1. Some Practice with the Lagrangian formalism. In class, we have made use of
the Euler-Lagrange equation (Eq. 1) to obtain the equation of motion for a
few target systems, and we have also used the equation to derive
conservation laws for energy, momentum, and angular momentum.

\[ \frac{\partial L}{\partial q_i} = \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} \quad (1) \]

In this equation, \( L \) is the Lagrangian, defined as the kinetic energy minus the
potential energy, \( L = T - V \), \( q_i \) is a generalized coordinate of the \( i \)th particle in
our system, \( \dot{q}_i \) is a generalized velocity, and \( t \) is time. In Cartesian
coordinates, \( q_i \) is just one component of a position vector, say \( x \), and \( q_i \) is one
component of a velocity vector, say \( v_x \). All together, a free particle will have
three \( q_i \) and three \( \dot{q}_i \). So in general for a point particle, the Euler-Lagrange
equation as written in Eq. 1 will yield up three separate equations depending
on the number of degrees of freedom of the particle.
(a) Write down the Lagrangian for a free particle in three dimensions in
Cartesian coordinates. Use Eq. 1 to show that the momentum of the
particle is conserved.
(b) Write the Lagrangian for a single particle including a position-dependent
potential energy. Use the Euler-Lagrange equation to derive Newton’s
Second Law.
(c) Finally, write the Lagrangian for a one-dimensional mass on a spring and
use the Euler-Lagrange equation to write down the system’s equation of
motion.