Physics 411—Winter 2015
Problem Set #6:

Taylor 6.1, 6.16, 6.19, 6.24, 7.3, 7.10, 7.16, 7.31, 7.41

1. You have been hired by an Atomic Force Microscopy (AFM) probe company to design a new line of AFM probes. The probes need to be 100 micrometers in length and will be made of single crystal silicon. Also, the cross section of the probe needs to be rectangular. What dimensions will yield fundamental resonance frequencies of 100 kHz and 1 MHz? How much larger is frequency of the second mode than the first? Estimate the fundamental resonance frequency of a diving board. Hint: The “angular” resonance frequencies for a cantilever are given by:

\[ \omega_n = \left( \frac{EI}{mL^3} \right)^{\frac{1}{2}} \beta_n^2 \]

where \( \beta_n = 1.875, 4.694, 7.855, \ldots \)

2. BIG TIME EXTRA CREDIT: Use the wave equation for beams

\[ EI \frac{\partial^4 u(x, t)}{\partial x^4} = -\rho A \frac{\partial^2 u(x, t)}{\partial t^2} \]

to solve for the beam profiles, \( u_n(x) \), and resonance frequencies, \( \omega_n(x, t) \), of a doubly-clamped beam. How does the fundamental frequency compare to that of the cantilever? Plot the profiles for the first few modes using Mathematica. Estimate the fundamental frequency of a span of the Golden Gate Bridge.