Bringing Inclusivity to Mathematics by Mauricio Gómez-López

I first learned about the idea of inclusive teaching when I started working at the University of Oregon, where I'm currently a postdoc. Before coming to the United States, I lived in Denmark for eight years, during which time I completed my graduate studies and got my first academic position.

Denmark is a small country with a very homogenous society where almost everybody grows up with optimal social-economic conditions. Most notably, everybody in Denmark has access to an outstanding education system, which prepares students well for the challenges of college. For this reason, a concept like inclusivity was not extremely relevant in Denmark, and I never heard any of my colleagues bring it up.

When I then first heard about inclusive teaching, I didn't see its relevance immediately in mathematics. I thought, somewhat naively, that mathematics couldn't be affected by factors outside of it. If you want to be good at math, you need to be passionate about it, and if a student shows promise, teachers will notice it.

The Turning Point

Recently though, I had an experience which made me realize the importance, and urgency, of inclusive teaching in mathematics. In the Spring term of 2018, I had the chance to teach an introductory course in discrete mathematics, usually taken by math and computer science majors at the University of Oregon. However, in this course, I also had a business student who was interested in doing a minor in computer science. When this student first came to my office and told me he was majoring in business, in my mind, I could picture myself rolling my eyes. This was a proof-based course, and even math majors occasionally struggled with it. So my immediate mental reaction was: 'This student is not going to make it.' But soon after this first encounter, I was utterly and hilariously proven wrong. While other students in the course struggled with assignments, this student seemed to have a natural ability for proofs. And during office hours, he consistently impressed me with his eloquence to articulate mathematical concepts.

I was impressed, and I wondered why a student with such potential was not majoring in math in the first place. But one year later, when he took a second course with me, I learned the sad reason why he had not considered a career in mathematics. This student had a slight attention-deficiency problem, and for this reason, he would seldom participate in class. Also, instead of taking his exams with his classmates, he needed to be in a separate room with reduced distractions.

At the end of the second course, this student told me that, before having me as a teacher, he never felt he was any good at math. In high school, he struggled with math lectures, and his teachers never thought he had the qualifications for AP courses. I was shocked by this revelation. I immediately told him that I found his story hard to believe, and I assured him that he had great potential for mathematics. After saying goodbye to him, I thought to myself: "How is it possible that nobody noticed such an obvious talent before? Was it because his condition made him seem 'slow'?'"
I realized at that moment that my student had been the victim of non-inclusive attitudes. Teachers had judged him unfairly for not having the typical 'math student' image, and because of this, never saw himself as a capable math learner. This incident motivated me to reevaluate my opinions about inclusive teaching and its relevance in mathematics. And the more I learned about inclusivity, the more I realized that my student's story was not an isolated case. During my time at the University of Oregon, I realized that there are many factors that prevent students from having an enriching and enjoyable experience with math.

Even more worrying, many of these factors are a result of the way we usually teach mathematics. Instead of empowering students, many methods and habits in mathematical education tend to have a discouraging and exclusionary effect. And frequently, these factors tend to have a more profound impact on students coming from underrepresented groups in the sciences.

With this piece, I want to share my experiences with inclusive teaching and how it has transformed my practice as an educator. But also, I want to point out those aspects of math education which don't foster inclusive environments and describe some of my strategies for counteracting these.

**My personal experience with non-inclusivity**

The irony of my experience with inclusivity is that, even though I didn't initially see its relevance, I was affected by non-inclusive attitudes and environments when I was a student.

I grew up in Colombia, where I did my undergraduate degree at the Universidad Nacional, considered to be Colombia's top public university. I was happy, and fortunate, that I got admitted to this school. However, even though I enjoyed studying and doing mathematics, I remember my college experience being pretty stressful. La Universidad Nacional in Colombia is famous for its high academic standards, and for being demanding on its students. Instead of designing courses that would promote learning and broaden one's mind, teachers would often focus more on making exams unnecessarily complicated, making students feel that the risk of flunking was always close. Pressure can sometimes lead to good results in academics. But if not presented with support and a healthy learning environment, it can have a devastating impact on students. Several of my classmates, even some who were genuinely good at math, failed courses several times. Some of them were discouraged to the point of abandoning the idea of a career in academics.

After a lot of hard work and stress, I managed to graduate second in my class and win a scholarship to study at the University of Copenhagen in Denmark. Even though my experience at the Universidad Nacional was overall a positive one, I still sometimes remember it with some bitterness. I thought it was still possible to motivate students to high academic achievement, but without the unnecessary pressure. My education experience in Denmark would turn out to be substantially different.

Universities in Denmark strike a great balance between offering a relaxed learning environment and instilling ambition to its students. For a student in Denmark, the risk of failing a course is
low, and their government subsidy does not depend on their GPA. So not only do students receive ample learning resources, but they are also relieved from the usual academic and financial pressures of college, which allows them to engage with their studies more freely.

Nevertheless, in all the eight years I spent in Denmark, I never heard anybody talk about things such as 'inclusivity' or 'diversity.' At Danish universities, there was undoubtedly a lot of focus on how to teach well and efficiently, but the question of whether or not these methods worked for all students was somehow never brought up. Maybe since Denmark is such a small country and everybody studies the same curriculum in high school, it might be natural to assume that all students come with the same background and qualifications when starting college. Therefore, making adjustments for some students might not seem necessary.

But Danish schools could benefit immensely from inclusive teaching practices. At the University of Copenhagen, I did encounter students who, even though they were good at math, still struggled to get good grades. Some of them lacked skills that college students are just expected to have, like writing well or fast note-taking skills. Some suffered from anxiety, which could cripple their performance in tests. For these students, having the right resources to address these issues would have made a world of difference.

My experience in both Copenhagen and the United States made me realize that it's crucial to make students feel that they are part of a community that supports them, and that they can count on the help and guidance of teachers in their academic growth.

**Experiencing diversity in classrooms**

And the need for this sense of community and support is especially pressing in the United States. It is a well-documented fact that the number of students coming from historically underrepresented minorities in this country is alarmingly low. Many of these students seem to face challenges and barriers that other students do not experience, including the bias, intentional or not, from teachers. On some occasions, the sense of rejection is so strong that many of them receive adverse comments or gestures from their peers. Naturally, this kind of environment can lead these students to feel that they are not valuable members of the community, which will chip away at their ambition to succeed.

Moreover, students who don't necessarily come from minorities, but still do not fit into the 'standard math student' image, can also face obstacles. My student from the discrete mathematics course had teachers who didn't view him as a good student, perhaps because he wasn't as fast as gifted math students typically are. Also, unlike Denmark, many students in the United States didn't have the privilege of receiving a rigorous high school education. These students might not come with the usual skills assumed of math students, like dissecting technical books and fast note-taking skills, but none the less have the potential to grasp and appreciate mathematical ideas. However, mathematicians seem to adhere to a strict tradition when it comes to teaching. Math teachers typically follow the usual format of lectures, homework, and exams, while expecting students to read the book and keep up. While this approach prompts students to be independent, it has the effect of leaving behind students who don't have a thorough high school preparation, even those who are talented for math.
Inclusive teaching means being mindful of all these diverse profiles and create an atmosphere where all students can have an enjoyable experience with mathematics. More precisely, to create an inclusive environment, I believe that one must do the following:

- Make students feel that they are capable of learning math, regardless of their background.
- Provide all the necessary academic resources and support to help students in their learning process.
- Create an academic environment where each student feels that their participation is valuable.

These principles have enriched my teaching experience and have had a meaningful positive impact on many of my students. They might seem simple, or even obvious, but the reality is that many students perceive math courses as an impossible struggle, and not as an experience of growth.

Where the problems are, and how to fix them

What exactly are the factors that prevent students from having an empowering experience with mathematics? While working in the United States, I've realized that many of the barriers that students face are a product of teaching habits and routines that many mathematicians consider standard. However, I've also realized that as a teacher, I have the power to alleviate these obstacles. Even small adjustments in our teaching methods can have a significant impact on our students' learning and sense of engagement.

The following are simple practices that I've learned through either my own teaching experience or other scientists who have done a lot of work in education.

**Language matters.** The language we use in our classroom or during office hours contributes significantly to the kind of atmosphere we create. As Federico Ardila points out in [FA], calling things 'trivial', 'stupid', or 'easy to see' might seem harmless to some mathematicians, but feel exclusive and discouraging to many students.

A first step to create an environment that fosters learning and growth is to convince students that they are not deficient in case they're not getting something. It merely means that they require an alternative approach. However, if a professor calls something 'trivial' when perhaps it isn't, students might be lead to believe that they don't have the aptitude to study mathematics.

**Books don't often offer the best explanations.** Many mathematicians seem to believe that the best approach to learn math is by reading textbooks, which in mathematics tend to be somewhat technical. But I recently had a crucial insight: being able to understand a math book is not the same as being good at math. Some students may not have the academic skills to decipher the technical discussions given in textbooks, but none the less can grasp deep mathematical concepts.

The tradition of presenting mathematics with heavy use of notation and jargon is often a barrier for students to succeed in mathematics. For this reason, I make a substantial effort in class to
strip a mathematical concept from its technical scaffolding so that students can appreciate its meaning. For example, most books on basic set theory will first introduce 'indexing sets' to explain the notion of 'partition.' Adding such a technical prerequisite will often confuse students and give them the impression that some concepts are more abstract than they are.

Simplifying explanations from textbooks is especially beneficial for students who didn't have an ideal high school education. For these students, realizing that a concept is more accessible than they initially thought can be an empowering experience.

Lecturing is not the same as teaching: inviting students to office hours. Lectures are an appropriate space for giving an overview of the material and giving broad explanations. But in my experience, I have better teaching results when I talk with my students. Through more personal interactions, I can gauge easier where students are feeling shaky and design explanations that best fit their approach to math. For this reason, I consistently invite my students to office hours and let them know that this is perhaps their most valuable resource in the course.

Some students might not want to go to office hours because they fear being judged by the teacher for not understanding something. For this reason, it is indispensable to let students know that the purpose of office hours is not to evaluate them but to guide them better in their learning process.

Also, I believe that offering students these kinds of interactions will help them feel that they have a voice and that their role in the course matters.

Change the way we assess and evaluate students. Exams in mathematics are perhaps the factor with the most exclusionary and discouraging effect among students. For many students, the pressure of performance presents a severe barrier in their learning growth. Also, often unintentionally, professors will use exams as a mechanism to weed out 'weak' students.

In my recent courses, I have tried to explain to my students that their grades in midterms and quizzes are a tool for me to gauge their progress. I let my students know that a course is not a platform for evaluation but a learning process. In particular, if I notice that a student has gradually improved throughout the course, I'm willing to award the student a better grade than the one they should get based solely on the average of their test scores.

I have achieved amazing results with some students by reducing the amount of pressure from exams. Instead of feeling that they are on the brink, students go into the exam knowing that it's just step of the process. Paradoxically, this carefree mindset often leads to better test scores.

Create equitable spaces. By researching more about inclusivity and attending teaching workshops, I've learned that not all students experience the classroom environment in the same way. Many of them might see it as an intimidating experience and a place where they compete with other students. Also, many students feel that going to classes is not an enriching experience because the instructor is not applying methodologies that work for diverse learning styles.
I'm still in the process of learning how to create an equitable environment for my students. Nevertheless, I have found the articles [KT] by K. Tanner and [FA] by F. Ardila to be valuable resources.

**Concluding remarks**

Inclusivity has become one of the central components of my teaching, especially after I've realized that many students struggle not because they lack the potential, but because the college experience presents many barriers that might be invisible to teachers. While teaching in the United States, I've become more aware of these barriers. The harsh structural inequalities in this environment produce conditions that are adverse for many students, and undeniably affects their academic experience.

But I've also realized that the way we approach our teaching and how we interact with students can have a truly transformative effect. It might not be possible to tackle all the structural problems in the classroom, but we can guide students to an empowering experience with math. And the first step is to be mindful of the diversity of our students. Traditional teaching in mathematics has historically served only a small portion of students, ignoring the needs of those with different backgrounds, working styles, and temperaments. The methods that I described in this piece might not meet the needs of all students, but I have witnessed very positive results by applying them.

And I have also benefited immensely from my network of colleagues who are also engaged in education. I've been lucky that the University of Oregon frequently hosts workshops that address the challenges of teaching. Through them, I have become aware of the nuances of diversity and learned strategies for creating equitable conditions for students. For anybody aiming to create a more inclusive space in their courses, I highly recommend participating in these types of events.

I'm aware that investing a significant amount of time and effort into teaching might seem out of reach for many mathematicians. The pressures of an academic career drastically reduce the amount of time that we can invest in education and the growth of students. But in my experience, I've learned that even small adjustments in our teaching habits can have a profound impact on students and transform their experience with mathematics.

**References**
