Public Goods and the Dissolution of States

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Abstract

We present a simple model of the dissolution of states. We combine elements of the literature on the break-up and integration of states based on models in the fiscal federalism tradition, with elements of the literature that explains the determination of a state’s political institutions as a means to credibly promise redistributions in the face of costly, perhaps violent, redistributive conflict. We are able to characterize when the equilibrium involves the dissolution of a country, and when it involves continued unity. We are also able to explore some aspects of the linkages between political institutions and the determination of national boundaries.

Keywords: Dissolution of States, Fiscal Federalism, Redistributive Conflict

JEL classification: D02; D77; H77.

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1 Introduction

From 1815 to 2012 the number of sovereign states in the world has risen from 38 to 195. Since 1861 this increase has been almost monotonic (the exception being a brief small decline around the start of WWII).\(^1\) The average geographic size of a state has fallen monotonically since 1881, with average state size today being approximately half of what it was in 1881.\(^2\) There are several factors that explain these dramatic changes, such as the creation of states from "peripheral" territories, and the break up of European colonial empires with the subsequent establishment of new sovereign states.\(^3\) However part of the explanation for these important changes lies with the dissolution of pre-existing sovereign states. Examples include Czechoslovakia into The Czech Republic and Slovakia, Yugoslavia into Serbia, Bosnia and Herzegovina, Croatia, Montenegro, Slovenia, Macedonia, and Kosovo, Ethiopia into Ethiopia and Eritrea, India into India and Pakistan, and Pakistan itself into Pakistan and Bangladesh, and very recently Sudan into Sudan and South Sudan.\(^4\) These trends towards a greater number of smaller sovereign states appear to be sufficiently pronounced so as to constitute stylized facts. In this paper we provide one explanation for these stylized facts.

The traditional literature on the size and number of states treats individuals as having heterogeneous but single-peaked preferences over a public good. The individuals’ most preferred provision levels are then distributed on the unit line (or circle) according to some continuous distribution function. The literature thus employs a spatial competition model of the Hotelling (1929) or Salop (1979) type. See for example Bolton, Roland and Spolaore (1996) or Alesina and Spolaore (2003) for nice reviews of this literature. The key element in this literature is that states are treated as clubs in the tradition of Tiebout (1956) and Buchanan (1965), the advantage to greater size is the sharing of a fixed cost, but the disadvantage is that the level of provision or mix of public goods supplied is further from the marginal citizens most preferred allocation.\(^5\) That is, there are costs to heterogeneity that increase with size. One of the key questions addressed in this literature is; how does the optimal size of a nation (and therefore number of nations), compare to the size achieved under democracy or a Leviathan? That is, how does state size relate to the political institutions under which states are determined? This too will be one of our focuses.

In this paper we shall adopt many of the features of the previous literature, but our analysis will differ in two important regards. The key observation that motivates our departure from

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\(^1\) We follow the practice in international law by defining a sovereign state according to the Montivedeo Convention of 1933, which defines a state by as having (a) a permanent population; (b) a defined territory; (c) government; and (d) capacity to enter into relations with the other states.


\(^3\) See Lake and O’Mahony (2004) for more details

\(^4\) New states currently being created from the dissolution of older states include Azwad in Mali, Palestine in Israel, Nagorno-Karabkh in Azerbaijan, The Republic of Somaliland in Somalia, The Western Sahara Free Zone in Morroco, Abkhazia and South Ossetia in Georgia, and Transnistria and Gagauzia in Moldova. Further there are currently separatist movements operating in 28 African countries.

\(^5\) See Sandler and Tschirhart (1997) for a review of club theory.
previous contributions, is that in many of the cases of the dissolution of a state, the populations of the various new sovereign states are *very different* from their previous partners on at least one important dimension. Either they are ethnically different; for example Sudan is primarily populated by individuals of Arabic decent, while South Sudan is populated by individuals who are ethnically from the Dinka, Nuer, Azande, Bari and Shiluk/Anwak tribes. They may perhaps be religiously different; India is predominantly Hindu while Pakistan and Bangladesh are predominantly Muslim. There may be linguistic differences; the Czechs and Slovaks speak different languages. We argue that these ethno-linguistic and cultural differences are sufficiently pronounced in many cases, such that we may usefully view populations as being divided into separate "tribes". Further, the differences in preferences between members of the two tribes are more important than any heterogeneity of preferences within a tribe. Hence, rather than modeling preferences as single peaked with the peaks distributed according to some continuous density function, we assume that the density is discontinuous and bimodal. Indeed for simplicity we shall assume that the two tribes are internally homogenous.

This leads us to our second point of departure from the established literature, which essentially determines state size by the indifference of a marginal individual between membership of one state and the next adjoining state, and the absence of the incentive for a new state to "enter" in a spatial competition sense. We instead think of individuals as belonging to tribes that act as single entities able to play as individual players in a non-cooperative game; in our analysis it is the equilibrium of this game that determines the number and size of nations. Given this, it seems natural to link the literature on the size of nations to that on the economic origins of democracy as developed by Acemoglu and Robinson (2000, 2001, 2006). In much of this literature there is redistributive conflict between different income classes, and the less affluent group pose a revolutionary threat to the wealthier group. The introduction of democracy is then a means by which the affluent group make credible redistributive promises to the poorer group, and by doing so avoid the cost of a revolution. In our analysis there will be redistributive conflict, but between the tribes rather than income classes, the conflict arises because higher taxes and expenditures on public goods will reallocate resources towards the tribe with a stronger preference for public goods. Rather than a threat of revolution as in the "Democracy" literature there will be the threat of a war of succession. We believe that this may capture some of the fundamental features of several episodes of the dissolution of states. While the model we develop is quite simple in structure it

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6It is also the case that Sudan is Muslim while South Sudan is primarily Christian, further prior to the dissolution of Sudan the North had attempted to impose Sharia Law on the South

7Up to this point our setup resembles that of Desmet, Le Breton, OrtúñO-Ortín and Weber (2011) quite closely. They consider a model of countries consisting of coalitions of regions, where the regions are internally homogeneous but are different from each other, that is coalitions of regions are heterogeneous and have thus to resolve issues of redistributive conflict. They then analyze coalitional stability under different voting rules. The key difference between their analysis and our own will be that they have tax rates determined by sincere majority voting by citizens, we allow for taxes and the boundaries of the state to be set strategically by tribes in response to the threat of conflict.

is amenable to a variety of interpretations. In the analysis the notion of a public good may be interpreted quite broadly; any expenditure or costly action by a government that promotes the ideological, religious, or ethnic interests of one tribe relative to the other may be thought of as providing a public good. Hence it follows that redistributive conflict is also subject to different interpretations, and can be considered shorthand for some cases of ethnic, cultural, and religious conflicts. For example part of the explanation for the conflict in and eventual dissolution of Sudan involved attempts by the Islamic north to impose Sharia Law on the non-Islamic south.

In many ways our set-up fits very nicely with some of the literature on the relationship between ethnic fractionalization and conflict. This literature tends to find that what is important for predicting conflict is a high level of polarization when the ”prize” is a public good such as the imposition of religious or cultural conformity, but when the prize is private then fractionalization predict conflict. In our analysis the prize is a pure public good, and it is conflict over determining the level of provision of this good that becomes more intense with an increase in polarization. This motivates many of our conclusions concerning the dissolution of states.

In the next section we present our basic model of the dissolution of states. In section 3 we consider the possibility that direct transfers between the tribes are feasible. In section 4 we allow for the devolution of the powers to tax and spend as an alternative to dissolution. In section 5 we introduce the possibility that one tribe may appropriate all of the endowment of the other tribe, and show that in some circumstances this can lead to violence in equilibrium. In section 6 we allow for ”enclaves” of one tribe to be left behind within the territory of the other tribe after dissolution, this allows us to consider some issues concerning the political institutions established after the break-up. In section 7 we discuss the implications of migration and assimilation for our analysis. Finally section 8 provides a concluding discussion and some suggestions for further analysis.

2 A Model of the Dissolution of Countries

In the following, we introduce our model environment, and the structure of the non-cooperative game developed to model the decision of countries whether or not to dissolve.

2.1 Model Environment

The economy is populated by two “tribes” indexed $i = 1, 2$. The tribes are internally homogeneous, and of respective sizes $N_i$. Each member of each tribe receives a unit endowment per period. The citizens of this economy derive utility from two goods; $x_i$ a composite private good, and $z$ a pure public good. We intend that the public good be interpreted quite broadly, so as to capture both the usual examples such as defense, laws and their enforcement, and also to capture elements of

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ethnicity, culture, language, or religion.\textsuperscript{10} \textsuperscript{11} The tribes are externally heterogeneous; they differ from one another in terms of Cobb-Douglas preferences over the pure public good. The utility function of a member of tribe $i$ is

$$u_i(x_i, z) = x_i^{1-\beta_i} z^{\beta_i}$$

We assume $\beta_1 = 1/2 + \varepsilon/2$ and $\beta_1 = 1 - \beta_2$, so $\varepsilon \in [-1, 1]$ is a measure of the distance between the preferences of the two tribes, where $\varepsilon > (\varepsilon) 0$ indicates tribe 1 has the stronger (weaker) preference for the public good.\textsuperscript{12}

The public good is supplied by a government that levies a proportionate tax $t$ on endowments and runs a balanced budget. Hence, the government budget constraint is

$$z = t \sum_i N_i$$

The assumptions of a simple constant common proportionate tax rate and balanced budget condition will obviously play an important role in this analysis. They should be interpreted as a shorthand for the sort of problems that arise when a government does not have a complete set of tax instruments at its disposal, and must deal with issues of redistributive conflict between different groups.\textsuperscript{13} With these assumptions the indirect utility function of a member of tribe $i$ with income $y_i = 1$ is then

$$v_i(t, 1, \sum_i N_i) = (1 - t)^{1-\beta_i} (t \sum_i N_i)^{\beta_i}$$

From equation (3) we can derive the first order conditions which in turn allow us to derive the most preferred tax rate of a member of tribe $i$, which is

$$t_i^* = \beta_i$$

Hence, the tribe with a relatively greater taste for the public good prefers a higher tax rate.\textsuperscript{14} \textsuperscript{15}

\textsuperscript{10}The results that follow do not hinge exclusively on their being a pure public good, but rather require that the government supplies a good to its citizens under conditions of decreasing cost. This may be a pure or impure public good, or even a publicly provided private good.

\textsuperscript{11}That apparently small cultural differences are very important to the groups concerned is illustrated by the almost surreal "hyphen war" between the Czechs and Slovaks over the spelling of the name of their country, with the former preferring Czechoslovakia and the latter Czecho-Slovakia.

\textsuperscript{12}Note that $\varepsilon$ may be thought of as the one dimensional version of Bossert, D’Ambrosio and La Ferrara’s (2011) similarity matrix.

\textsuperscript{13}Providing the public good is not a public bad to some members of the economy, then a government with a complete set of tax instruments could simply choose the efficient level of public good provision and finance the expenditure via personalized lump-sum transfers/taxes, unity, indeed global unity, would then seem to be the only equilibrium!

\textsuperscript{14}The preferences are clearly single-peaked in the tax rate

\textsuperscript{15}The use of Cobb-Douglas preferences implies that the most preferred tax rates do not depend upon endowments, this allows us to focus on redistributive conflict between tribes that is not conflated with conflict that derives from differences in the tribes’ incomes
To simplify notation we hereafter write \( \sum_i N_i \equiv N \) and \( (1 - \beta_j)^{1-\beta_i} (\beta_j)^{\beta_i} \equiv B_{ji} \) and \( B_{ji} = B_j = B_i \) if \( i = j \). Further our assumptions ensure \( B_j = B_i \geq B_{ji} = B_{ij} \).

### 2.2 The Simple Dissolution Game

We begin with a very simple version of our model, in which the key question addressed is whether, in equilibrium, the country will remain unified or divide along tribal lines. We model this question as an infinitely repeated noncooperative game \( \Gamma = (N, S, P, \delta, T) \), where \( N = \{N_1, N_2\} \) is the set of players with each tribe treated as a single unitary player, \( S = S_1 \times S_2 \) is the strategy space, \( P = \{P_1, P_2\} \) are the payoff functions, \( \delta \) is the discount factor, and \( T \leq \infty \) is the number of repetitions of the game.

We assume that, initially, the tribes are in a single country in which one tribe controls the government. Political institutions are represented in a very simple way. We assume that if the government pursues the interests of the majoritarian tribe then this corresponds to democracy. Whereas if it represents the minority, then it is an oligarchic regime.\(^{16}\)

Without loss of generality we assume that the tribe in government is tribe 1. In each period, if the tribes are united, the strategy set for the government, \( S_1 \), involves their choosing between; keeping the country united and setting the tax rate and public good provision levels, peacefully and costlessly dissolving the country into two along tribal lines, or engaging in appropriation. In the latter case tribe 1 appropriates all of tribe 2’s current endowment provided that the the country remains united for the period. If the country is divided, the game effectively ends and each tribe thereafter sets their own independent tax rate. If the game does not end through dissolution the country remains united and the strategy set for the tribe out of government, \( S_2 \), involves their potential responses to tribe 1 choosing unity or appropriation. In each case they may choose either acceptance or to engage in a secessionist civil war. Should tribe 2 choose acceptance the stage payoffs are realized and the game repeats, should they choose civil war the stage payoffs are realized the country dissolves and again the game effectively ends.\(^{17}\)

The extensive form game for one period of this decision making process is depicted in Figure 1.

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\(^{16}\)Clearly there are many interesting features of political institutions from which we are abstracting, and there is no operational difference between a majoritarian oligarchy or a democracy in our analysis.

\(^{17}\)We interpret the circumstances where the tribe out of government chooses to dissolve the union against the wishes of the tribe in government as a secessionist conflict (civil war). More broadly this represents the idea that if the dissolution of the country is forced by one tribe on another then this is more costly than mutually voluntary dissolution. The cost may follow from civil disobedience etc. rather than outright war.
Given the indirect utility functions of the players, we can now calculate their stage payoffs at each decision node in every period. If there is civil war (node 1) then in the period of violence the members of each tribe lose a $1-\gamma$ share of their endowment giving
\[ v_i(\beta_i, \gamma, N_i) = \gamma B_i(N_i) \beta_i \quad i = 1, 2 \] (5)

If the country remains unified (node 2), the per period payoffs for members of tribe $i$ are
\[ v_i(t_1, 1, N_i) = (1 - t_1)^{1-\beta_i} (t_1 N_i) \beta_i \quad i = 1, 2 \] (6)

If the country is dissolved (node 3), then the members of the tribes receive
\[ v_i(\beta_i, 1, N_i) = B_i(N_i) \beta_i \quad i = 1, 2 \] (7)
in every period of the game.\(^{19}\) If the tribe 1 choose appropriation and tribe 2 civil war (node 4), the

\[^{18}\text{The reader may be uncomfortable with the assumption that the destructiveness of a civil war is the same when tribe 1 first chooses unity and when they first choose appropriation. It might seem natural to suggest that destruction will be greater in the appropriation sub-game. We tend to agree, but since the appropriation sub-game is then less attractive to tribe 1 and is already not selected in either of the possible sub-game perfect equilibria, then this would not change the equilibria of the model qualitatively or quantitatively. But it will add algebraic complexity. We therefore omit this possible detail.}\]

\[^{19}\text{In the case of the dissolution of Czechoslovakia the two nations agreed to divide all public assets in the same proportions as their respective populations, with the Czechs being twice as numerous as the Slovaks receiving two}\]
first period payoffs of the members of the tribes are again given by expression (5) above. However, if tribe 1 choose appropriation (node 5) and tribe 2 do not rebel then the payoffs become

\[ v_1(\beta_1, N/N_1, N) = \frac{B_1 N}{N_1^{1-\beta_1}} \]

\[ v_2(\beta_2, 0, N) = 0. \] (8)

Since the game in non-stochastic, and since we are only considering pure strategies, it follows that if a decision node is reached in any period of the game is also reached in each subsequent period. Hence, we can write the continuation value for a member of tribe \( i \) at the decision nodes (1,2,3,4,5) as follows.

1. \( V_i(\beta_i, 1, \gamma, N_i) = \gamma B_i(N_i)^{\beta_i} + \frac{\delta B_i(N_i)^{\beta_i}}{1-\delta} \quad i = 1, 2 \)
2. \( V_i(t_1, 1, N) = \frac{(1-t_1)^{1-\beta_i}(t_1 N_i)^{\beta_i}}{1-\delta} \quad i = 1, 2 \)
3. \( V_i(\beta_i, 1, N_i) = \frac{B_i(N_i)^{\beta_i}}{1-\delta} \quad i = 1, 2 \)
4. \( V_i(\beta_i, 1, \gamma, N_i) = \gamma B_i(N_i)^{\beta_i} + \frac{\delta B_i(N_i)^{\beta_i}}{1-\delta} \quad i = 1, 2 \)
5. \( V_1(\beta_1, N/N_1, N) = \frac{B_1 N}{(1-\delta)N_1^{1-\beta_1}} \)
   \( V_2(\beta_2, 0, N) = 0 \)

2.3 Equilibrium

The continuation values of the game allow us to find the equilibrium.

We note that a subgame perfect equilibrium cannot involve civil war; were tribe 1 to choose unity followed by the choice of civil war by tribe 2, then tribe 1 would be better off choosing dissolution. It follows that if tribe 1 choose unity, then they will set the tax rate that makes tribe 2 indifferent between civil war and unity. This No Civil War Condition, (NCW) defines \( t_1 \) as satisfying

\[ \frac{(1-t_1)^{1-\beta_2}(t_1 N_i)^{\beta_2}}{1-\delta} \geq \left[ \frac{\gamma(1-\delta) + \delta}{1-\delta} \right] B_2 \omega_2^{\beta_2} \] (9)

where \( \omega_2 = \frac{N_2}{N} \) is the share of tribe 2 in total endowment. Proposition 1 now defines the equilibrium of the game.

**Proposition 1** For the noncooperative game \( \Gamma = (N, S, P, \delta, T) \) a subgame perfect equilibrium \( \exists \) and consists of strategies \( s = (s_1, s_2) \in S \) which may take one of two forms dependent on parameter values; either tribe 1 chooses to set taxes which satisfy the NCW condition and the country remains unified, or they choose dissolution and the game ends.

thirds of state assets.
Proof The proof of this and subsequent propositions and lemmas may be found in the appendix.

We represent the equilibria graphically in figures 2a and 2b. In each the black line represents the trade-off frontier between the two tribes continuation values as the tax rate varies from zero to one with the two tribes unified. The intersection of the red lines defines the no civil war constraint. The tangencies between the green lines and the continuation values trade-off frontier gives the payoffs associated with each tribes most preferred tax rate.

Figure 2a: Dissolution
Figure 2b: Unity

Figure 2a depicts the case in which tribe 1 chooses dissolution, because tax concessions to tribe 2 would make tribe 1 worse off than it would be after dissolving the country. Figure 2b shows the case in which tax concessions make tribe 1 better off than dissolution, hence the country remains united.

Our proof of proposition 1 (see the appendix) tells us that for certain extreme values of the parameters each of the possible equilibria may be realized. To provide a more comprehensive division of the parameter space we employ numerical methods; these reveal regions where the equilibrium involves unity and regions where it involves dissolution, a typical set of simulation results is represented in Figure 3.

The simulations were performed using Mathematica, the actual programs used are available from the authors on request. For Figure 3 we set $\gamma = 0.9$, $\delta = 0.9$ and $n=1,000,000$, the general properties reported are robust to very wide variations in the simulation parameters. That the lines for $\gamma = 0.1$ and $\gamma = 0.5$ terminate before reaching $\omega_2 = 1$ in the lower panel is an artifact of Mathematica’s root finding algorithm failing to converge at the specified level of precision, and has no economic meaning.
The simulations show that the equilibrium tends to involve the dissolution of the country if $\varepsilon$ is absolutely large, that is if the tribes have quite different preferences over the ideal tax rate and thus level of public good provision; furthermore this is true whichever tribe has the greater taste for the public good. Mechanically in our model this is quite intuitive. If $\varepsilon$ is absolutely large the tax concessions required to avoid civil war must be large, that is they must be further from the most preferred tax rate of tribe 1, and this causes tribe 1 to choose dissolution.

Alternatively expressed, when $\varepsilon$ is large, then for a given tax rate tribe 2 will choose civil war for a greater range of parameter values (see expression (9)), which we might term a greater threat of conflict. Dissolution is then chosen to avoid conflict. There is a considerable literature that explores the links between the likelihood of conflict and various measures of ethno-linguistic fractionalization or polarization. Many studies find that standard indexes of ethno-linguistic fractionalization struggle to adequately explain conflict while indexes of polarization are better predictors of conflict (see for example Montalvo and Reynal-Querol (2005)). However, recent careful work by Esteban, Mayoral and Ray (2012) provides further insights into these linkages. They find that how well fractionalization relative to polarization explains conflict depends on the degree to which the prize in the conflict is public or private. When the prize is public, and there is strong cohesion within groups, then polarization indexes do a good job of explaining conflict. However, when the prize is private, and again within group cohesion is high, then fractionalization indexes explain conflict well.\textsuperscript{21} Our analysis involves a model where the prize is public, groups are cohesive, and are highly polarized. Indeed, in our analysis, a larger absolute value of $\varepsilon$ implies both a greater threat of conflict for any given tax rate, and larger values of Bossert, D’Ambrosio and La Ferrara’s (2011) Generalized

\begin{figure}[!h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{A Typical Division of the Parameter Space}
\end{figure}

\textsuperscript{21}Examples they give of public prizes include ”ideological or religious supremacy, or political power” while private prizes include ”the capture of oil resources or mining revenues”. In their work they also find that the Greenberg-Gini coefficient explains conflict well when group cohesion is low - albeit with the opposite of the predicted sign
Index of Fractionalization (GELF) and Esteban and Rays’ (1994) Polarization Index (PI). Indices which include some notion of the distance between the preferences of ethno-linguistically different individuals. However, it does not imply any increase in the standard Ethno-Linguistic Fractionalization Index (ELF) of Easterly and Levine (1997), or in Montalvo and Reynal-Querol’s (2005) Index of Ethnic Polarization (IEP), neither of which include any notion of the distance between the preferences of ethno-linguistically different individuals.\textsuperscript{22} Our analysis thus provides a theoretical underpinning for the results on the relationship between polarization and conflict.

The simulations also indicate that the implications of the relative sizes of the two tribes (and hence relative endowments) differ depending on whether the tribe making the initial decision has the stronger or weaker preference for the public good. There are two countervailing effects of the size of the tribe not making the dissolution decision (tribe 2) on that decision. First, the larger is tribe 2 the greater must be the tax concession made to them to satisfy the no civil war constraint. This tends to promote dissolution. However, second, the larger is tribe 2 the more valuable are they to tribe 1 in contributing to the tax base that funds the public good. This tends to promote unity. If the two tribes are not too similar, which in our simulations means $|\varepsilon| > 0.65$, and if tribe 1 has relatively stronger preference for the public good, that is when $\varepsilon \in (0, 1]$, then the tax base effect dominates and dissolution is preferred as tribe 2 becomes sufficiently small. However when tribe 1 has the relatively weaker preference for the public good, that is when $\varepsilon \in [-1, 0)$, the effect working through the no civil war constraint dominates and dissolution is preferred as tribe 2 becomes sufficiently large.

Notice also that the greater is the relative size of the tribe not setting taxes, $\omega_2$, the greater is the tendency towards conflict in the sense that the no civil war constraint is satisfied for a smaller set of parameter values (again see expression (9)). This relationship is monotonic, but the implications for whether or not dissolution will occur depends on the specific relative preferences of the two tribes, that is whether $\varepsilon \gtrless 0$. This implies that the relationship between ELF and the likelihood of dissolution is quite complicated. For any given $\varepsilon$, ELF, GELF, PI and the IEP increase as $\omega_2 \to 1/2$ either from above or below. In the case of $\varepsilon > 0.65$, and where $\omega_2 \uparrow 1/2$, this promotes unity in the sense that the equilibrium may switch from dissolution to unity, but if $\omega_2 \downarrow 1/2$ this promotes dissolution in the sense that the equilibrium may switch from unity to dissolution. When $\varepsilon < -0.65$ then these conclusions are precisely reversed.

Further, the simulations also reveal that the area of the parameter space involving dissolution grows larger as $\gamma$ increases. That an increase in $\gamma$ might increase the likelihood of dissolution is unsurprising as it implies that a civil war would be less destructive, making tribe 2 more willing to adopt this strategy, with the implication that the ruling tribe needs to make a greater tax concession to the other tribe to avert conflict. It follows that they are more likely to choose dissolution. The circumstances where $\gamma$ might be larger include when the two tribes are already

\textsuperscript{22}The IEP measures the distance of the distribution of the population from two internally homogeneous identically sized groups.
largely segregated into separate areas. Surprisingly this appears to have been the case with the
dissolution of Czechoslovakia, where in the first census after dissolution it was revealed that the
Slovak minority in the Czech republic was just below 2% and the Czech minority in Slovakia was
only 0.8%. So either the groups were geographically largely segregated at the time of dissolution
or migrated quite rapidly thereafter.

It is interesting to speculate that the decisions such as that of the Israelis to locate Jewish
settlers in Palestinian areas might be interpreted as an attempt to lower $\gamma$ and hence discourage
the dissolution of Israel into separate Jewish and Palestinian states. It is also interesting to note that
the efforts by external supporters to provide financial support or even arm compatriots potentially
engaged in a separatist conflict, in as much as this lowers $\gamma$, actually tends to be counterproductive.

2.4 Equilibrium and Democracy

The conclusions reached in the preceding section are based on the assumption that one tribe makes
the taxation and public good provision decisions when faced by a threat of a secessionist civil war by
the other tribe. Suppose instead that we consider the possibility that the tax rate is democratically
determined, and that voting is strategic. There are then at least two interesting questions that
arise; would a switch from oligarchy to democracy tend to promote unity or dissolution, and,
if decision making is democratic, how might differential changes in the population sizes of the
two tribes effect the boundaries of the country? We investigate these questions by considering the
economically interesting case of $|\varepsilon| > 0.65$.

Consider first the introduction of democracy when the regime is originally autocratic, that is
tribe 1 are in the minority, and let $|\varepsilon| > 0.65$, to give the two cases depicted is figures 4a and 4b.

\begin{figure}[h]
\centering
\begin{subfigure}{0.45\textwidth}
\includegraphics[width=\textwidth]{figure4a.png}
\caption{Democratization with $\varepsilon > 0$}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\includegraphics[width=\textwidth]{figure4b.png}
\caption{Democratization with $\varepsilon < 0$}
\end{subfigure}
\end{figure}

\textsuperscript{23} When voting is sincere our analysis does not add much extra to the insights gained by Desmet, Le Breton, Ortúñ-Ortín and Weber (2011).
In figure 4 the axis labelled $\omega_2$ must be reinterpreted as the relative size of the tribe not setting taxes, hence this represents a different tribe for the upper panel of the diagram (where it is tribe 2 as before) compared to the lower panel (where it represents tribe 1). For both 4a and 4b the line $A \rightarrow B$ captures the effect of switching from an autocratic regime with taxes set by tribe 1 to a democratic regime with taxes set by tribe 2. In 4a tribe 1 has the stronger relative preference for the public good, $\varepsilon > 0$, in 4b it has the weaker preference $\varepsilon < 0$. The introduction of democracy has a somewhat surprising implication in that it does not have a significant impact on the equilibrium of the game, irrespective of the relative preferences of the two tribes. This seemingly paradoxical finding has a simple explanation. When tribe 1 have the relatively greater preference for the public good, $\varepsilon > 0$, and when tribe 2 are large, $\omega_2 > 0.5$, then tribe 1 value the contribution of tribe 2 to the tax base sufficiently greatly such that they are willing to make the tax rate concessions necessary to ensure unity. With the introduction of democracy tribe 2 set taxes, but as they are large, and because tribe 1 have the stronger relative reference for the public good, then tribe 1 do not find a secessionist civil war very attractive, hence tribe 2 need only make a limited tax rate concession to tribe 1 to preserve the unity of the state and they find this optimal. Alternatively when tribe 1 have a weaker relative preference for the public good, as in Figure 4b, and are autocratic as at point A, then the value of a larger tax base is outweighed by the tax concessions they must make to keep the country unified and they thus choose dissolution. If the country were democratic taxes would be set by tribe 2, who also prefer dissolution but for the reason that they are relatively numerous and thus are not willing to suffer the decrease in tax revenues and public good provision necessary to prevent tribe 1 engaging in civil war.

Now suppose that the tax rate would be set democratically, and that the relatives size of the two tribes changes such that the median voter switches from being a member of tribe 1 to being a member of tribe 2. In our simulations the effects of such a change is numerically small. If tribe 1 has the stronger preference for the public good, then as the right to set taxes switches to tribe 2, the line dividing the parameter space between unity and dissolution displays a small downward discontinuity, that is the region of dissolution becomes slightly larger then it would have been should tribe 1 have continued to set taxes. If tribe 1 has the lesser relative preference for the public good then the line dividing the parameter space between unity and dissolution again displays a small downward discontinuity, but here this implies that the region of unity becomes slightly larger than it would have been should tribe 1 have continued to set taxes.

Clearly one of the fears of the indigenous populations in several European nations is that large rapidly growing immigrant populations will significantly change the nature of public good provision and perhaps the political structures in those states. The simulations above suggest these fears are

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24 This representation of the numerics is not ideal, but was necessary because of the structure of the simulation programs in Mathematica.

25 Diagrams and simulation programs for these exercises are available on request from the authors, but were judged not to warrant committing additional space to in the paper.
largely unfounded.

3 Transfers

It may be argued that one response by a government faced with the problem of redistributive conflict embedded in the taxation public good expenditure scheme modeled above is to introduce redistributive transfers. It follows immediately from the Le Chatelier Principle that the introduction of transfers cannot lower the payoff of the ruling tribe in a united country. However, the payoffs after dissolution are necessarily unaffected as between-tribe transfers are unavailable in this circumstance.

It follows then that the introduction of transfers makes the region of the parameter space for which dissolution is the equilibrium smaller. Below we demonstrate that while the introduction of transfers mitigates the tendency to dissolve the country it does not eliminate it. This is perhaps intuitive. What may be less clear is the relationship between taxes and transfers in equilibrium.

Writing the per-person transfer from an individual of tribe 1 as $\tau$, then the per period payoffs of members of the two tribes in a united country become

$$v_1(t_1, \tau, 1, N) = (1 - t_1 - \tau)^{1-\beta_1} (t_1N)^{\beta_1}$$

$$v_2(t_1, \tau, 1, N) = \left(1 - t_1 + \left(\frac{N_1}{N_2}\right)\tau\right)^{1-\beta_2} (t_1N)^{\beta_2}$$

It follows that in the presence of transfers the no-civil-war condition must be rewritten

$$\left(1 - t_1 + \frac{N_1}{N_2}\tau\right)^{1-\beta_2} (t_1N)^{\beta_2} \geq (\gamma(1 - \delta) + \delta) B_2 N_2^{\beta_2}. \quad (12)$$

Maximizing (10) subject to (12) and manipulating the resultant first order conditions gives us the relationship between the optimal tax rate and level of transfers

$$t_1 = \frac{\beta_1^2 + (1 - \beta_1)^2 \frac{N_2}{N_1} + \epsilon \tau}{\beta_1 + (1 - \beta_1) \frac{N_2}{N_1}}. \quad (13)$$

Notice that if the ruling tribe has the stronger preference for the public good, $\epsilon > 0$, then the relationship between taxes and transfers is positive, whereas if they have the weaker preference this relationship is negative. In essence the ruling tribe wish to extract all rents from the other tribe up to the point where they are indifferent to engaging in a secessionist civil war. The method by which they do this depends on their relative preferences for the public good. When $\epsilon > 0$ there tends to be higher taxes and levels of public good provision accompanied by compensating transfers to satisfy the NCW constraint. When $\epsilon < 0$ the ruling tribe tend to set low taxes and public good provision levels and choose negative transfers to extract rents from the other tribe.

Our analysis strikes a note of caution for the literature on the relationships between polarization
and/or fractionalization and redistribution. First, it should be noted that redistribution may be
direct in terms of transfers, or indirect as a consequence of the tax and public good expenditure
decisions. Second, as the preference distance between the tribes becomes larger, that is \( \epsilon \) becomes
absolutely larger, then for any given tax rate transfers may rise or fall depending on the sign of \( \epsilon \). Hence both the distance between tribes matters, and what that distance represents in terms of
preferences.

If we write \( \Gamma' = (N, S', P, \delta, T) \) as the non-cooperative game with the expansion of the strategy
set to \( S' \) which includes the choice of transfers by tribe 1. We may immediately prove

**Proposition 2** For the noncooperative game \( \Gamma' = (N, S', P, \delta, T) \) a subgame perfect equilibrium \( \exists \)
and consists of strategies \( s' = (s'_1, s'_2) \in S' \) which may take one of two forms dependent on parameter
values; either tribe 1 chooses to set taxes and transfers which satisfy the NCW condition and the
country remains unified, or they choose dissolution and the game ends.

Intuitively we can see that this must be the case. As the two tribes become sufficiently similar
their most preferred tax rates and public good provision levels approach each other and hence the
tax base effect must dominate leading to the equilibrium involving unity. Alternatively, if the tribes
are sufficiently dissimilar, then the transfers from tribe 1 to tribe 2 necessary to satisfy the NCW
constraint become so large that they cost the ruling tribe more than the benefits of a larger tax
base and dissolution is preferred.

Note that in this structure the introduction of transfers never reduces the payoffs of the ruling
tribe or increases the payoffs of the other. This is intuitive, when the NCW condition binds the
payoffs to tribe 2 are always equal to their post civil war payoff \( \left( \gamma + \frac{\delta}{1-\delta} \right) B_2 N_2^{\beta_2} \). In the presence
of transfers tribe 1 has an instrument that always allows it to reduce the payoff of tribe 2 to this
level. Prior to the introduction of transfers whenever the NCW constraint did not bind, that is
when \( \left( \gamma + \frac{\delta}{1-\delta} \right) B_2 N_2^{\beta_2} < \frac{B_1 N_1^{\beta_1} - \delta}{1-\delta} \), then tribe 2 could enjoy rents that tribe 1 had no mechanism
to extract.

## 4 Devolution

A response that is often observed as an alternative to full dissolution is devolution of the powers to
tax and spend to the regions of a country. Here this implies that both tribes may independently set
their own tax rate, and thus level of public good expenditure. If devolution is costless then it can
easily be shown that in the game with devolution as a possible strategy for tribe 1 the existence of
the option to appropriate does not affect the set of equilibria associated with different parameter

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See for example Desmet, Ortúñoo-Ortuñ and Weber (2005)

The devolution of “tax-varying powers” to the Scottish parliament in Edinburgh in 1999 involved both a new set
of policy instruments and the establishment of a new political institution, namely a unicameral elected parliament
based in Holyrood, Edinburgh.
values. So, for simplicity, we modify the strategy space by excluding this option and adding the devolution option to the strategy set of tribe 1, $S_1$. The strategy set of tribe 2 remains as before such that the game becomes as shown in Figure 5.

![Figure 5: The Devolution Game](image)

The continuation payoffs associated with nodes 1-4 are just as described in section 2.2, where the payoff associated with appropriation followed by civil war is relabeled as devolution followed by civil war. The new option is where tribe 1 chooses devolution and tribe 2 accepts this (node 5 in Figure 5). In this case each tribe takes the contribution of the other to public good supply as given and chooses $t_i$ so as to maximize

$$v_i(t_i, 1) = (1 - t_i)^{1 - \beta_i} \left( \sum_i t_i N_i \right)^{\beta_i}$$

(14)

Performing the optimization problems and rearranging the resultant first order conditions we get the following best responses

$$t_i = \beta_i - (1 - \beta_i) t_{-i} \left( \frac{N-i}{N_i} \right)$$

(15)

It might be argued that the destructiveness of conflict under devolution or under unity would be different. It is unclear which should be the more destructive, but note that if conflict is more destructive following devolution then our results would be unchanged as this only serves to reduce payoffs along those branches of the game tree that are not chosen in equilibrium.
If there is devolution we may characterize the outcome of the tax setting game between to two tribes as follows

**Lemma 3** The stable Nash equilibrium in the tax setting subgame between the two tribes involves

1. \( t_i = \frac{\beta_i}{1-(1-\beta_i)\frac{N_i}{N-N_i}} \) \( \beta_i > \frac{\beta_i}{1-(1-\beta_i)\frac{N_i}{N-N_i}} \) \( i = 1, 2 \)
2. \( t_1 = \beta_1 \) and \( t_2 = 0 \) \( \beta_1 > \frac{\beta_2 N_2}{\beta_1 N_1} \) and \( \frac{\beta_1 N_1}{\beta_2 N_2} > \beta_2 \),
3. \( t_1 = 0 \) and \( t_2 = \beta_2 \) \( \beta_1 > \frac{\beta_2 N_2}{\beta_1 N_1} \) and \( \beta_2 > \frac{\beta_1 N_1}{\beta_2 N_2} \).

**Lemma 4** In a stable equilibrium of the devolution subgame the continuation payoffs to the two tribes involve

1. \( V_i(t_i, t_{-i}, 1, N_i, N_{-i}) = \frac{B_i(N)^{\beta_i}(1-(1-\beta_i)^{\frac{N_i}{N-N_i}})(1+\frac{N_i}{N-N_i})^{1-\beta_i}}{1-(1-\beta_i)^{\frac{N_i}{N-N_i}}} \) \( \beta_i > \frac{\beta_{-i} N_{-i}}{\beta_i N_i} \) \( i = 1, 2 \),
2. \( V_1(\beta_1, 1, N_1) = \frac{B_1(N_1)^{\beta_1}}{1-\delta} \) and \( V_2(0, 1, N_1) = \frac{(\beta_1 N_1)^{\beta_2}}{1-\delta} \) \( \beta_1 > \frac{\beta_2 N_2}{\beta_1 N_1} \) and \( \frac{\beta_1 N_1}{\beta_2 N_2} > \beta_2 \),
3. \( V_1(0, 1, N_2) = \frac{(\beta_2 N_2)^{\beta_1}}{1-\delta} \) and \( V_2(\beta_2, 1, N_2) = \frac{B_2(\beta_2 N_2)^{\beta_1}}{1-\delta} \) \( \beta_1 > \frac{\beta_2 N_2}{\beta_1 N_1} \) and \( \beta_2 > \frac{\beta_1 N_1}{\beta_2 N_2} \).

We are now able to examine when the option of devolution is an equilibrium in the game. Employing numerical methods we are able to show that the division of the parameter space remains exactly as described in figure 3 that is

**Proposition 5** There do not exist parameter values for which the subgame perfect equilibrium to the game involves devolution.

Superficially this result might appear quite counterintuitive. Casually we might think that devolution must always dominate dissolution, because each tribe could set the same tax rate in both circumstances yet when there is dissolution each enjoys the provision of the public good by the other. There are positive spillovers between the two regions under devolution, whereas under dissolution these spillovers do not exist. This argument neglects the fact that public good provision in the two regions involves strategic substitutes, that is there is an incentive for each tribe to free
ride on the provision of the public good by the other. We can develop some intuition on the failure of devolution to be an equilibrium by considering the limiting cases; as $\varepsilon \to 0$ the preferred tax rates and public good provision levels of the two tribes approach each other, such that under unity each experiences only a small deviation from their most preferred outcome, whereas if there is devolution there is still a significant amount of free riding behavior. In fact if the two tribes are of equal size it is straightforward to show that with unity $t_1 = t_1^* = t_2^* = 1/2$ whereas with devolution $t_1 = t_2 = 1/3$. When $\varepsilon \to 1$ (or $-1$) then tribe 2(1) are complete free riders in the tax setting subgame under devolution, hence tribe 1 are indifferent between devolution and dissolution.

The game in this section resembles quite closely the one currently being played over Scotland’s status within Great Britain. Currently the Scottish Parliament receives a block grant from the British Government in Westminster, and then sets its own spending priorities. However in 2014 this will change and Scotland will vote on some form of independence from Britain. While the exact nature of the questions on the referendum is yet to be formally determined, three options are being mooted, full independence, unity, and an option known as ”devo max” which is unity but with full devolution of the powers to tax and spend from the British Government in Westminster to the Scottish Parliament in Holyrood. The British Parliament are the first movers in this game as Westminster must transfer to Holyrood the powers needed to hold the referendum. The British Prime Minister David Cameron is on record of only being willing to transfer these powers if the choice is between independence and unity. That is he is ruling out devolution as an equilibrium in the game. Presumably the subsequent vote in Scotland between dissolution and unity will be respected in the British Parliament, since were a vote for dissolution not be respected the Scots might engage in some form of civil disobedience and perhaps even conflict. This seems very close to the structure and predictions of our simple model.

5 Appropriation and Civil War in Equilibrium

A simple criticism that may be leveled at the preceding analysis is that violence is never observed in equilibrium. However, amongst the set of countries that have actually experienced dissolution violence and civil war has been very common.

The explanation for this may lie in the timing of the model, above we assume that if a tribe chooses civil war this is immediately enacted, successful, and destructive. It might be that we have treated the rebels of tribe 2 as too well organized if they decide to engage in civil war. If tribe 1 choose either to set taxes or to engage in appropriation we have assumed that tribe two can

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29 Whether Scotland currently receives a net subsidy from the rest of Britain or pays a net tax is controversial, the issue of property right ownership over North Sea oil and gas tax revenues being one of the major points of contention.

30 This option was recently proposed by Alex Salmond, Scottish First Minister and leader of the Scottish Nationalist Party

31 For a nice summary of the current state of this game see The Telegraph newspaper article, ”Alex Salmond: Devo max instead of independence is 'very attractive', Friday 28th of September 2012.
rebel immediately, the result is that in these cases tribe 1 would always prefer dissolution and no violence would be observed. Suppose instead that civil war must be committed to one period in advance, and hence can only be carried out with a one period delay, perhaps because the rebellion takes time to fund, organize, and gain support. Now the tribe in power has the further option of appropriating the other tribe’s endowment for a period in the correct anticipation that the other tribe will engage in civil war. If this is the case the extensive form of the game becomes

![Figure 6: Appropriation](image-url)

Where the continuation payoffs at the various nodes are given by

1. \( V_i(t_1, 1, \gamma, N, N_i) = (1 - t_1)^{1-\beta_i}(t_1N)^{\beta_i} + B_i(N_i)^{\beta_i} \left[ \delta \gamma + \frac{\delta^2}{1-\gamma} \right] \) \( i = 1, 2 \)

2. \( V_i(t_1, 1, N) = \frac{(1-t_1)^{1-\beta_i}(t_1N)^{\beta_i}}{1-\delta} \) \( i = 1, 2 \)

3. \( V_i(\beta_i, 1, N_i) = \frac{B_i(N_i)^{\beta_i}}{1-\delta} \) \( i = 1, 2 \)

4. \( V_1(\beta_1, 1, \gamma, N, N_1) = \frac{B_1(N)}{(N_1)^{1-\beta_1}} + B_1(N_1)^{\beta_1} \left[ \delta \gamma + \frac{\delta^2}{1-\gamma} \right] \)
\( V_2(\beta_2, 1, \gamma, N_2) = B_2(N_2)^{\beta_2} \left[ \delta \gamma + \frac{\delta^2}{1-\gamma} \right] \)

5. \( V_1(\beta_1, N/N_1, N) = \frac{B_1 N}{(1-\delta)N_1^{1-\beta_1}} \)
\( V_2(\beta_1, 0) = 0 \)
Notice again that unity followed by civil war cannot constitute a subgame perfect equilibrium, but here this follows not from tribe 1’s preference for dissolution, but rather from its preference for appropriation.

*Proposition 2* defines the equilibrium of the game.

**Proposition 6** When a civil war takes a period to enact then the game admits 3 possible equilibria each of which exist for a set of parameter values. These are unity and tax setting by the incumbent tribe, peaceful dissolution, and appropriation followed by violent dissolution (civil war).

That appropriation followed by civil war now becomes an equilibrium for some set of parameter values is intuitive, if the tribes discount the future sufficiently then the possibility for tribe 1 to both appropriate all resources for one period and set their most preferred tax rate must yield their highest payoff.

In this context we see that appropriation followed by violent dissolution tends to occur if tribe 1 is small. This follows immediately from inspection of the payoffs associated with nodes 2, 3 and 4 in figure 6.\(^\text{32}\) Intuitively, as tribe 1 become small they receive lower payoffs under unity because satisfying the no-civil-war requires they make greater tax concessions, whilst dissolution involves lower payoffs because of the smaller tax base. However the first period returns to appropriation are invariant with respect to relative tribe size. We conclude that dissolutions are more prone to occur violently if the initial regime is an oligarchy, whereas if the initial regime is democratic they tend to occur peacefully.

Comparing the results of this section to those of section 2 is quite interesting. First, interpret the timing of the basic dissolution game as one where the potential rebels are efficient, that is they are able to initiate conflict quickly should that be the best strategy. Then interpret the timing structure of the game in this section as involving inefficient rebels that take a period to initiate conflict. We see that the prediction is that conflict will arise in the case where tribe 2 is relatively large but is inefficient at initiating conflict. It is tempting to suggest that this might give some insight into why the break up of states like Czechoslovakia were peaceful, but the break up of states like Sudan involved conflict.

### 6 Enclaves

In each of our analysis’ above the tribes are internally homogeneous, and if dissolution occurs they separate perfectly into two internally homogeneous countries. It follows that the form of governance within the separate countries would not matter for the determination of the tax rates. Suppose instead that a perfect separation of the two tribes cannot be achieved at any reasonable cost, and that a subset of one tribe must remain as an enclave within the other after separation. It follows

\[^{32}\text{We see that as } N_1 \to 0 \text{ then } V_1(\beta_1, 1, \gamma, N, N_1) > \max\{V_1(t_1, 1, N), V_1(\beta_1, 1, N_1)\}\]
that this country will be heterogeneous and thus the form of governance adopted may well matter for resource allocations, that is for the tax rate chosen and the level of public good provision.

Figure 7: An Enclave

Figure 7 illustrates a simple starting configuration where a small enclave of agents from tribe 1 are located within the region populated by tribe 2. The peaceful, or otherwise, dissolution of the state involves a redrawing of the borders between the tribes. It is natural to think of a border as being geographic, although its true function in our analysis is to provide demarkation between "areas" of public good provision. We thus make the assumption that public good provision may only be shared between groups that are contiguous, on our diagram this rules out the possibility that tribe 1 and their enclave may enjoy a jointly provided level of pubic good provision that differs from that of tribe 2. Obviously there are circumstances where this assumption is better than others, common defense might be a good example.

In this setting we extend the strategy set of tribe 1 in a very natural way. We assume that they may enforce, if they choose, a dissolution between themselves and tribe 2 together with the enclave, but they further have the option of choosing whether to hand the apparatus of government in the new region over to their fellow tribesmen in the enclave or to tribe 2. If we assume those in the enclave are fewer in number than the members of tribe 2 then this amounts to choosing between installing an autocratic or majoritarian (democratic?) regime in the new country.\textsuperscript{3334} Of course the

\textsuperscript{33}There seems to be quite a lot of evidence that after the break up of the Soviet Union the Russian Federation interfered in the domestic politics of the post-Soviet States, frequently on behalf of and by exploiting enclaves of ethnic Russians "left behind". See Cameron and Orenstein (2012).

\textsuperscript{34}Russia also employs "soft-power" through supposedly cultural organizations such as the Russkiy Mir Foundation, which promote the political position of ethnic Russians in the Post-Soviet States. See for example Conley and Gerber
new decision makers in the new country then play the game analyzed in section 2.2. This leads to the extensive form game illustrated in Figure 8.

![Figure 8: Dissolution and Regime Choice](image)

A question that immediately arises is how to represent the preferences of the members of tribe 1. We assume that the tribe weight the preferences of the main group and enclave according to their relative sizes, hence if $N_1 = N_1^m + N_1^e$ where superscripts $m$ and $e$ indicate main group and enclave respectively, then the preferences of tribe 1 are written in the form 35.

$$v_1(t, y) = \frac{N_1^m}{N_1} v_1^m(t, y) + \frac{N_1^e}{N_1} v_1^e(t, y) \quad (16)$$

With this modification we may now define the continuation values of the game at each of the ten nodes as follows.

First define $\psi_0 \equiv 1 + \frac{\delta}{1-\delta}$, $\psi_1 \equiv \gamma + \frac{\delta}{1-\delta}$ and $\psi_2 \equiv \gamma^2 + \frac{\delta}{1-\delta}$

1. $V_1^i(\beta_1, 1, \gamma, N_1^m, N_1^e) = \frac{B_1}{N_1} \left[ \psi_1(N_1^m)^{1+\beta_1} + \psi_2(N_1^e)^{1+\beta_1} \right] \quad i = m, e$

2. $V_2(\beta_2, 1, \gamma, N_2) = \psi_2 B_2(N_2)^{\beta_2}$

The results are not particularly sensitive to how we treat the attitudes of one part of tribe 1’s members to the utilities of the others. This is because in the sub games following dissolution the payoffs of the main group become a constant in the payoffs of the enclave which is common across all subsequent nodes.

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2. \( V^i(\beta_1, t_2, 1, \gamma, N_1^m, N_1^e) = \frac{\psi_1}{N_1} [B_1(N_1^m)^{1+\beta_1} + (1-t_2)^{1-\beta_1}(t_2(N_1^e + N_2))]^{\beta_1}N_1^e \) \( i = m, e \) \\
\( V_2(t_2, 1, \gamma, N_1^e, N_2) = \psi_1(1-t_2)^{1-\beta_2}(t_2(N_1^e + N_2))^{\beta_2} \)

3. \( V^i(\beta_1, 1, \gamma, N_1^m, N_1^e) = \frac{\psi_1B_1}{N_1} [(N_1^m)^{1+\beta_1} + (N_1^e)^{1+\beta_1}] \) \( i = m, e \) \\
\( V_2(\beta_2, 1, \gamma, N_2) = \psi_1B_2(N_2)^{\beta_2} \)

4. \( V_i(t_1, 1, N) = \psi_0(1-t_1)^{1-\beta_i}(t_1N)^{\beta_i} \) \( i = 1^m, 1^e, 2 \)

5. \( V^i(\beta_1, 1, \gamma, N_1^m, N_1^e) = \frac{B_1}{N_1}N_1 [\psi_0(N_1^m)^{1+\beta_1} + \psi_1(N_1^e)^{1+\beta_1}] \) \( i = m, e \) \\
\( V_2(\beta_2, 1, \gamma, N_2) = \psi_1B_2(N_2)^{\beta_2} \)

6. \( V^i(\beta_1, t_2, 1, \gamma, N_1^m, N_1^e, N_2) = \frac{\psi_0}{N_1} [B_1(N_1^m)^{1+\beta_1} + (1-t_2)^{1-\beta_1}(t_2(N_1^e + N_2))]^{\beta_1}N_1^e \) \( i = m, e \) \\
\( V_2(t_2, 1, N_1^e, N_2) = \psi_0(1-t_2)^{1-\beta_2}(t_2(N_1^e + N_2))^{\beta_2} \)

7. \( V^i(\beta_1, 1, N_1^m, N_1^e) = \frac{\psi_1B_1}{N_1} [(N_1^m)^{1+\beta_1} + (N_1^e)^{1+\beta_1}] \) \( i = m, e \) \\
\( V_2(\beta_2, 1, N_2) = \psi_1B_2(N_2)^{\beta_2} \)

8. \( V^i(\beta_1, 1, \gamma, N_1^m, N_1^e) = \frac{B_1}{N_1} [\psi_0(N_1^m)^{1+\beta_1} + \psi_1(N_1^e)^{1+\beta_1}] \) \( i = m, e \) \\
\( V_2(\beta_2, 1, \gamma, N_2) = \psi_1B_2(N_2)^{\beta_2} \)

9. \( V^i(\beta_1, t_1^e, 1, \gamma, N_1^m, N_1^e, N_2) = \frac{\psi_0}{N_1} [B_1(N_1^m)^{1+\beta_1} + (1-t_1^e)^{1-\beta_1}(t_1^e(N_1^e + N_2))]^{\beta_1}N_1^e \) \( i = m, e \) \\
\( V_2(t_1^e, 1, N_1^e, N_2) = \psi_0(1-t_1^e)^{1-\beta_2}(t_1^e(N_1^e + N_2))^{\beta_2} \)

10. \( V^i(\beta_1, 1, N_1^m, N_1^e) = \frac{\psi_1B_1}{N_1} [(N_1^m)^{1+\beta_1} + (N_1^e)^{1+\beta_1}] \) \( i = m, e \) \\
\( V_2(\beta_2, 1, N_2) = \psi_0B_2(N_2)^{\beta_2} \)

Despite the relative complexity of the game with an enclave we can immediately rule out six of the ten nodes as potential equilibria, leaving the set of equilibria as described by the following proposition.

**Proposition 7** When the tribe initially in power must leave an enclave of their members behind after dissolution, but have the option of choosing who to endow with the apparatus of government, then the following are equilibria for certain parameter configurations: (i) Full unity, (ii) Dissolution of tribe 1 from tribe 2 and the enclave, with a autocratic government in the union of tribe 2 and the enclave, (iii) Dissolution of the country into three independent countries, consisting of the main body of tribe 1, tribe 2, and the enclave of tribe 1 members.

Notice that if tribe 1 choose dissolution followed by democracy, this will only be transitory as tribe 2 will then elect to choose dissolution between themselves and the enclave in the subsequent subgame. Following dissolution from tribe 1, tribe 2 and the enclave will only remain unified in equilibrium if the enclave are installed as an oligarchy.
Figure 9: Division of the Parameter Space

Figure 9 illustrates a typical division of the parameter space, the novel element, as compared to the case without an enclave, is the region of the parameter space annotated Unity-Oligarchy (9) where the (9) refers to the equilibrium node in Figure 8. We see that for a large positive $\varepsilon$, and when tribe 2 is of intermediate size, then tribe 1 will choose dissolution and install its co-tribalists in government in the new state consisting of tribe 2 and the enclave of tribe 1 members. For this parameter configuration tribe 1 have the stronger preference for the public good. When $\omega_2$ is small the contribution of tribe 2 to the tax base is insufficient to offset the tax concessions needed to prevent them rebelling, so both the tribe 1 and the tribe 1 members who comprise the enclave successively choose dissolution. When $\omega_2$ is large the contribution of tribe 2 to the tax base is also large, and tribe 1 are willing to make the necessary tax rate concession to preserve unity. For intermediate values of $\omega_2$, tribe 2 do not constitute a sufficiently large proportion of the tax base in a fully united country for tribe 1 to be willing to make the requisite tax rate concession to preserve unity. However, after the first round of dissolution the members of tribe 2 are now relatively large contributors to the tax base in the new state, so the tribe 1 enclave members are willing to make the tax rate concessions necessary to preserve unity.

In Figure 10 we demonstrate the effects of changes in the relative size of the enclave and the destructiveness of conflict on the division of the parameter space. Here again the interest is on the effect of these changes on the proportion of the parameter space that involves the equilibrium Unity-Oligarchy. Figure 10a shows that as the proportionate size of the enclave increases the region of the parameter space that involves the equilibrium Unity-Oligarchy shrinks.  

36 For this simulation $\gamma = 0.9$ and the enclave constitute 10% of tribe 1.
37 The diagram appears incomplete in the sense that the broken read lines seem to simply terminate at some point,
A larger enclave implies a smaller main group who thus have a smaller tax base if they form a separate state, therefore as the enclave becomes larger, and if tribe 1 is already relatively small compared to tribe 2, then the main group tend to prefer unity. On the diagram the region Unity-Oligarchy shrinks from the right. Alternatively, if tribe 1 is relatively large compared to tribe 2, then as the enclave gets larger, the main group of tribe 1 continue to prefer dissolution, but the enclave have a larger tax base and tend to prefer dissolution. It follows that on the diagram the region Unity-Oligarchy also shrinks from the left. The prediction of the model then is that, everything else equal, we should except to see relatively more small enclaves as these exist as equilibria for a greater range of parameter values than do large enclaves.

*Figure 10b* reveals that the region of parameter values for which Unity-Oligarchy is the equilibrium shrinks as civil war becomes more destructive, with the region of the parameter space "lost" being added to the region of full unity. Here the threat of civil war by tribe 2 diminishes, allowing tribe 1 to make lesser tax rate concessions to satisfy the the NCW constraint, hence the initial dissolution takes place for a smaller range of values of the parameters ε and ω₂.

There are numerous examples of enclaves being left behind after a state splits, although these frequently involve quite complex situations. Perhaps the closest examples to the simple theory presented above are the cases of Transnistria an enclave in Moldova, and South Ossetia an enclave in Georgia. Both of these enclaves became disputed territories after the break up of the Soviet Union and subsequently achieved independent status in 1990 with the support and contrivance of Russia. South Ossetia is populated primarily by Ossetians, a group of Iranian origin who speak an Indo-European language of Iranian origin. The Georgians are a Caucasian indigenous ethnic group that speak a language from the Kartvelian language group. Approximately 70,000 Ossetians live in South Ossetia while over 500,000 live in Russia. The Eastern Orthodox church is the this is just an artifact of *Mathematica*’s root finding algorithm being unable to converge at the given degree of precision, and has no real economic meaning.

38 The Russin role in Transnitrian/Moldovan politics was highlighted in 2008 when at a conference at the Russian Black Sea resort of Sochi Russian President Dmitry Medvedev publicaly warned Moldovan President Vladimir Voronin against attempting to regain control over Transnistria
primary religion in both Georgia and South Ossetia, however the later has large Suni Muslim and Pagan (Etseg Din) minorities (approximately 30% each). Transnistria is also akin to the situation described in our model, it is populated by 32% ethnic Moldovans, 30% ethnic Russians and 29% ethnic Ukrainians, the official language is Russian. The Russian and Ukrainian languages are from the same Balto-Slavic language group and their speakers find each other intelligible. Moldavian is a Romance language close to Romanian. This seems quite close to the two tribe scenario explored above.

While it is slightly beyond the scope of our present analysis, it seems plausible to suggest that the creation of enclaves might arise in the form of a strategic move made by tribe 1 anticipating dissolution from tribe 2. The choices made by China to relocate ethnic Chinese to Tibet, the strategic location of Jewish settlements in Palestinian areas, and the settlement of ethnic Russians in both the Baltic Countries and in countries of Central Asia, all seem to potentially fall into this category. This does of course beg the question of why these enclaves are of value to the main body of the tribe, however if additionally there were spillovers between the nations in terms of public good provision then the creation of these enclaves seems quite explicable. 39

7 Migration and Assimilation

The theoretical and simulation results presented above assume both that the populations of the two tribes are fixed in terms of location, and that their preferences are immutable. However, millions of people migrate each year and the preferences of tribes evolve over time, particularly from one generation to the next. Modeling these phenomena formally is beyond the scope of our analysis, but we can develop some insights into the implications they may have for our conclusions, and in turn how our theoretical structure throws light on these phenomena.

Consider first assimilation in the simple dissolution model of section 2.2. This involves some subtleties. First, we need to ask who is assimilating whom. Is it the case that the larger tribe assimilates the smaller whoever controls the government? This might be some form of natural cultural assimilation involving the smaller group adopting the mores and practices of the larger group so as to reduce transactions costs. Alternatively do members of the tribe out of power choose strategically to become like the members of the tribe in power to obtain some economic advantage? In the former case assimilation may involve the convergence in the preferences of all members of both tribes, that is a fall in the absolute value of $\varepsilon$, in the latter it may involve individuals essentially switching tribes such that $\varepsilon$ remains constant but $\omega_2$ falls. 40 Inspection of

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39 The Russian enclave (or Oblast - and technically an enclave) of Kaliningrad is an interesting case, surrounded by Poland and Lithuania it is ethnically 80% Russian and provides the Russian Federation with an ice free Baltic port. However it is one of the most prosperous regions of Russia and potentially viable as an independent nation. Some of its politicians and population have recently discussed the possibility of it forming an independent Baltic State. Indeed in Kaliningrad the Baltic Republican Party exists with the objective of independence from Russia.

40 Or, of course, some combination of the two.
figure 3 allows us to get some insights into these processes. Clearly as $|\varepsilon| \to 0$ this tends to promote unity. There is then little redistributive conflict in the form of different preferences over public good provision levels and tax rates while there remain significant gains from a larger tax base.

When individuals essentially switch tribes, but tribal preferences are far apart, then the issue is more complex. In our model there are two effects of a reduction in $\omega_2$, first tribe 2 receive a lower payoff after dissolution and thus less of a tax concession to them is required to satisfy the NCW constraint, tending to promote unity. However as tribe 2 become smaller they are of less value to tribe 1 in terms of their contribution to the tax base, so this effect promotes dissolution. If $\omega_2$ falls over time due to assimilation its effect will depend on which tribe has the stronger preference for the public good. When $\varepsilon > 0$ we know that as $\omega_2$ falls the importance of the tax base effect diminishes relative to the cost of satisfying the NCW condition and this tends to lead to dissolution. However if it is known that $\omega_2$ will fall further in the next period it may be worthwhile for tribe 1 to delay dissolution since the costs of satisfying the NCW condition are short run, whereas the benefits from a larger tax base after dissolution are permanent. When $\varepsilon < 0$ we know that a reduction in $\omega_2$ tends to promote unity, here clearly the anticipation that $\omega_2$ will fall further in the further through assimilation reinforces this effect.

The effects of migration are more naturally discussed in the context of the model of enclaves presented in section 6. Here members of the enclave may migrate back to rejoin their cotribalists after the dissolution of the state. There is considerable evidence that this takes place. For example, since the break up of the Soviet Union ethnic Russians have been flooding back into the Russian Federation from other states. In 2012 almost 12 million ethnic Russians born outside the Russian Federation actually resided within its borders.\(^41\) Migration to the Russian Federation from 2004-2011 totaled 1,888,500.\(^{42}\) Furthermore, the Russian Federation is the most popular destination for emigrants from Moldova, although we cannot identify how many of these are from the enclave of Transnistria.\(^{43}\) From our analysis it seems likely that migration will promote the initial dissolution between tribes 1 and 2. The advantages to dissolution are that the main group of tribe 1 may set exactly the tax rate they wish, but at the cost of losing some of the tax base. Migration restores some of this tax base. However assuming that not all co-tribalists in an enclave migrate back to the “mother” region implies that the gains of the main group and migrants may be offset by the losses of the members of the enclave who do not migrate. Presumably this mitigates the pressures for an initial dissolution. However, whether this promotes unity between the enclave and the members of tribe 2 or a second round of dissolution as the enclave itself becomes a sovereign state, will depend on the relative preferences of the two tribes as in section 2.2. If tribe 1 have the stronger preference for the public good, that is $\varepsilon > 0$, then migration would seem to promote a second round of dissolution, if $\varepsilon < 0$ then the opposite seems likely.

\(^{41}\) Source: http://www.migrationinformation.org/
\(^{42}\) Source: oecd.org
\(^{43}\) Source: http://www.migrationpolicy.org
8 Conclusions

In a lot of the literature on political violence and its relationships to political institutions the boundaries of the state are taken as given. In this paper we have argued that the political and economic institutions and the boundaries that define a country are jointly determined. We developed a theoretical model of the dissolution of countries based on the idea that dissolution of a state may be an equilibrium response to the threat of secessionist conflict. This model allows us to characterize when a country will remain unified and when it will divide. We have shown that the determination as to whether the equilibrium in the simple dissolution game involves dissolution or unity depend on the interactions of two effects; a tax base effect which tends to promote unity, and tax rate concession effect needed to avert secessionist conflict which tends to promote dissolution. We find that a significant difference in preferences for the public good is a necessary condition for the equilibrium to involve dissolution. A sufficient condition for dissolution is both that preferences be different enough and one of the following; if the tribe setting taxes has the stronger preference for the public good then the other tribe must be sufficiently small (this reduces the tax base effect) or if the tribe setting taxes has the weaker preference for the public good, then the other tribe must be sufficiently large, that is the tax rate concession effect is large.

We extend the simple basic model in a number of directions. First, we change the assumption that one tribe makes oligarchic decisions over the tax rate, to one where the tax rate is determined by the majority, that is we switch from oligarchy to a simple version of democracy. Using numerical method we show that this does not have a measurable effect on whether the equilibrium involves dissolution or not. We also explore the role of transfers, and find that these tend to reduce the range of parameter values for which the equilibrium involves dissolution but do not eliminate this possibility. Transfers offer the ruling tribe an extra instrument to extract rents from the other tribe and allow them to make the no civil war condition just-and-just binding for all parameter values. Of interest is the relationship between transfers and measures of polarization/fractionalization. We find that whether equilibrium transfers rise or fall as the preference distance between the two tribes, $\varepsilon$, increases depends on which tribe has the stronger preference for the public good. That is whether $\varepsilon \gtrless 0$.

Next we examine the possibility of devolution of the powers to tax and spend to the two tribes. We find that devolution is not an equilibrium to the game because the players then experience a large free-rider problem. We also examine the effects of modifying the timing of the model, and show that if civil war can only occur with a one period lag then appropriation followed by secessionist conflict becomes an equilibrium for some parameterizations of the model. Finally we examine the possibility that the two tribes cannot perfectly separate from each other, and that if the tribes separate then an enclave of the members of one tribe will either remain within the region of public good provision of the other tribe or must form a third independent state. If we further argue that the tribe introducing the initial dissolution may also choose to whom to pass the apparatus of
government, then there is a natural link between political institutions and a nation’s boundaries. In our model the tribe initially controlling the apparatus of government may hand control of the new state after dissolution to either its co-tribalists of the other tribe, but only in the case where the co-tribalists take control does the country subsequently remain unified in equilibrium.

In a discussion section we explore how migration and assimilation might affect the results of our analysis. In the context of our basic model we argue that if assimilation involves a convergence of the preferences of the two tribes this tends to promote unity. However, if assimilation involves members of the tribe out of power gradually switching preferences then assimilation can promote either unity or dissolution depending on which tribe have the strong preference for the public good. This depends on the effect of changing the size of tribe 2 on the trade off between the tax base effect and the need to satisfy the no civil war condition. Migration seems more pertinent in the context of the model with enclaves. We argue that if there is significant migration from an enclave back to the mother state, then this will likely promote the initial dissolution of the state between tribes 1 and 2 with an enclave. Whether this promotes a subsequent dissolution between tribe 2 and the enclave of tribe 1 members depends on their relative preferences as in our basic model.

There are several directions in which we might extend this analysis in future work. A model that has both redistributive conflict between tribes and between income classes within tribes seems to offer the prospect of gaining further insights into the joint determination of national boundaries and political institutions. A more sophisticated measure of the degree of integration between the two tribes (as opposed to the stylized enclaves) might also bring the analysis closer to the empirical literature on conflict, fractionalization, and polarization. It might also be interesting to consider a set up with a wider menu of public goods and institutional structures. For example where there is a mix of local and pure public goods in a federal setting, but where the center is reluctant to let the periphery succeed as in the old Soviet Union. We hope to explore these ideas in the near future.
References


Appendix A

Proof of Proposition 1 To obtain the subgame perfect equilibria note first that should tribe 1 choose appropriation then tribe 2 will choose civil war because \( \gamma B_2(N_2)^{\beta_2} + \frac{\delta B_2(N_2)^{\beta_2}}{1-\delta} \geq 0 \). Given this tribe 1 can always improve their payoff by choosing dissolution because \( \frac{B_1(N_1)^{\beta_1}}{1-\delta} > \gamma B_1(N_1)^{\beta_1} + \frac{\delta B_1(N_1)^{\beta_1}}{1-\delta} \). Hence we can rule out appropriation as an equilibrium in the game. Notice also we can rule out tribe 1 choosing unity followed by tribe 2 choosing civil war as each chooses from the identical budget set with identical budget sets. Now suppose that the two tribes are in separate countries such that \( N_1 = N_2 = \frac{N}{2} \). Now suppose that the two tribes are in separate countries such that each chooses from the identical budget set \( x \leq 1 - \frac{2z}{N} \). It is straightforward to show that each selects taxes \( t_i = \beta_i \ i = 1, 2 \) giving the indirect utilities \( v_i(\beta_i, 1|D) = B_i(N_i)^{\beta_i} \ i = 1, 2 \). Now if we can show that there does not exist a tax rate in the united country that yields both tribes at least the utility levels just defined then we have proven this part of the proposition. To accomplish this consider the indifference curves \( x_i \leq \beta_1 \ i = 1, 2 \), and let \( \varepsilon \rightarrow 1 \Rightarrow \beta_1 \rightarrow 1 \) and \( \beta_2 \rightarrow 0 \), so the indifference curves reduce to \( z = B_1(N_1)^{\beta_1} \) and \( x_2 = B_2(N_2) \). So for the separate countries with identical budget sets \( x \leq 1 - \frac{2z}{N} \), the at least as good as sets must satisfy \( x \geq 1 \) and \( z \geq N/2 \). For a united country the tax rate selects a point on the boundary of the budget set \( x \leq 1 - \frac{1}{N} \), but

\begin{align*}
\frac{(1 - t_1)^{1-\beta_2}(t_1)^{\beta_2}}{1-\delta} & \geq \left[ \frac{\gamma(1-\delta) + \delta}{1-\delta} \right] B_2 \omega_2^{\beta_2} \quad (17) \\
\frac{(1 - t_1)^{1-\beta_1}(t_1)^{\beta_1}}{1-\delta} & \geq \frac{B_1 \omega_1^{\beta_1}}{1-\delta} \quad (18)
\end{align*}

and

\begin{align*}
\frac{(1 - t_1)^{1-\beta_2}(t_1)^{\beta_2}}{1-\delta} & \geq \left[ \frac{\gamma(1-\delta) + \delta}{1-\delta} \right] B_2 \omega_2^{\beta_2} \rightarrow B_2 \geq 0 \quad (19) \\
\frac{(1 - t_1)^{1-\beta_1}(t_1)^{\beta_1}}{1-\delta} & \geq \frac{B_1 \omega_1^{\beta_1}}{1-\delta} \rightarrow 1 \geq \omega_1^{\beta_1} \quad (20)
\end{align*}

which are clearly satisfied.

For the case where dissolution constitutes the equilibrium the proof is straightforward. First suppose that \( N_1 = N_2 = \frac{N}{2} \). Now suppose that the two tribes are in separate countries such that each chooses from the identical budget set \( x \leq 1 - \frac{2z}{N} \). It is straightforward to show that each selects taxes \( t_i = \beta_i \ i = 1, 2 \) giving the indirect utilities \( v_i(\beta_i, 1|D) = B_i(N_i)^{\beta_i} \ i = 1, 2 \). Now if we can show that there does not exist a tax rate in the united country that yields both tribes at least the utility levels just defined then we have proven this part of the proposition. To accomplish this consider the indifference curves \( x_i^{1-\beta_i} z^{\beta_i} = B_i(N_i)^{\beta_i} \ i = 1, 2 \), and let \( \varepsilon \rightarrow 1 \Rightarrow \beta_1 \rightarrow 1 \) and \( \beta_2 \rightarrow 0 \), so the indifference curves reduce to \( z = B_1(N_1)^{\beta_1} \) and \( x_2 = B_2(N_2) \). So for the separate countries with identical budget sets \( x \leq 1 - \frac{2z}{N} \), the at least as good as sets must satisfy \( x \geq 1 \) and \( z \geq N/2 \). For a united country the tax rate selects a point on the boundary of the budget set \( x \leq 1 - \frac{1}{N} \), but
to satisfy the requirement that the choice from this set lies in the at least as goods as sets from the independent countries requires \( x \geq 1 \) and \( z \geq N/2 \), we immediately see that these requirements violate the unified budget set and the proof is complete. 

**Proof of Lemma 2** Substituting from one best responses from (12) into the other we have

\[
t_i = \beta_i - (1 - \beta_i) \left( \beta_{-i} - (1 - \beta_{-i}) t_i \left( \frac{N_i}{N_{-i}} \right) \left( \frac{N_{-i}}{N_i} \right) \right)
\]

which reduces to

\[
t_i = \frac{\beta_i - (1 - \beta_i)^2 (N_i/N_{-i})}{1 - (1 - \beta_i) \beta_i}
\]

and provides lemma 2 part (1) iff the resultant solutions \( t_i \in (0, 1], \ i = 1, 2 \). For parts (2) and (3) set \( t_{-i} = 0 \) and solve the best responses, then set \( t_i = 0 \) and repeat the procedure. The conditions for each case are the intercept conditions such that the best responses give stable Nash equilibria. 

**Proof of Proposition 2** From the NCW condition with transfers we have

\[
\left( 1 - t_1 + \frac{N_1}{N_2} \tau \right)^{1-\beta_2} (t_1 N)^{\beta_2} \geq (\gamma(1 - \delta) + \delta) B_2 N_2^{\beta_2}
\]

and the relationship between transfers and taxes satisfies

\[
t_1 = \left( \frac{\beta_1^2 + (1 - \beta_1)^2 \frac{N_2}{N_1}}{\beta_1 + (1 - \beta_1) \frac{N_2}{N_1}} \right) - \left( \frac{1 - 2 \beta_1}{\beta_1 + (1 - \beta_1) \frac{N_2}{N_1}} \right) \tau
\]

Now let \( \frac{N_i}{N_2} \to 1 \) and \( \beta_1 \to 1 \) we immediately get from the tax/transfer condition \( t_1 = 1 - \tau \). Now it immediately follows that the left hand side of the NCW condition is zero, and since civil war cannot be prevented it then follows that tribe 1 will select dissolution. Which is then the sub-game perfect equilibrium.

Alternatively let \( \beta_1 \to 1/2 \) we get from the tax/transfer condition \( t_1 = \beta_1 = 1/2 \) which is then the most preferred tax rate of both tribes. The transfer is then set by tribe 1 so that the NCW condition is satisfied with an equality, implying \( \tau < 0 \). Hence given tribe 1 receive transfers in this case and \( N > N_1 \) implies a lower cost of any given level of public good provision it follows that tribe 1 must prefer unity, which is the sub-game perfect equilibrium in this case.

**Proof of Lemma 3** For part (1) substitute the tax rate solutions into the indirect utility functions to gives
\[ v_i(t_i, t_{-i}, 1, N_i, N_{-i}) = \left( 1 - \frac{\beta_i - (1 - \beta_i)^2(N_i/N_{-i})}{1 - (1 - \beta_i)\beta_i} \right)^{-\beta_i} \times \left( \frac{\beta_i - (1 - \beta_i)^2(N_i/N_{-i})}{1 - (1 - \beta_i)\beta_i} \right) N_i \left[ 1 + \frac{N_{-i}}{N_i} \right]^{1-\beta_i} \]

which reduces to

\[ v_i(t_i, t_{-i}, 1, N_i, N_{-i}) = B_i(N)^{\beta_i} \left[ 1 - \frac{\beta_i}{1 - (1 - \beta_i)\beta_i} \right] \left[ 1 + \frac{N_{-i}}{N_i} \right]^{1-\beta_i} \]

which if discounted gives the result in the text.

For parts (2) and (3) set \( t_i = 0, \ t_{-i} \neq 0 \ i = 1, 2 \) and substitute the tax rate solutions into the indirect utility functions, discount the resultant expressions and we have the results.

**Proof of Proposition 4** Established numerically. *Mathematica* simulation programs available on request.

**Proof of Proposition 5** Consider the continuation values

1. \( V_i(t_1, 1, \gamma, N, N_i) = (1 - t_1)^{1-\beta_i}(t_1N)^{\beta_i} + B_i(N_i)^{\beta_i} \left[ \delta \gamma + \frac{\delta^2}{1-\delta} \right] \quad i = 1, 2 \)

2. \( V_i(t_1, 1, N) = \frac{(1-t_1)^{1-\beta_i}(t_1N)^{\beta_i}}{1-\delta} \quad i = 1, 2 \)

3. \( V_i(\beta_i, 1, N_i) = B_i(N_i)^{\beta_i} \quad i = 1, 2 \)

4. \( V_i(\beta_1, 1, \gamma, N, N_1) = \frac{B_1(N)}{(N_1)^{1-\beta_1}} + B_1(N_1)^{\beta_1} \left[ \delta \gamma + \frac{\delta^2}{1-\delta} \right] \)

   \[ V_2(\beta_2, 1, \gamma, N_2) = B_2(N_2)^{\beta_2} \left[ \delta \gamma + \frac{\delta^2}{1-\delta} \right] \]

5. \( V_1(\beta_1, N/N_1, N) = \frac{B_1 N}{(1-\delta)N_1^{1-\beta_1}} \)

   \[ V_2(\beta_1, 0) = 0 \]

It is immediately obvious that only continuation values 2, 3 and 4 can be equilibrium payoffs. This follows since for all admissible parameter values 5 is dominated by 4 for tribe 2, and 1 is dominated by 3 for tribe 1.

**Proof of Proposition 6** Given the complexity of this game it is useful to reproduce Figure 5 with a little extra notation as follows
Further to simplify the exposition of the proof write $P_i(j)$ as the continuation payoff to the tribe $i$ at node number $j$ in the diagram. The actual continuation values are as in the text.

We shall now demonstrate that we can eliminate nodes 1-3, 5-6 and 8 as potential equilibria.

1. Elimination of nodes 1-3 as possible equilibria
   This follows immediately from noting that the sub game B Pareto dominates the sub game A because endowment has destroyed by civil war in sub game A, but in all other respects these sub games are identical. This leaves 4 as a possible equilibrium to the game plus the possible equilibria of the sub games B and C.

2. Elimination of nodes 5,6 and 8 as possible equilibria
   We shall establish that subgame C weakly dominates subgame B for tribe, the weakly part following from $P_1(7) = P_1(10)$.

   Assume tribe 1 has chosen dissolution and consider all the possibilities

   (a) In subgame B tribe 2 choose unity and in subgame C tribe 1e choose unity
   Then
\[ P_1(6) = P_1(7) \] - by the NCW condition
\[ P_1(7) > P_1(5) \] - because civil war is destructive
\[ P_1(7) = P_1(10) \] - same outcomes
\[ P_1(9) > P_1(10) \] - revealed by choice
\[ \Rightarrow P_1(9) > P_1(6) \]

(b) In subgame B tribe 2 choose dissolution and in subgame C tribe 1e choose unity
Then
\[ P_1(7) = P_1(10) \] - same outcomes
\[ P_1(9) > P_1(10) \] - revealed by choice
\[ \Rightarrow P_1(9) > P_1(7) \]

(c) In subgame B tribe 2 choose unity and in subgame C tribe 1e choose dissolution
Then
\[ P_1(6) = P_1(5) \] - by the NCW condition
\[ P_1(7) > P_1(5) \] - because civil war is destructive
\[ P_1(7) = P_1(10) \] - same outcomes
\[ \Rightarrow P_1(10) > P_1(6) \]

(d) In subgame B tribe 2 choose dissolution and in subgame C tribe 1e choose dissolution
\[ P_1(10) = P_1(7) \] - same outcomes

(a)-(d) jointly demonstrate that from tribe 1’s perspective subgame C weakly dominates sub game B. Subgame C is effectively identical to the the game analyzed in section 2.2 which demonstrates that nodes 9 and 10 are the only potential equilibria.

3. Parts 2 and 3 jointly supply the result that the equilibrium of the game must be one of nodes 4, 7 or 9 dependent on the values of the parameters as reported in the text