
- $f(12) = 12$

Find $f(9)$.

(5) Prove that for any integers $a_1, a_2, \cdots, a_7$, there exist a pair of them a_i and a_j $(1 \leq i, j \leq 7)$ such that $a_i + a_j$ or $a_i - a_j$ is divisible by 10.

(6) Prove the following for any $a, b > 0$:

$$\frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2}{\frac{1}{a} + \frac{1}{b}}.$$