

Exam #: _____

Printed Name: _____

Signature: _____

PHYSICS DEPARTMENT
UNIVERSITY OF OREGON
Unified Graduate Examination

PART II

Tuesday, April 3, 2012, 12:30 to 16:30

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

You are encouraged to use the integrals, constants, and other information on the following two pages where appropriate to help you solve the problems.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed.** Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed.** **No other papers or books may be used.**

When you have finished, come to the front of the room, put all problems in numerical order and staple them together with this sheet on top. Then hand your examination paper to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19} \text{ C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31} \text{ kg}$ ($0.511 \text{ MeV}/c^2$)
Proton rest mass (m_p)	$1.673 \times 10^{-27} \text{ kg}$ ($938 \text{ MeV}/c^2$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27} \text{ kg}$ ($940 \text{ MeV}/c^2$)
Atomic mass unit (AMU)	$1.7 \times 10^{-27} \text{ kg}$
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	$6.63 \times 10^{-34} \text{ J}\cdot\text{s}$
Speed of light in vacuum (c)	$3.00 \times 10^8 \text{ m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23} \text{ J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$
Permeability of free space (μ_0)	$4\pi \times 10^{-7} \text{ H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12} \text{ F/m}$
Mass of earth (M_E)	$5.98 \times 10^{24} \text{ kg}$
Equatorial radius of earth (R_E)	$6.38 \times 10^6 \text{ m}$
Radius of sun (R_S)	$6.96 \times 10^8 \text{ m}$
Classical electron radius (r_0)	$2.82 \times 10^{-15} \text{ m}$
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	$1.01 \times 10^5 \text{ Pa}$

Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Trigonometric identities

$$1 - \cos \theta = 2 \sin^2(\theta/2)$$

$$1 + \cos \theta = 2 \cos^2(\theta/2)$$

$$\sin \theta = 2 \sin(\theta/2) \cos(\theta/2)$$

Stirling's formula

$$\log(N!) \approx N \log N - N$$

for large N , with an error that grows only like $\log N$.

Problem 1

Compute numerical values for the following (squared) Clebsch–Gordan coefficients. The notation convention for the coefficients is $\langle j_1, m_1; j_2, m_2 | j_3, m_3 \rangle$.

(a) $|\langle 1, 1; 1, 0 | 3, 1 \rangle|^2$.

(b) $|\langle 1, 1; 1, 0 | 1, -1 \rangle|^2$.

(c) $|\langle 0, 0; 1, 0 | 1, 0 \rangle|^2$.

(d) $|\langle 1, 0; 1, 0 | 0, 0 \rangle|^2$.

Problem 2

Consider a particle with mass m in the attractive delta-function potential in one dimension,

$$V(x) = -\beta \delta(x)$$

with $\beta > 0$ a constant. Consider the following family of normalized trial wavefunctions, $\psi_a(x) \equiv \langle x | \psi_a \rangle$, of the (triangular) form

$$\psi_a(x) = \begin{cases} \sqrt{\frac{3}{2a^3}} (a - x) & \text{for } 0 \leq x \leq a \\ \sqrt{\frac{3}{2a^3}} (x + a) & \text{for } -a \leq x \leq 0 \\ 0 & \text{for } |x| > a \end{cases}$$

You may use units with $\hbar = 1$ if you like.

- Calculate the expectation value of the potential energy in the states $|\psi_a\rangle$, i.e., calculate $\langle \psi_a | \hat{V} | \psi_a \rangle$. (Here \hat{V} denotes the operator associated with the potential V given above.)
- Calculate the expectation value of the kinetic energy in the states $|\psi_a\rangle$, i.e., calculate $\langle \psi_a | \hat{T} | \psi_a \rangle$. Hint: be careful calculating this, as the derivative of $\psi_a(x)$ is not continuous at $x = a$, nor at $x = -a$, nor at $x = 0$. You may want to sketch the function $\psi_a(x)$.
- Would you expect $\langle \psi_a | \hat{T} + \hat{V} | \psi_a \rangle$ to be bigger or smaller than the ground state energy for the Hamiltonian $\hat{H} = \hat{T} + \hat{V}$?
- Using the variational method, estimate the ground-state energy by varying a .
- The exact ground state energy is $E_{\text{gs}} = -m\beta^2/(2\hbar^2)$. Is your bound in part (c) consistent with your answer to part (d)?

Problem 3

A crude model potential for a molecule can be written

$$U(r) = -2U_0 \left[\frac{a}{r} - \frac{1}{2} \left(\frac{a}{r} \right)^2 \right] .$$

Here $U_0 > 0$ is a characteristic energy, a is a characteristic length, and $r = |\vec{x}|$ is the distance from the potential center. You may use units with $\hbar = 1$ if you like.

- (a) If we write the wave function in spherical polar coordinates in the form

$$\psi(r, \theta, \phi) = R(r)Y_l^m(\theta, \phi) ,$$

what form does the Schrödinger equation $H|\psi\rangle = E|\psi\rangle$ take as a differential equation for $R(r)$? What is meant by the centrifugal barrier term in this equation?

- (b) Sketch and briefly discuss both $U(r)$ and the effective potential $U_{\text{eff}}(r)$ that is obtained in the radial equation once the centrifugal barrier is taken into account. Compare U_{eff} with the effective potential V_{eff} that results from a Coulomb potential, $V(r) = -e^2/r$, plus the centrifugal barrier.

- (c) Recall that the energy spectrum for a particle with mass m in a Coulomb potential is

$$E_{n_r, \ell}^{(c)} = \frac{-E_R}{(n_r + \ell + 1)^2} ,$$

where $E_R = me^4/2\hbar^2$, $n_r = 0, 1, \dots$ is the radial quantum number, and $\ell = 0, 1, 2, \dots$ is the angular momentum quantum number. Find the energy spectrum for the potential $U(r)$. Write the energy levels as a function of a radial quantum number n_r and the angular momentum quantum number ℓ . Please use the parameter γ where

$$\gamma^2 = 2U_0ma^2/\hbar^2 .$$

- (d) Is the degeneracy of the energy levels for $U(r)$ the same as in the case of a Coulomb potential? If not, what is the difference?

Problem 4

A gas consisting of an enormous number of indistinguishable, non-interacting, spin 0 particles is in equilibrium with an atomic trap in which each of the particles can be in one of two single particle states, one with energy $-E_1 - \epsilon$, the other with energy $-E_1$. (Here E_1 and ϵ are both > 0 .) The mean number of particles in the trap is N . Find the temperature T at which there are, on average, twice as many particles in the ground state as in the excited state.

Problem 5

- (a) When a small amount ΔQ of heat is transferred to a body at temperature T , its entropy changes by an amount ΔS . What is the relation between ΔS , ΔQ , and T ?
- (b) A block of material with a temperature independent heat capacity $C = 500 \text{ J/K}$ is initially at a temperature $T_0 = 600 \text{ K}$. It is then placed in a large lake at a temperature $T_L = 300 \text{ K}$ and allowed to come into thermal equilibrium with the lake. Find the change in entropy of the block, the lake, and the total system. (Give numerical answers).
- (c) An identical block at an initial temperature $T_0 = 600 \text{ K}$ is placed in a large geothermally heated pool at $T_P = 373 \text{ K}$ and allowed to come to thermal equilibrium with the pool. The block is then placed in the lake at $T_L = 300 \text{ K}$ and allowed to come into thermal equilibrium with the lake. Find the change in entropy of the block, the pool, the lake, and the total system. (Give numerical answers).
- (d) Explain how the total entropy change involved in cooling the block from 600 K to 300 K could be made as small as one wished.

Problem 6

A surface contains a number $B \gg 1$ of sites to which the molecules of a gas in contact with the surface can bind. Each site can be occupied by only one particle (or none). The energy of a particle bound to a site is $-\epsilon$. (Here $\epsilon > 0$.)

- (a) Will all the sites be occupied at a non-zero temperature? Justify your answer, in words.
- (b) Suppose N particles bind to the surface. How many ways can they be arranged? What is the entropy S associated with this? Simplify your expression for S in the limit that N , B , and $B - N$ are all $\gg 1$.
- (c) What is the energy of the surface when $N \gg 1$ particles are on it? What is its Helmholtz free energy F ? Finally, what is the chemical potential μ for particles on the surface? Express your answers in terms of the occupied fraction of sites: $f = N/B$, B , and ϵ .
- (d) If the adsorbed particles are in thermal equilibrium with the particles in the (non-degenerate, ideal) gas, what is f ? Express your answer in terms of ϵ , the temperature T , the number density n of particles in the ideal gas, and fundamental constants.

Hint: The chemical potential of a non-degenerate ideal gas is

$$\mu = k_B T \ln \left(n \left(\frac{2\pi\hbar^2}{mk_B T} \right)^{3/2} \right)$$